Answers to Dr. Z.'s Math 421 (Calc5) Homework assignments (when applicable)

Disclaimer: not responsible for any errors. The first finder of any error will get \$1.

Version of Nov. 12, 2024 (thanks to Olakunle David Oyedele)

Assignment 1: 1. $\frac{1}{s^2}+2\frac{1}{s}$ 2. $\frac{4}{s-1}+\frac{2}{s+1}$ 3. $\frac{2(1-2e^{-3s})}{s}$ 5. $\frac{6}{s^4}+\frac{2}{s^2}+\frac{11}{s}$ 6. $\frac{2}{s^3}-\frac{1}{s^2}-\frac{2}{s}$ 7. $\frac{1}{s-1}+\frac{2}{s-3}+\frac{6}{s^4}$ 8.$\frac{2}{s}+\frac{1}{s-2}+\frac{3}{s-1}$ 9.$\frac{2}{s-1}-\frac{3}{s^2+1}+\frac{2s}{s^2+25}$.

Assignment 2: 1. $\frac{t^3}{6}$ 2. t^2-6t 3. $1+2t+\frac{1}{2}t^2$ 4. $\frac{1}{4}e^t-\frac{1}{4}e^{-3t}$ 5. $\frac{1}{5}-\frac{1}{5}\cos(\sqrt{5}t)$ 6. $\frac{1}{3}e^{-2t}-\frac{1}{2}e^{-t}+\frac{1}{6}e^t$ 7. $y(t)=-\frac{1}{2}+\frac{1}{2}e^{2t}$ 8. $y(t)=2e^{2t}+e^{-2t}$ 9. $y(t)=e^{2t}+e^t$ 10. $y(t)=-e^{-t}+e^t+e^{2t}$ 11. $y(t)=\frac{2}{3}\sin t-\frac{1}{3}\sin 2t$.

Assignment 3: 1. $\frac{120}{(s+2)^6}$ 2. $\frac{2}{(s-3)^2+4}$ 3. $\frac{s-4}{(s-4)^2+4}-\frac{2}{(s-2)^3}$ 4. $\frac{1}{3}e^t\sin 3t$ 5. $e^{3t}(2\cos t+7\sin t)$ 6. $y(t)=(t-1)e^t$ 7. $y(t)=\frac{1}{12}t^4e^t$ 8. $(t+1)-(t+1)\mathcal{U}(t-3)$ 9. $e^{-3s}(\frac{9}{s}+\frac{6}{s^2}+\frac{2}{s^3})$ 10. $\frac{se^{-(\pi/2)s}}{s^2+1}$ 11. $\frac{1}{6}(t-2)^3\mathcal{U}(t-2)$ 12. $(t-3)e^{t-3}\mathcal{U}(t-3)$ 13. $(e^{2t}-1)-\frac{3}{2}(e^{2(t-1)}-1)\mathcal{U}(t-1)$.

Assignment 4: 1. $\frac{4s}{(s^2+4)^2}$ 2. $\frac{2}{(s-2)^3}$ 3. $\frac{6}{(s+1)^4}$ 4. $\frac{1}{s^2(s-1}$ 5. $\frac{1}{s^2+1}$ 6. $f(t)=-\frac{1}{8}e^{-t}+\frac{1}{8}e^t(2t^2+6t+1)$ 7. $f(t)=e^{-t}$ 8. $y(t)=\frac{1}{2}t\cos t+\frac{1}{2}(t-1)\sin t$.

Assignment 5: 1. $y(t)=-e^{2t}+e^{2t-6}\,\mathcal{U}(t-3)$ 2. $y(t)=-e^{-t}+e^{-t+1}\,\mathcal{U}(t-1)+2e^{-t+2}\mathcal{U}(t-2)+3e^{-t+3}\,\mathcal{U}(t-3)$ 3. $y(t)=\frac{1}{2}\sin 2t\,\mathcal{U}(t-\pi)$ 4. $y(t)=(t-2)e^{-2t+4}\,\mathcal{U}(t-2)$ 5. $y(t)=-e^{-t+\pi}\sin t\,\mathcal{U}(t-\pi)$.

Assignment 6: 1. $x(t)=\frac{1}{3}e^t-\frac{1}{3}e^{-2t}$, $y(t)=\frac{2}{3}e^t+\frac{1}{3}e^{-2t}$ 2. $x(t)=-\frac{7}{3}\sin 3t+2\cos 3t$, $y(t)=-\frac{5}{3}\sin 3t-\cos 3t$ 3. $x(t)=-e^{-t}+\frac{1}{2}t^2+t+3$, $y(t)=\frac{t+1}{3}e^{-t}-\frac{1}{3}$ 4. $x(t)=e^t+e^{2t}$, $y(t)=e^t-e^{2t}$.

Assignment for Linear Algebra Refresher: 1. $1:\{\begin{bmatrix}3\\2\end{bmatrix}\}$; $2:\{\begin{bmatrix}2\\1\end{bmatrix}\}$. 2. $1:\{\begin{bmatrix}1\\-1\end{bmatrix}\}$; $-1:\{\begin{bmatrix}1\\-2\end{bmatrix}\}$.

Assignment 7: 2. No 3. No 4. norm: $\sqrt{\frac{\pi}{2}}$ 5. norm: $\sqrt{\frac{3}{2}}$ 7.(b) $c_1=14, c_2=6, c_3=1$.

Assignment 8: 1.$\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n\pi}\sin nx$, or better: $\frac{1}{2}+\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\sin(2n+1)x$. 2. $\frac{3}{2}-\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\sin(\frac{(2n+1)x}{3})$ 3. $\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)x}{(2n+1)^2}$ 4. $\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$ 5.

$$\frac{e^{\pi}-e^{-\pi}}{2\pi}+\frac{e^{\pi}-e^{-\pi}}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}\cos nx-\frac{e^{\pi}-e^{-\pi}}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n n}{n^2+1}\sin nx \quad .$$

Assignment 9: 1. odd,even, neither, even, odd, neither. 2. $\frac{\pi}{2} - \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1-(-1)^n}{n^2}\cos nx$ or better: $\frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos(2n+1)x$ 3. $\frac{1}{2} - \frac{2}{\pi^2}\sum_{n=0}^{\infty}\frac{1-(-1)^n}{n^2}\cos(n\pi x)$ or better: $\frac{1}{2} - \frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos((2n+1)\pi x)$ 4. $\frac{3}{2}+\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos(2n+1)x$ 4'. $\frac{3}{2}+\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos\frac{(2n+1)x\pi}{8}$ 5. $\sum_{n=1}^{\infty}\frac{8(-1)^{n+1}}{n}\sin\frac{nx}{4}$ 6. $2\pi - \frac{16}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\frac{(2n+1)x}{4}$.

Assignment 10: 1. $\sum_{n=-\infty,n\neq 0}^{\infty}(-1)^n\frac{i}{n}e^{inx}$ 2. $\sum_{n=-\infty,n\neq 0}^{\infty}(-1)^n\frac{2i}{n}e^{inx/2}$ 3. $\sum_{n=-\infty,n\neq 0}^{\infty}i(-1)^n\frac{n^2\pi^2-6}{n^3}e^{inx}$ 4. $64i\sum_{n=-\infty,n\neq 0}^{\infty}(-1)^n\frac{n^2\pi^2-6}{n^3}e^{inx/4}$ 5. $\frac{1}{2}-\frac{i}{\pi}\sum_{n=-\infty,n\neq 0}^{\infty}\frac{1}{2n+1}e^{(2n+1)ix}$ 1'. $\{(n,\frac{1}{|n|})\}$ $(n\neq 0)$ and $(0,0)$ 2'. $(n/2,\frac{2}{|n|})$ $(n\neq 0)$ and $(0,0)$ 3'. $(n,|\frac{n^2\pi^2-6}{n^3}|)$ $(n\neq 0)$ and $(0,0)$ 4'. $(n/4, 64|\frac{n^2\pi^2-6}{n^3}|)$ $(n\neq 0)$ and $(0,0)$ 5'. $(2n+1, 1/|2n+1|)$ (n integer) and $(0,\frac{1}{2})$.

Assignment 11: 1. $\{\sin\lambda_n x\}$, $\tan\lambda_n = -2\lambda_n$ 2. $\{\cos\lambda_n x\}$, $\cot 2\lambda_n = -3\lambda_n$ 3. $\{\cos(n+\frac{1}{2})x\}$, $n+\frac{1}{2}$ (n integer) 4. $\{\sin(\frac{n}{2})x\}$, $\frac{n}{2}$ (n integer) 5. $\{\sin\lambda_n x\}$, $\tan 10\lambda_n = -5\lambda_n$

Assignment 12: 1. $2x^2-1, 4x^3-3x, 8x^4-8x^2+1$ 2. $\frac{1}{2}x^2-2x+1, -\frac{1}{6}x^3+\frac{3}{2}x^2-3x+1, \frac{1}{24}x^4-\frac{2}{3}x^3+3x^2-4x+1$ 3. $27x^4-4x+9, 1728x^6-256x^3+571x^2+16x$ 4. $-\frac{1}{2}, \frac{3}{4}, 0, -\frac{7}{16}$ 5. $\frac{3}{5}P_1(x)+\frac{2}{5}P_3(x)$ 6. $\frac{e}{2}-\frac{1}{2e}, \frac{3}{e}, \frac{5e}{2}-\frac{35}{2e}$.

Assignment 13: 1. $e^{5kx}e^{-ky}$ 2. $e^{kx/11}e^{ky/5}$ 3. $x^k y^k$ 4. $e^{kx}e^{k^2y/9}$, $e^{-kx}e^{k^2y/9}$, c_1x+c_2, $e^{-k^2y/9}(\cos kx)$, $e^{-k^2y/9}(\sin kx)$ 5. $e^{\sqrt{k+1}x}e^{kt/9}$, $e^{-\sqrt{k+1}x}e^{kt/9}$ (when $k>-1$) $\cos\sqrt{-(k+1)}xe^{kt/9}$, $\sin\sqrt{-(k+1)}xe^{kt/9}$, (when $k<-1$), $e^{-t/9}$ (when $k=-1$). 6. impossible 7. hyperbolic 8. hyperbolic.

Assignment 14: (Warning: Do not peek at the answers before you do them yourself! In these problems, the answers and solutions coincide. Just copying the answers will not do you any good!)

1.a: $ku_{xx}=u_t, 0<x<10, 0<t<\infty$; $u(0,t)=1000, u_x(10,t)=0$; $u(x,0)=100$.

1.b: $ku_{xx}=u_t, 0<x<10, 0<t<\infty$; $u(0,t)=130, u_x(10,t)=0$; $u(x,0)=x^2$.

1.c: $ku_{xx}=u_t, 0<x<10, 0<t<\infty$; $u_x(0,t)=0, u_x(10,t)=0$; $u(x,0)=\sin x$.

2.a: $\alpha^2u_{xx}=u_{tt}, 0<x<100, 0<t<\infty; u(0,t)=0, u(100,t)=0 (0\leq t<\infty); u(x,0)=0, u_t(x,0)=\sin 20x (0<x<100)$

2.b: $\alpha^2u_{xx}=u_{tt}, 0<x<100, 0<t<\infty; u(0,t)=\sin 5t, u(100,t)=\cos 3t (0\leq t<\infty); u(x,0)=x^3, u_t(x,0)=\cos 5x (0<x<100)$.

3.a: $u_{xx}+u_{yy}=0 (0<x<10, 0<y<20); u(x,0)=150, u(x,20)=50 (0<x<10); u(0,y)=150, u(10,y)=g(y) (0<y<20)$

3.b: $u_{xx}+u_{yy}=0(0<x<10,0<y<20); u_y(x,0)=0, u(x,20)=f(x)(0<x<10); u_x(0,y)=0, u_x(10,y)=0(0<y<20)$

3.c: $u_{xx}+u_{yy}=0(0<x<10,0<y<20); u(x,0)=50, u(x,20)=50(0<x<10); u(0,y)=\sin 2y, u_x(10,y)=0(0<y<20)$.

Assignment 15: 1. $-32\sum_{n=1}^{\infty}\frac{1+2(-1)^n}{n^3}e^{-n^2t/4}\sin(nx/2)$ 2. $-\frac{1200}{\pi}\sum_{k=0}^{\infty}\frac{1}{2k+1}e^{-3(2k+1)^2t/4}\sin((2k+1)x/2)$ 3. $1+\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}e^{-7(2k+1)^2t}\cos((2k+1)x)$ 4. $u(x,t)=\frac{A_0}{2}+\sum_{n=1}^{\infty}A_ne^{-(n^2+1)t}\cos nx$, where $A_0=\frac{2}{\pi}\int_0^{\pi}f(x)\,dx$, $A_n=\frac{2}{\pi}\int_0^{\pi}f(x)\cos nx\,dx$.

Assignment 16: 1. $u(x,t)=\cos 3t\sin 3x+\frac{1}{4}\sin 4t\sin 4x$ 2. $u(x,t)=10\cos t\sin x+\frac{2}{3}\sin 3t\sin 3x$ 3. $u(x,t)=-4\sum_{n=1}^{\infty}\frac{1+2(-1)^n}{n^3}\cos nt\sin nx$ 4. $u(x,t)=4\sum_{n=1}^{\infty}\frac{2+(-1)^n}{n^4}\sin nt\sin nx$ 5. $u(x,t)=-\frac{72}{\pi}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^3}\cos(2k+1)t\sin\frac{(2k+1)x}{3}$.

Assignment 17: 1. $u(x,y)=\sum_{n=1}^{\infty}A_n\sin nx\sinh ny$, where $A_n=\frac{2}{\pi\sinh n}\int_0^{\pi}f(x)\sin nx\,dx$

2. $u(x,y)=\sum_{n=1}^{\infty}A_n\sin nx\cosh ny$, where $A_n=\frac{2}{\pi\cosh n}\int_0^{\pi}f(x)\sin nx\,dx$

3. $u(x,y)=A_0+\sum_{n=1}^{\infty}A_n\cos nx\cosh ny$, where $A_0=\frac{1}{\pi}\int_0^{\pi}f(x)\,dx$, $A_n=\frac{2}{\pi\cosh n}\int_0^{\pi}f(x)\cos nx\,dx$.

4. $u(x,y)=u_1(x,y)+u_2(x,y)$, where $u_1(x,y)$ is a solution of:

$$(u_1)_{xx}+(u_1)_{yy}=0 \quad , \quad 0<x<3 \quad , \quad 0<y<4$$

$$u_1(0,y)=y^3 \quad , \quad u_1(3,y)=\cos 7y \quad , \quad 0<y<4 \quad .$$

$$u_1(x,0)=0 \quad , \quad u_1(x,4)=0 \quad , \quad 0<x<3 \quad .$$

and $u_2(x,y)$ is a solution of:

$$(u_2)_{xx}+(u_2)_{yy}=0 \quad , \quad 0<x<3 \quad , \quad 0<y<4$$

$$u_2(0,y)=0 \quad , \quad u_2(3,y)=0 \quad , \quad 0<y<4 \quad .$$

$$u_2(x,0)=x^3 \quad , \quad u_2(x,4)=e^x \quad , \quad 0<x<3 \quad .$$

Assignment 18: 1. $r^7\cos 7\theta$. 2. $\frac{3}{32}(\cos 5\theta)r^5+\frac{11}{8}(\sin 3\theta)r^3-\frac{19}{2^{29}}(\sin 29\theta)r^{29}$
3. $50+\frac{200}{\pi}\sum_{k=0}^{\infty}\frac{r^{2k+1}\sin(2k+1)\theta}{(2k+1)2^{2k+1}}$.
4.

$$-\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{r^{2k+1}}{(2k+1)^2 3^{2k+1}}\cos(2k+1)\theta+2\sum_{k=0}^{\infty}\frac{r^{2k+1}}{(2k+1)3^{2k+1}}\sin(2k+1)\theta$$

5.

$$u(r,\theta) = A_0 \quad + \sum_{n=1}^{\infty} (A_n \cos(n\theta) + B_n \sin(n\theta)) r^{-n}$$

where

$$A_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) \quad , \quad A_n = \frac{c^n}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta \, d\theta \quad , \quad B_n = \frac{c^n}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta \, d\theta \quad .$$

Assignment 19: 1. $19U_{xxx} - 7U_{xx} = (s^2 + 4s - 11)U - (s+4)u(x,0) - u_t(x,0)$ 2. $29U_{xxx} - 17U_{xx} = (s^3 + 4s^2 - 11s + 5)U - (s^2 + 4s - 11)u(x,0) - (s+4)u_t(x,0) - u_{tt}(x,0)$ 3. $29U_{xx} + 11s^2U + 17sU_x - 17u_x(x,0) - 11su(x,0) - 11u_t(x,0)$ 4. $u(x,t) = 3\sin x \sin t - \frac{5}{4} \sin 4t \sin 4x$ 5. $(\frac{3}{8} \sin t - \frac{1}{8} \sin 3t) \sin x$ 6. $(2 \sin t - \sin 2t) \sin 2x$

Assignment 20:

1.

$$\frac{1}{\pi} \int_0^{\infty} \frac{\sin \alpha}{\alpha} \cos \alpha x \, d\alpha - \frac{3}{\pi} \int_0^{\infty} \frac{\cos \alpha - 1}{\alpha} \sin \alpha x \, d\alpha \quad .$$

2.

$$\frac{1}{\pi} \int_0^{\infty} \frac{\cos 2\alpha + 2\alpha \sin 2\alpha - 1}{\alpha^2} \cos \alpha x \, d\alpha + \frac{1}{\pi} \int_0^{\infty} \frac{\sin 2\alpha - 2\alpha \cos 2\alpha}{\alpha^2} \sin \alpha x \, d\alpha \quad .$$

3.

$$\frac{1}{\pi} \int_0^{\infty} \frac{\alpha^2 \sin \alpha - 2 \sin \alpha + 2\alpha \cos \alpha}{\alpha^3} \cos \alpha x \, d\alpha + \frac{1}{\pi} \int_0^{\infty} \frac{2\alpha \sin \alpha - (\alpha^2 - 2) \cos \alpha - 2}{\alpha^3} \sin \alpha x \, d\alpha \quad .$$

4.

$$\frac{1}{\pi} \int_0^{\infty} \frac{1}{\alpha^2 + 1} \cos \alpha x \, d\alpha + \frac{1}{\pi} \int_0^{\infty} \frac{\alpha}{\alpha^2 + 1} \sin \alpha x \, d\alpha$$

5.

$$\frac{4}{\pi} \int_0^{\infty} \frac{1 - 3\alpha^2}{(\alpha^2 + 1)^3} \cos \alpha x \, d\alpha \quad , \quad \frac{4}{\pi} \int_0^{\infty} \frac{(3 - \alpha^2)\alpha}{(\alpha^2 + 1)^3} \sin \alpha x \, d\alpha$$

6.

$$\frac{4}{\pi} \int_0^{\infty} \frac{1}{\alpha^2 + 4} \cos \alpha x \, d\alpha \quad , \quad \frac{2}{\pi} \int_0^{\infty} \frac{\alpha}{\alpha^2 + 4} \sin \alpha x \, d\alpha$$

Assignment 21: 1.

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{e^{-\alpha^2 t} \cos \alpha x}{\alpha^2 + 1} \, d\alpha$$

2.

$$\frac{10}{\pi} \int_{-\infty}^{\infty} \frac{(\cos \alpha - 1) e^{-\alpha^2 t/4} \sin(\alpha x)}{\alpha} \, d\alpha$$

3.

$$\frac{2}{\pi}\int_0^\infty \frac{\sinh(2\alpha x)\cos(\alpha y)}{(\alpha^2+1)\sinh(4\alpha)}\,d\alpha$$

4.

$$\frac{4}{\pi}\int_0^\infty \frac{\alpha\sinh(\alpha x)\sin(\alpha y)}{(\alpha^2+1)^2\sinh(2\alpha)}\,d\alpha$$

Assignment 22: 1. 26 2. $\frac{577}{8}$ 3. 1.452 4. 1.5179106 5. 1.521952697

Assignment 23: 1. $u(1,1) \approx \frac{5}{4}$ 2. $u(2,2) \approx \frac{15}{4}$ 3. $u(\frac{1}{2},\frac{1}{2}) \approx \frac{19}{24}, u(\frac{1}{2},1) \approx \frac{29}{24}, u(1,\frac{1}{2}) \approx \frac{23}{24}, u(1,1) \approx \frac{37}{24}$

4.

$$u_{2,3} + u_{3,2} - 4\,u_{3,3} = 3/2 \quad ,$$

$$u_{3,2} + u_{2,1} - 4\,u_{3,1} = -3/4 \quad ,$$

$$u_{2,1} + u_{1,2} - 4\,u_{1,1} = -5/4 \quad ,$$

$$u_{2,3} + u_{1,2} - 4\,u_{1,3} = -1/2$$

$$u_{3,3} + u_{2,2} + u_{3,1} - 4\,u_{3,2} = 1/2 \quad ,$$

$$u_{2,2} + u_{1,3} + u_{1,1} - 4\,u_{1,2} = -1/2 \quad ,$$

$$u_{3,1} + u_{2,2} + u_{1,1} - 4\,u_{2,1} = -1 \quad ,$$

$$u_{3,2} + u_{2,3} + u_{1,2} + u_{2,1} - 4\,u_{2,2} = 0 \quad ,$$

$$u_{3,3} + u_{1,3} + u_{2,2} - 4\,u_{2,3} = 1/2 \quad .$$