

Solutions to the “QUIZ” for Lecture 25

1. Let

$$F(x, y, z) =$$

$$\langle \cos(\sqrt{1+x^7} + zy^9) \quad , \quad \tan(x^7 + y^2 + 1/z) \quad , \quad \tan^{-1}(e^{xyz} + \cos^6(x^8 - y + 3z)) \rangle \quad ,$$

and let $\langle P, Q, R \rangle = \text{curl } \mathbf{F}$. Compute

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \quad .$$

Be sure to explain everything. (Note: there was a typo in the problem that has now been corrected).

Sol. We have to compute $\text{div}(\langle P, Q, R \rangle)$ but $\langle P, Q, R \rangle = \text{curl}(\mathbf{F})$ so we have to compute $\text{div}(\text{curl}(\mathbf{F}))$. But a famous theorem says that this is always the *zero* function.

Ans.: 0.

2. Calculate the surface integral

$\int \int_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = \langle 2x + y + z, x + 2y + z, x + y + 2z \rangle$$

where S is the surface of the box bounded by the planes $x = 0, x = 1, y = 0, y = 4, z = 0, z = 5$.

Sol.: We are supposed to use the **Divergence Theorem**:

$$\int \int_S \mathbf{F} \cdot d\mathbf{S} = \int \int \int_E \text{div}(\mathbf{F}) dV \quad ,$$

where E is the **inside** of the box. We have

$$\text{div}(\mathbf{F}) = \frac{\partial(2x + y + z)}{\partial x} + \frac{\partial(x + 2y + z)}{\partial y} + \frac{\partial(x + y + 2z)}{\partial z} = 2 + 2 + 2 = 6 \quad .$$

So

$$\int \int_S \mathbf{F} \cdot d\mathbf{S} = \int \int \int_E 6 dV \quad ,$$

Taking 6 out, we get:

$$6 \int \int \int_E 1 dV \quad .$$

But a **volume-integral** whose integrand is 1 equals the volume, so this equals

$$6 \cdot \text{Volume}(\text{Box})$$

The volume of the box is $1 \cdot 4 \cdot 5 = 20$, so the answer is $6 \cdot 20 = 120$.

Ans.: 120.