NAME: (print!) ____

Section: ____ **E-Mail address:** ____________________

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Exam 2, Monday, Nov. 23, 2020, 8:40-10:40am

Email the completed test, renamed as `mt2FirstLast.pdf` to DrZcalc3@gmail.com no later than 10:40am, (or, in case of conflict, two hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

1.

2.

3.

4.

5.

6.

7. (a) (b) (c) (d)

8.

9.

10.

Types: Number, Function of *variable*(s), 2D vector of numbers, 3D vector of numbers, 2D vector of functions, 3D vector of functions, equation of a plane, parametric equation of a line, equation of a line, equation of a surface, equation of a line, DNE (does not exist), parametric equation of surface, double integral of an abstract function.

Sign the following declaration:

I Hereby declare that all the work was done by myself. I was allowed to use Maple, calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 90 minutes on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed:

1. (10 pts.) Compute the line integral

$$\int_C yz\,dx \;+\; (xz+z)\,dy \;+\; (xy+y+1)\,dz \quad ,$$

over the path

$$\mathbf{r}(t) = \langle e^{t^3}, t^2 e^{t^4}, t e^{t^7} \rangle \quad , \quad 0 \le t \le 1 \quad .$$

Explain!

The **type** of the answers is:

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ans.

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2. (10 points) By changing the order of integration, if necessary, evaluate the double-integral

$$\int_0^5 \int_{(y/5)^{1/3}}^1 \sin x^4 \, dx \, dy$$

The **type** of the answer is:

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ans.

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3. (10 points) Find the equation of the tangent plane at the point $(1,1,1)$ to the surface given parametrically by

$$x(u,v) = u^3 v \quad , \quad y(u,x) = uv \quad , \quad z(u,v) = uv^3 \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

The **type** of the answer is:

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ans. $z =$

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4. (10 points) Let $f(x,y,z) = e^{\cos x^2 + \sin xyz + \cos xz}$, and let

$$\mathbf{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$$

Let C be the curve

$$r(t) = \langle \cos t, t, \sin t \rangle \quad , \quad 0 \leq t \leq 2\pi \quad .$$

Find the value of the line-integral

$$\int_C \mathbf{F}.d\mathbf{r} \quad .$$

Explain! Just giving the answer will give you no credit.

The **type** of the answer is:

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ans.

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5. (10 points) Evaluate the triple integral

$$\int_R (x^2 + y^2 + z^2)^3 \, dx \, dy \, dz \quad ,$$

where R is the region in 3D space given by

$$\{(x, y, z) \,|\, x^2 + y^2 + z^2 \leq 1 \quad , \quad x, y, z \geq 0\} \quad .$$

The **type** of the answer is:

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ans.

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6. (10 points) Evaluate the double integral

$$\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y^2)^2 \, dy \, dx$$

The **type** of the answer is:

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ans.

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7. (10 points altogether) Decide whether the following limits exist. If it does find them. If it does not **Explain** why not?

$$(a)\ (2\ points)\ \lim_{(x,y)\to(\pi/2,\pi/2)} \frac{\cos x + \sin x}{x+y} \quad , \quad (b)\ (2\ points)\ \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} \quad ,$$

$$(c)\ (2\ points)\ \lim_{(x,y)\to(0,0)} \frac{x-y}{x^2-y^2} \quad , \quad (d)\ (4\ points)\ \lim_{(x,y)\to(1,1)} \frac{x+y-2}{2x+y-3} \quad ,$$

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ans. (a) (b) (c) (d)

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8. (10 points) Compute the line integral $\int_C f\, ds$ where

$$f(x, y, z) = xyz$$

and C is the line segment starting at $(0, 0, 0)$ and ending at $(1, 2, -3)$

The **type** of the answer(s) is:

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ans.

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9. (10 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle z, z, x \rangle \quad ,$$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , x \geq 0, y \geq 0, z \geq 0$$

with **downward pointing** normal.

The **type** of the answer is

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ans.

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10. (10 points) Find the **point** on the plane $x + 2y + 3z = 18$ where the function $f(x, y, z) = xyz$ is **as large as possible**.

The **type** of the answer is:

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ans.
