"QUIZ" for Lecture 16

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q16FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 2, 8:00pm

1. Compute the Jacobian of the transfomation

$$\overset{x\ y}{\Phi(r,s) = (rs, r+s)}$$

$\hookrightarrow \Rightarrow (x(r,s), y(r,s))$

① $x = rs$
$y = r+s$

$$\begin{vmatrix} \frac{dx}{dr} & \frac{dx}{ds} \\ \frac{dy}{dr} & \frac{dy}{ds} \end{vmatrix} = \begin{vmatrix} s & r \\ 1 & 1 \end{vmatrix} = s(1) - r(1) = s - r$$

$\boxed{s-r}$ = the Jacobian

2. Let $\mathcal{D} = \Phi(\mathcal{R})$ where $\Phi(\overbrace{u,v}^{\text{translated to a function of } u \text{ and } v}) = (u+v, v^2)$ and $R = \underbrace{[0,6] \times [1,2]}_{\text{changing this region } (R)}$. Calculate

$u = d$
$u = c$
$c \Rightarrow d$ is a u-axis
(changing the axis)

$$\int\int_D y\,dA \ .$$

(Note: it is not necessary to compute D).

$y = r\sin\theta$

$x = u+v$
$y = v^2$

$$J = \begin{vmatrix} 1 & 1 \\ 0 & 2v \end{vmatrix} = 2v - 0 = \underbrace{2v}_{\text{the Jacobian}}$$

① $\int_0^6 \int_1^2 y\,dy\,dx$

$x = u+v$
$x =$

$= \int_{-1}^{6-\sqrt{2}} \int_1^{\sqrt{2}} v^2 (2v)\,dv\,du =$

② $\int_1^{\sqrt{2}} 2v^3\,dv$

$= \frac{1}{2} v^4 \Big|_1^{\sqrt{2}} = 2 - \frac{1}{2} = 1.5$

③ $\int_0^{6-\sqrt{2}} \frac{3}{2}\,du = \frac{3}{2}x \Big|_0^{6-\sqrt{2}} = \frac{3}{2}(6-\sqrt{2}) - 0$

$= \boxed{9 - 3\sqrt{2}}$

$0 = u+v$
$1 = v^2$
$v = 1$
$u = -1$

$6 = u+v$
$2 = v^2$
$v = \pm\sqrt{2}$
$u = 6 - \sqrt{2}$