No.

Date

Exercise 13.3

Q3. $r(t)=(2t, \ln t, t^2)$

$1 \le t \le 4$

$r'(t)=(2, \frac{1}{t}, 2t)$

$\|r'(t)\| = \sqrt{2^2+(\frac{1}{t})^2+4t^2}$

$=\sqrt{4+\frac{1}{t^2}+4t^2}$

$=2t+\frac{1}{t}$

$\int_1^4 \|r'(t)\| dt = t^2+\ln t \Big|_1^4$

$= 16+\ln 4-1-\ln 1$

$= 15+\ln 4$

Q9. $s(t)=\int_a^t \|r'(u)\| du$

$r(t)=(t^2, 2t^2, t^3), a=0$

$r'(t)=(2t, 4t, 3t^2)$

$\cancel{L=\int_0^t \sqrt{(2t)^2+16t^2+9t^4}\,dt}$

$\cancel{=\int_0^t \sqrt{4u^2+16u^2+9u^4}\,du}$

$\|r'(t)\| = \sqrt{20t^2+9t^4}$

$s(t)=\int_a^t \|r'(u)\| du$

$=\int_0^t (20t^2+9t^4)^{\frac{1}{2}}\, du$

$=\int_0^t (20u^2+9u^4)^{\frac{1}{2}}\, du$

$= \cancel{(20u^2+9u^4)^{\frac{3}{2}}}\ \frac{1}{27}\left((20+9t^2)^{\frac{3}{2}}-20^{\frac{3}{2}}\right)$

Q11. $r(t)=(2t+3, 4t-3, 5-t)$

$t=4$

$r'(t)=(2, 4, -1)$

$|r'(t)|=\sqrt{2^2+4^2+1^2}$

$=\sqrt{21}$

Q13. $r(t)=(t, \ln t, (\ln t)^2)$

$t=1$

$r'(t)=(1, \frac{1}{t}, \frac{1}{t}\cdot 2(\ln t))$

$r'(1)=(1, 1, 0)$

$\therefore |r'(t)|=\sqrt{1^2+1^2+0}$

$=\sqrt{2}$

Q15. $r(t)=(\sin 3t, \cos 4t, \cos 5t)$

$t=\frac{\pi}{2}$

$r'(t)=(\cos 3t, -\sin 4t, -\sin 5t)$

$\|r'(t)\| = \sqrt{(\cos\frac{3\pi}{2})^2+(\sin 2\pi)^2+(\sin\frac{5}{2}\pi)^2}$

$=5$

KOKUYO

No.

Date

Exercise 13.4

Q1: $r(t) = (4t^2, 9t)$

$r'(t) = (8t, 9)$

$T(t) = \dfrac{r'(t)}{\|r'(t)\|}$

$= \dfrac{(8t, 9)}{\sqrt{64t^2+81}}$

$T(1) = \left(\dfrac{8}{\sqrt{145}}, \dfrac{9}{\sqrt{145}}\right)$

Q5: $r(t) = (\cos\pi t, \sin\pi t, t)$

$r'(t) = (-\pi\sin\pi t, \pi\cos\pi t, 1)$

$T(t) = \dfrac{(-\pi\sin\pi t, \pi\cos\pi t, 1)}{\sqrt{(-\pi\sin\pi t)^2+(\pi\cos\pi t)^2+1}}$

$= \dfrac{(-\pi\sin\pi t, \pi\cos\pi t, 1)}{\sqrt{\pi^2+1}}$

$T(1) = \dfrac{-\pi\sin\pi, \pi\cos\pi, 1}{\sqrt{\pi^2+1}}$

$= \left(0, \dfrac{-\pi}{\sqrt{\pi^2+1}}, \dfrac{1}{\sqrt{\pi^2+1}}\right)$

Q7. $r(t) = 1, e^t, t$

$r'(t) = (0, e^t, 1)$

$r''(t) = (0, e^t, 0)$

$r'(t) \times r''(t) = \begin{vmatrix} 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix}$

$= (-e^t)i - 0 + 0k$

$= -e^t i$

$|r'(t) \times r''(t)|$

$= \sqrt{(-e^t)^2}$

$= e^t$

$|r'(t)| = \sqrt{(e^t)^2+1}$

$= (e^{2t}+1)^{\frac{1}{2}}$

$k(t) = \dfrac{e^t}{(e^{2t}+1)^{\frac{3}{2}}}$

Q11. $r(t) = \left(\frac{1}{t}, t^{-\frac{1}{2}}, t^2\right)$

$t = -1$

$r'(t) = (-t^{-2}, -2t^{-3}, 2t)$

$r'(-1) = (-1, 2, -2)$

$|r'(t)| = 3$

$r''(t) = (2t^{-3}, 6t^{-4}, 2)$

$|r''(-1)| = (-2, 6, 2)$

$r''(-1) \times r'(-1) = \begin{vmatrix} -1, & 2, & -2 \\ -2, & 6, & 2 \end{vmatrix}$

$= 16i + \cancel{2}6j + (-2k)$

$\sqrt{16^2+6^2+4} = \sqrt{296} = 2\sqrt{74}$

$k(t) = \dfrac{2\sqrt{74}}{3^3}$

$= \dfrac{2\sqrt{74}}{27}$

Q17. $y=t^4$, $t=2$

$r(t)=(0,t^4,0)$

$r'(t)=(0,4t^3,0)$

$r'(2)=(0,32,0)$

$|r'(2)|=\sqrt{32^2}=32$

$r''(t)=(0,12t^2,0)$

$r''(2)=(0,48,0)$

$r'(2)\times r''(2)=\begin{vmatrix} 0,32,0 \\ 0,48,0 \end{vmatrix}$

Q21. $r(t)=(t-\tanh t, \operatorname{sech} t)$

$k(t)=\operatorname{sech}(t)$

$r'(t)=(1-h\operatorname{sech} t, -h\tanh t)$

$r''(t)=\cancel{(1+h^2\cosh t,}$
$(-\operatorname{sech} t+h^2 \cancel{\cos}\tan ht, \cancel{+}-\tanh t+(h^2\operatorname{sech} t))$

$r'(t)\times r''(t)=\begin{vmatrix} 1-h\operatorname{sech} t, & -h\tanh t \\ -\operatorname{sech} t+h^2\tanh t, & -\tanh t-h^2\operatorname{sech} t \end{vmatrix}$

$=(1-h\operatorname{sech} t)(-\tanh t-h^2\operatorname{sech} t)-(-\operatorname{sech} t+h^2\tan$
$\cdot(-h\tanh t)$

No.

Date

Exercise 13.5

Q3. $r(t) = (t^3, 1-t, 4t^2)$ $t=1$

$r'(t) = v(t) = (3t^2, -1, 8t)$

$= (3, -1, 8)$

$r''(t) = a(t) = (6t, 0, 8)$

$= (6, 0, 8)$

$|r'(t)| = |v(t)| = \text{speed} = \sqrt{3^2+1^2+64}$

$= \sqrt{74}$

Q5. $r(\theta) = (\sin\theta, \cos\theta, \cos 3\theta)$ $\theta = \frac{\pi}{3}$

$r'(\theta) = (\cos\theta, -\sin\theta, -3\sin 3\theta)$

$v(\theta) = (\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)$

$r''(\theta) = a(\theta) = (-\sin\theta, -\cos\theta, -9\cos 3\theta)$

$= (-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9)$

$|r(\theta)| = |v(\theta)| = \sqrt{\frac{1}{4} + \frac{3}{4} + 0}$

$= \sqrt{1} = 1$

Q15. $a(t) = (t, 4)$ $v(0) = (3, -2)$ $r(0) = (0, 0)$

$v(t) = \int a(t)\,dt$

$= \int (ti + 4j)\,dt$

$v(t) = \frac{1}{2}t^2 i + 4tj + C$

$3i - 2j = C$

$C = 3i - 2j$

$v(t) = (\frac{1}{2}t^2 + 3)i + (4t - 2)j$

$\int v(t)\,dt = r(t) = \int (\frac{1}{2}t^2 + 3)i + (4t-2)j\,dt$

$= (\frac{1}{6}t^3 + 3t)i + (2t^2 - 2t)j + C$

$r(0) = 0 = C$

$\therefore r(t) = (\frac{t^3}{6} + 3t)i + (2t^2 - 2t)j = (\frac{t^3}{6} + 3t, 2t^2 - 2t)$

Q17. $a(t) = tk$ $v(0) = i$ $r(0) = j$

$\int a(t)\,dt = v(t) = \frac{1}{2}t^2 k + C$

$i = C$

$\therefore v(t) = \frac{1}{2}t^2 k + i$

$\int v(t)\,dt = \int \frac{1}{2}t^2 k + 1i\,dt$

$r(t) = \frac{1}{6}t^3 k + ti + C$

$r(0) = \cancel{0}j = C$

$\therefore r(t) = ti + \frac{t^3}{6}k + j$

Q31.

$\because v = (12, 20, 20)$

$a = (2, 1, -3)$

$\therefore |v| = \sqrt{12^2 + 20^2 + 20^2}$

$= \sqrt{944} = 4\sqrt{59} > 0$

$|a| = \sqrt{2^2 + 1^2 + (-3)^2}$

$= \sqrt{14} > 0$

speed is always positive $|v|$.

$\therefore v = 12i + 20j + 20k$

$a = 2i + j - 3k$

so $v > 0, a < 0$

the speed is decreasing.

Campus

No.

Date

Q33. $r(t)=(t,\cos t,\sin t)$

$v(t)=r'(t)=(1,-\sin t,\cos t)$

$a(t)=r''(t)=(0,-\cos t,-\sin t)$

$$a_T=\frac{v\cdot a}{|v|}=\frac{(\sin t\cos t)+(-\sin t\cos t)}{\sqrt{1+\sin t^2+\cos t^2}}$$

$$=0$$

$$a_N=\frac{|v\times a|}{|v|}$$

$$|v\times a|=\begin{vmatrix} 1 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{vmatrix}$$

$$=(\sin t^2+\cos t^2)i$$

$$-(-\sin t)j+(-\cos t)k$$

$$a_N=\frac{\sqrt{(\sin t^2+\cos t^2)^2+\sin t^2+\cos t^2}}{\sqrt{1+\sin t^2+\cos t^2}}$$

$$=\frac{\sqrt{1+1}}{\sqrt{1+1}}=1$$

$\therefore a_N=1 \quad a_T=0$

No.

Date

Exercise 14.1

Q1. $f(x,y) = x+yx^3$

$\therefore f(2,2)=2+2^3\cdot 2=18$

$f(-1,4)=-1+(-1)^3\cdot 4$

$=-5$

Q3. $h(x,y,z)=xyz^{-2}$

$h(3,8,2)=3\cdot 8\cdot \frac{1}{2^2}$

$=24\cdot\frac{1}{4}$

$=6$

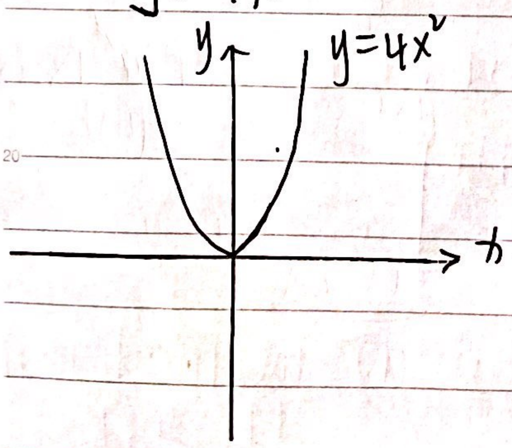
$h(3,-2,-6)=3\cdot(-2)\cdot\frac{1}{(-6)^2}$

$=-6\cdot\frac{1}{36}$

$=\frac{1}{-6}$

Q7. $f(x,y)=\ln(4x^2-y)$

$y=4x^2$



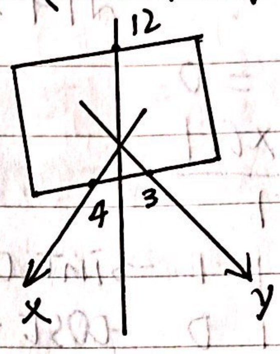
Q21. $f(x,y)=12-3x-4x$

horizontal trace

$12-c=3x+4x$ $z=c$.

vertical trace:

$z=(12-3a)-4y$ $x=a$, $y=a$.



Q23. when $x=a$

$z=a^2+4y^2$

when $y=a$

$z=x^2+4a^2$

Q33 & Q35

draw on the Maple.

Exercise 14.2

Q5. (no question on page).

Q23.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2} = \frac{0}{0}$$

∴ pick $x=t, y=t, z=t$

$$\therefore \frac{3t}{t^2+t^2+t^2} = \frac{3t}{3t^2} = \frac{1}{t}$$

$$\therefore \lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2} = \frac{1}{x}$$

$= \infty$

Q15. $f(x,y) = \dfrac{x^3+y^3}{xy^2}$

$$f(x,y) = \frac{x^3+(mx)^3}{x\cdot(mx)^2}$$

$$= \frac{x^3(m^3+1)}{x^3\cdot m}$$

$$= \frac{m^3+1}{m}$$

∴ the $f(x,y)$ is depend on the slope m.

therefore, $f(x,y)$ do not exist.

Q27. $$\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}}$$

$$= \frac{(-2)^4\cos\pi}{e^{-1}}$$

$$= \frac{-16}{e^{-1}}$$

$$= -16e$$

Q31. $$\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}$$

$$= \frac{1}{\sqrt{9+16}}$$

$$= \frac{1}{5}$$

Q21. $$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = \frac{0}{0}$$

∴ $y = mx$

$$\frac{x(mx)}{3x^2+2(mx)^2} = \frac{mx^2}{3x^2+2m^2x^2} = \frac{mx^2}{x^2(2m^2+3)}$$

$$= \frac{m}{2m^2+3}$$

∴ $\dfrac{xy}{3x^2+2y^2}$ is depend on slope m

∴ Do Not Exist.

Q35.

$$\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy)$$

$= (9(-8)+24)$

$= -48$

Campus