

“QUIZ” for Lecture 16

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q16FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 2, 8:00pm

1. Compute the Jacobian of the transformation

$$\Phi(r, s) = (rs, r + s)$$

$$\begin{aligned} x &= rs \\ y &= r+s \end{aligned}$$

$$\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} \end{bmatrix} = \begin{bmatrix} s & r \\ 1 & 1 \end{bmatrix} = \boxed{r-s}$$

2. Let $\mathcal{D} = \Phi(\mathcal{R})$ where $\Phi(u, v) = (u + v, v^2)$ and $R = [0, 6] \times [1, 2]$. Calculate

$$\iint_{\mathcal{D}} y \, dA$$

(Note: it is not necessary to compute \mathcal{D}).

$$(0, 6) = 0 = u + v, \quad 6 = u + v$$

$$(1, 2) = 1 = v^2, \quad 2 = v^2$$

$$v = -1, \quad v = \sqrt{2}$$

$$u = -1, \quad u = 6 - v$$

$$\int_1^{\sqrt{2}} \int_{-1}^{6-v} v^2 \, du \, dv = v^2 v \Big|_{-1}^{6-v} =$$

$$\int_{-1}^{\sqrt{2}} (7-v) v^2 \, dv = \frac{v^3(3v-28)}{12} \Big|_{-1}^{\sqrt{2}} \approx 8.183$$