"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y,z) = \overset{P}{(3x^2y^3z^3 + yz)}\,\mathbf{i} + \overset{Q}{(3x^3y^2z^3 + xz)}\,\mathbf{j} + \overset{R}{(3x^3y^3z^2 + xy)}\,\mathbf{k}$$

① curl $F = \langle 0, 0, 0 \rangle$:

$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ P & Q & R \end{vmatrix} = i\,(9y^2x^3\cancel{z^2} + x - 9y^2\cancel{x^3}z^2 + x) - j\,(9x^2y^3\cancel{z^2} + y - 9x\cancel{^2}y^3z^2 + y) + k\,(9x^2y^3\cancel{z^2} + y - 9x^2\cancel{y^3}z^2 + y) = \langle 0, 0, 0 \rangle \checkmark$$

② $\int (3x^2y^3z^3 + yz)\,dx = (x^3y^3z^3 + yzx + g(y,z))$

$\frac{d}{dy}(x^3y^3z^3 + yzx + g(y,z)) = 3y^2x^3z^3 + xz + g'(y,z) = 3x^3y^2z^3 + xz$

so $g'(y,z) = 0$ so $g(y,z) = h(z)$

$d/dz(x^3y^3z^3 + yzx)$
$= 3x^3y^3z^2 + yx + h'(z) = 3x^3y^3z^2 + xy$
$h'(z) = -3x^3y^3z^2 + xy + \ldots$
$h'(z) = 0$

$f = x^3y^3z^3 + yzx$

$\boxed{f = x^3y^3z^3 + yzx}$

2. Evalute

$$\int_C \overset{P}{5y}\,dx + \overset{Q}{10x}\,dy \quad ,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x,y)\,|\,0 \le x \le 1 \quad , \quad 0 \le y \le 1\}.$$

$\iint_D \left(\frac{\partial}{\partial x}(10x) - \frac{\partial}{\partial y}(5y)\right) dA$

$= \iint_D (10 - 5)\,dA$

$= \int_0^1 \int_0^1 5\,dx\,dy$

$\int_0^1 5\,dx = 5x \Big|_0^1 = 5$

$\int_0^1 5\,dy = 5y \Big|_0^1 = \boxed{5}$