"QUIZ" for Lecture 19

NAME: (print!) Racnel Baji **Section:** 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x,y,z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

① $\frac{dP}{dy} = 2yz^3$; $\frac{dQ}{dx} = 2yz^3$] =

$\frac{dR}{dz} = 6xy^2z$

② curl

i	j	k
d/dx	d/dy	d/dz
y^2z^3	$2xyz^3$	$3xy^2z^2$

$= i(6xyz^2 - 6xyz^2) - j(3y^2z^2 - 3y^2z^3) + k(2yz^3 - 2yz^3) = 0+0+0 = \langle 0,0,0\rangle$

So it is conservative

Step 1: Check if it is conservative ✓

③

Step 2:

Integrate fx w/ respect to x

constant depends on remaining variables of y, z

set = to the second component of F

(P) $f_x = y^2z^3 \longrightarrow xy^2z^3 + g(y,z)$

(Q) $f_y = 2xyz^3 \longrightarrow \frac{d}{dy}(xy^2z^3 + g(y,z)) = 2xyz^3 + g_y(y,z)$

$2yxz^3 + g_y(y,z) = 2xyz^3$

$g_y(y,z) = 0$

$g(y,z) = \int (0)\,dy + c = 0 + c = 0 + h(z)$

(R) $f_z = 3xy^2z^2 \longrightarrow \frac{d}{dz}(xy^2z^3 + h(z)) = 3xy^2z^2 + h'(z)$

$f(x,y,z) = xy^2z^3 + 0 + h(z)$

$xy^2z^3 + h'(z) = h(z)$ $h(z) = 0$ $\boxed{f(x,y,z) = xy^2z^3}$

2. Show that the line integral

If this is true then $\int_C F(r)\cdot dr = 0$

$$\int_C 2xy^2\,dx + 2x^2y\,dy \quad , = U(B) - U(A)$$

is independent of the path C, and evaluate it if C is *any* path from (1,0) to (0,1).

A B

①

$\frac{dP}{dy} = \frac{dQ}{dx}$

$\frac{dP}{dy} = 4xy$; $\frac{dQ}{dx} = 4xy$] = ✓ so they are conservative

(so it is independent of the path C).

Integrate each one

$\int 2xy^2\,dx + 2x^2y\,dy$

$= d(x^2y^2) + d(x^2y^2)$

$= d(2x^2y^2) = du$

$U = (2x^2y^2)$

$= U(B) - U(A)$

$= 0 - 0 = 0$

$3x^2y\,dx + x^3\,dy$

$x^3y + x^3y$

$= 2x^3y$

$yx + xy = 2xy$