

Solutions to the “QUIZ” for Lecture 2

1. Determine whether the two vectors are orthogonal and if not, whether the angle between them is acute or obtuse. **a.** $\langle 1, 1, 1 \rangle$, $\langle 3, -2, -1 \rangle$.

b. $\langle 4, 3 \rangle$, $\langle 2, -4 \rangle$.

Solution: a.

$$\langle 1, 1, 1 \rangle \cdot \langle 3, -2, -1 \rangle = 1 \cdot 3 + 1 \cdot (-2) + 1 \cdot (-1) = 3 - 2 - 1 = 0 \quad .$$

Ans.: Since the dot-product is 0, the vectors are perpendicular.

b.

$$\langle 4, 3 \rangle \cdot \langle 2, -4 \rangle = 4 \cdot 2 + 3 \cdot (-4) = 8 - 12 = -4 \quad .$$

Ans.: Since the dot-product is **not** 0, the vectors are **not** perpendicular. Since the dot-product is **negative**, the angle is **obtuse**.

Comments: Most people got it right. Quite a few people, for **b**, volunteered more information, actually finding the angle. This is nice, but not necessary! Answer what you have been asked, not less, but also not more!

2. Calculate $\mathbf{v} \times \mathbf{w}$, if

$$\mathbf{v} = \langle 0, 1, -1 \rangle \quad , \quad \mathbf{w} = \langle 1, -1, 0 \rangle \quad .$$

$$\begin{aligned} v \times w &= \\ & \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix} = \\ & \mathbf{i} \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} \\ &= \mathbf{i}(1 \cdot (0) - (-1) \cdot (-1)) - \mathbf{j}(0 \cdot (0) - (-1) \cdot (1)) + \mathbf{k}(0 \cdot (-1) - (1) \cdot (1)) = \\ & \quad -\mathbf{i} - \mathbf{j} - \mathbf{k} \quad . \end{aligned}$$

Finally converting to the **usual** notation, we get $\langle -1, -1, -1 \rangle$.

Ans.: $\langle -1, -1, -1 \rangle$.

Comments: About %80 of the people got it perfectly. Another %10 set-it up correctly, but messed up the arithmetic (be careful with the minus signs!: minus times minus is plus, minus times minus times minus is minus, etc.