"QUIZ" for Lecture 20

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2 \quad , \quad y = u + v \quad , \quad z = u^2 \quad ,$$

at the point $(1, 2, 1)$. Simplify as much as you can!

$1 = v^2 \longrightarrow v = \pm 1$

$2 = u + v$

$1 = u^2 \longrightarrow u = \pm 1$

$u = 1, v = 1$

$\vec{r}(u,v) = v^2\hat{i} + (u+v)\hat{j} + u^2\hat{k}$

① $r_u = 0\hat{i} + 1\hat{j} + 2u\hat{k} \rightarrow \langle 0, 1, 2u \rangle = \langle 0, 1, 2 \rangle$

$r_v = 2v\hat{i} + 1\hat{j} + 0\hat{k} \rightarrow \langle 2v, 1, 0 \rangle = \langle 2, 1, 0 \rangle$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{vmatrix} = (-2)i - 4j + 2k$$

$= \langle -2, -4, 2 \rangle$

② Equation =>

$N = \langle -2, -4, 2 \rangle$

$P = (1, 2, 1)$

$-2(x-1) + -4(y-2) + 2(z-1) = 0$

$-2x + 2 - 4y + 8 + 2z - 2 = 0$

$-2x - 4y + 2z + 8 = 0$

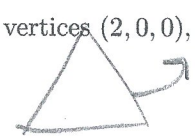
$-2(x + 2y - z - 4) = 0$

$\hookrightarrow \boxed{x + 2y - z - 4 = 0}$

2. Evaluate the surface integral

$$\iint_S z \, dS \quad ,$$

where S is the triangular region with vertices $(2, 0, 0), (0, 2, 0), (0, 0, 2)$.



$\sqrt{1^2 + 1^2 + 1} = \sqrt{3}$

$x + y + z = 2$

surface in explicit format $\rightarrow$ $z = 2 - y - x$

$0 = 2 - y - x$

$y + x = 2$

$$\int_0^2 \int_0^2 (2 - y - x)\sqrt{3} \, dx \, dy$$

$= \sqrt{3} \int_0^2 \left(\left[2x - 4x - \frac{x^2}{2} \right] \Big|_0^2 \right) dy$

$= \sqrt{3} \int_0^2 (4 - 2y - 2) \, dy$

$= \sqrt{3} \left[4y - y^2 - 2y \right] \Big|_0^2 = \sqrt{3}(8 - 4 - 4) = 0$ (0)