"QUIZ" for Lecture 22

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and oriented surface S.

$$\mathbf{F}(x,y,z) = \langle xy\,,\, yz\,,\, zx \rangle \quad ,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \le x \le 1\,,\, 0 \le y \le 1$ and has upward orientation.

$F(x,y,z) = \langle xy, yz, zx \rangle$ (P, Q, R)

$z = 1 - (x^2 + y^2)$, r^2

$z = 1 - r^2$

$= \int\int_D (2x^2y + (2y^2 + x)z)\,dA$

$= \int\int_D (2x^2y + (2y^2 + x)(1 - x^2 - y^2))\,dA$

$2y^2 - 2y^2x^2 - 2y^4 + x - x^3 - xy^2$

$= \int_0^1\int_0^1 (x^2y + 2y^2 - 2y^2x - 2y^4 + x - x^3)\,dx\,dy$

$= \int_0^1 \left(\left[\frac{x^3}{3}y + 2xy^2 - y^2x^2 - 2y^4x + \frac{x^2}{2} - \frac{x^4}{4}\right]\Big|_0^1\right) dy$

$= \int_0^1 \left(\frac{1}{3}y + 2y^2 - y^2 - 2y^4 + \frac{1}{2} - \frac{1}{4}\right) dy$

$= \frac{2}{3}y^2 + \frac{2}{3}y^3 - \frac{1}{3}y^3 - \frac{2}{5}y^5 + \frac{1}{2}y - \frac{1}{4}y\Big|_0^1$

$= \frac{2}{3} + \frac{2}{3} - \frac{1}{3} - \frac{2}{5} + \frac{1}{2} - \frac{1}{4}$

$= 1 - \frac{2}{5} - \frac{1}{4} = \frac{3}{4} - \frac{2}{5} = \boxed{\frac{7}{20}}$

$\frac{15}{20} - \frac{8}{20}$