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2017 Practice Exam

① $x = uv + w$
$y = uw + v$ @(1,1,2,1)
$z = vw + u$

$$\begin{vmatrix} v & u & 1 \\ w & 1 & u \\ 1 & w & v \end{vmatrix} = v\begin{vmatrix} 1 & u \\ w & v \end{vmatrix} - u\begin{vmatrix} w & u \\ 1 & v \end{vmatrix} + 1\begin{vmatrix} w & 1 \\ 1 & w \end{vmatrix}$$

$$= 1(2-1) - 1(2-1) + 1(1-1)$$
$$= 1 - 1 + 0 = \boxed{0}$$

② $F = \langle y^2z + 2xyz,\ 2xyz + x^2z,\ xy^2 + x^2y + 2z \rangle$

$$\text{curl } f = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ y^2z+2xyz & 2xyz+x^2z & x^2y^2+x^2y2z \end{vmatrix}$$

$= 0i - 0j + 0k = \langle 0,0,0 \rangle$

$f_x = y^2z + 2xyz = xy^2z + x^2yz + g(y,z)$

$f_y = 2xyz + x^2z \rightarrow g_y = 0$

$f_z = xy^2 + x^2y + 2z$ $= xy^2 + x^2y + 2z$

$xy^2 + x^2y + h'(z)$

$\boxed{f = xy^2z + x^2yz + z^2}$

$\int_c F \cdot dr = f(0,0,0) -$

③ $\int_1^2 \int_0^{e^x+1} F(x,y)\,dy\,dx \rightarrow \int_0^{e+1}\int_1^2 F(x,y)\,dx\,dy + \int_{e+1}^{e^2+1}\int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy$

$y=0$ and $y = e^x+1$

$y=1$ and $y = e+1 \rightarrow x=1$

$x = \ln(y-1) \rightarrow$ $\{(x,y) \mid 1 \le y \le e+1,\ 1 \le x \le 2\}$
$\{(x,y) \mid e+1 \le y \le e^2+1,\ \ln(y-1) \le x \le 2\}$

④ $x+y+z$ $xyz = 1$

$\nabla f = \langle f_x, f_y, f_z \rangle$ $\nabla g = \langle g_x, g_y, g_z \rangle$ $\rightarrow \nabla f = \lambda \nabla g$

$\nabla g = \langle yz, xz, xy \rangle$ $\langle 1,1,1 \rangle = \lambda \langle yz, xz, xy \rangle$

$\nabla f = \langle 1,1,1 \rangle$

$x = \lambda yz$
$y = \lambda xz$ $\rightarrow \lambda^3 = 1$ so $x=1$, $y=1$, $z=1$ $\rightarrow (1,1,1)$
$z = \lambda xy$

$\hookrightarrow$ plug into $f(x,y,z) = 1+1+1 = 3$

⑤ $\iiint_E 4z\cos(x^5)\,dV \quad E = \{(x,y,z) \mid 0 \le x \le 1,\ 0 \le y \le x,\ x < z < 2x\}$

$= \int_0^1 \int_0^x \int_x^{2x} 4z\cos(x^5)\,dz\,dy\,dx$

$= \int_x^{2x} 4z\cos(x^5)\,dz = \frac{4z^2\cos(x^5)}{2}\Big|_x^{2x} = \frac{3}{2}4x^2\cos(x^5)$

$\int_0^x 4\frac{3}{2}x^2\cos x^5 = \frac{3}{4}x^4\cos(x^5)$

$\frac{3}{4}\int_0^1 x^4\cos(x^5) = \frac{3\sin 1}{20} = \boxed{\frac{3\sin 1}{20}}$

⑥ $\iint_D e^{-x^2-y^2}\,dA \quad x = \sqrt{25-y^2}$

$D = \{(r,\theta) \mid -\frac{\pi}{2} \le \theta \le \pi/2,\ 0 \le r \le 5\}$

$\int_{-\pi/2}^{\pi/2} \int_0^5 e^{-r^2} r\,dr\,d\theta$

$= \int_0^5 e^{-r^2} r\,dr = \frac{(1-e^{-25})}{2}$

$\int_{-\pi/2}^{\pi/2} \frac{(1-e^{-25})}{2} = \boxed{\frac{\pi(1-e^{-25})}{2}}$

⑦ $F(x,y) = (2y+1)i + (2x+3)\hat{j} \quad A(0,0) \to B(-1,1)$

$\nabla f = 2y+1i + 2x+3\hat{j}$

$\int 2y+1\,dx = 2xy + x + g(y) \to g'(y) = 3$

$\int 3dy = 3y + c \to f(x,y) = 2xy + x + 3y \quad f(b) - f(a)$

$f(-1,1) - f(0,0) = 0 - 0 = 0$

⑧ $\int_0^3 \int_{\sqrt{y/3}}^1 e^{x^3}\,dx\,dy \to \Rightarrow \int_0^1 \int_0^{3x^2} e^{x^3}\,dy\,dx$

$\int_0^{3x^2} e^{x^3}\,dy = 3x^2 e^{x^3} \to \int_0^1 3x^2 e^{x^3} = e-1$

⑨ $\iiint_E x^2\,dV \quad y = \sqrt{1-x^2-z^2} + y = \sqrt{4-x^2-z^2}$

$\int_0^{\pi} \int_0^{\pi} \int_1^2 (\rho\sin\phi\cos\theta)^2 \rho^2 \sin\phi\,d\rho\,d\theta\,d\phi$

$\to \int_0^{\pi} \sin^3\phi\,d\phi = 4/3$

$\int_0^{4} \cos^2\theta\,d\theta = \pi/2$

$\int_1^2 \rho^4\,d\rho = \frac{\rho^5}{5}\Big|_1^2 = 31/5$

$\to 4/3 \cdot \pi/2 \cdot 31/5 = \boxed{62\pi/15}$

⑩ $\nabla F = \langle f_x, f_y, f_z \rangle$

$\nabla F = \langle y\cos(xy) + z\cos(yz) + x\cos(xz))$