**NAME: (print!)** ________________________ **RUID: (print!)** ______________________

**SSC:** (circle) None / I / II / I and II

MATH 251 (22,23,24 ) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as `finalFirstLast.pdf` to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

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WRITE YOUR FINAL ANSWERS BELOW

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Sign the following declaration:

I Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed:

**Possibly useful facts from school Geometry** (that you are welcome to use) : (i) The area of a circle radius $r$ is $\pi r^2$. (ii) The circumference of a circle radius $r$ is $2\pi r$ (iii) The parametric equation of an ellipse with axes $a$ $b$ and parallel to the $x$ and $y$ axes respectively is $x = a\cos\theta$, $y = b\cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes $a$ and $b$ is $\pi ab$ (v) The volume and surface area of a sphere radius $R$ are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

**Formula that you may (or may not) need**

If the surface $S$ is given in **explicit** notation $z = g(x, y)$, above the region of the $xy$-plane , $D$, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA \quad .$$

**1**. (12 pts.) **Without using Maple (or any software)** Compute the **vector-field line integral**

$$\int_C (cos\,(e^{\sin x}) + 5y)\,dx \;+\; (sin\,(e^{\cos y}) \;+\; 11x\,)\,dy \quad ,$$

over the path consisiting of the five line segments (in that order)

$$(1,0) \rightarrow (-1,0) \rightarrow (-1,1) \rightarrow (0,2) \rightarrow (1,1) \rightarrow (1,0) \quad .$$

Explain!

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**ans.**

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**2.** (12 points) Change the order of integration

$$\int_{\frac{1}{4}}^{1} \int_{0}^{\sqrt{x}} f(x,y)\,dy\,dx \qquad .$$

**ans.**

**3.** (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) \; = \; 7 \qquad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

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**ans.** $z =$

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**4.** (16 points) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k} \quad , \quad \mathbf{b} \times \mathbf{c} = \mathbf{i} - \mathbf{j} + \mathbf{k} \quad , \quad \mathbf{a} \times \mathbf{c} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad .$$

What is

$$(\mathbf{a} + \mathbf{b} + \mathbf{c}) \times (2\mathbf{a} - \mathbf{b} + 3\mathbf{c}) \qquad ?$$

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**ans.**

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**5.** (12 points) Find the three angles of the triangle $ABC$ where

$$A = (0,0,0) \quad , \quad B = (1,0,1) \quad , \quad C = (1,1,0) \qquad .$$

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**ans.** The angle at $A$ is: radians ;

The angle at $B$ is: radians ;

The angle at $C$ is: radians ;

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**6.** (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz \quad ,$$

at the point $(1, 1, 1)$ in a direction pointing to the point $(-1, -1, -1)$ .

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**ans.**

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**7.** (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2 \quad ,$$

and

$$x = e^u \cos v \quad , \quad y = e^u \sin v \quad .$$

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**ans.**

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**8.** (12 points) Without using Maple (or any other software), compute the vector-field surface integral $\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle\, 3x + cos(y^3 + yz)\,,\, -2y + e^{x+z^2}\,,\, 5z + \sin(xy^3 + e^x)\,\rangle \qquad ,$$

and $S$ is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \quad and \quad x > 0 \quad and \quad y < 0 \quad and \quad z > 0\} \quad .$$

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**ans.**

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**9.** (12 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} \;=\; \langle\, 3z\,,\, 2x\,,\, y+z\,\rangle \quad ,$$

and $S$ is the oriented surface

$$z = 2x + 3y \quad , \quad 0 < x < 1, \quad 0 < y < 1 \quad ,$$

with **upward pointing** normal.

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**ans.**

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**10.** (12 points) Without using Maple or software, find the critical point(s) of

$$f(x,y) = 4x - y^2 - \ln(2x+y) \quad ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Explain.

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**ans.**

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**11.** (12 points) Without using Maple or software, using a **Linearization** around the point $(1, 1, 2)$, approximate $f(1.001, 0.999, 2.001)$ if

$$f(x, y, z) = \sqrt{2x^2 + 3y^2 + z^2} \quad .$$

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**ans.**

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**12.** (12 points) Without using Maple (or any other software) and by using polar coordinates (no credit for doing it directly) find

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^x x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx \quad .$$

Explain!

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**ans.**

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**13.** (12 points) Convert the triple iterated integral

$$\int_0^2 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^0 x^2\, y\, z\, dx\, dy\, dz$$

to **spherical coordinates**. **Do not evaluate.**

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**ans.**

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**14.** (12 points) Find the curvature of the curve

$$\mathbf{r}(t) = \langle 5, 3\sin t, 3\cos t \rangle$$

at the point where $t = \frac{\pi}{3}$.

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**ans.**

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**15.** (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parametrically by

$$\mathbf{r}(u,v) = \langle u^2 \, , \, uv \, , \, v^2 \rangle \quad , \quad 0 < u < v < 1 \quad .$$

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**ans.**

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**16.** (12 points) Let

$$f(x, y, z) = xy^2z^3 \quad ,$$

and let

$$g(x, y, z) = x + y^2 + z^3 \quad .$$

compute the dot-product

$$grad(f) \, . \, grad(g) \qquad .$$

at the point $(1, 1, 1)$.

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**ans.**

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**17.** (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w} \quad .$$

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**ans.**

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