13.3

3. $r(t)=\langle 2t, \ln t, t^2\rangle$, $1\le t\le 4$

$r'(t)=\langle 2, \frac{1}{t}, 2t\rangle$

$|r'(t)|=\sqrt{4+\frac{1}{t^2}+4t^2}=2t+\frac{1}{t}$

$\int_1^4 |r'(t)|dt = \int_1^4 \sqrt{4+\frac{1}{t^2}+4t^2}\,dt$

$=\int_1^4 2t+\frac{1}{t}\,dt$

$=t^2+\ln t\Big|_1^4$

$=(4^2+\ln 4)-(1^2+\ln 1)$

$=16+\ln 4-1-\ln 1$

$=15+\ln 4$

9. $r(t)=\langle t^2, 2t^2, t^3\rangle$, $a=0$

$r'(t)=\langle 2t, 4t, 3t^2\rangle$

$|r'(t)|=\sqrt{4t^2+16t^2+9t^4}$

$\int_0^t |r'(t)|dt=\int_0^t\sqrt{4t^2+16t^2+9t^4}$

$=\frac{1}{27}\left((20+9t^2)^{\frac{3}{2}}-20^{\frac{3}{2}}\right)$

11. $r(t)=\langle 2t+3, 4t-3, 5-t\rangle$, $t=4$

$v(t)=r'(t)=\langle 2, 4, -1\rangle$

$|v(4)|=\sqrt{4+16+1}$

$=\sqrt{21}$

13. $r(t)=\langle t, \ln t, (\ln t)^2\rangle$ $t=1$

$v(t)=r'(t)=\langle 1, \frac{1}{t}, \frac{2}{t}\ln t\rangle$

$|v(1)|=\sqrt{1+\frac{1}{t^2}+(\frac{2}{t}\ln t)^2}$

$=\sqrt{1+1+(2\ln 1)^2}$

$=\sqrt{1+1+0}=\sqrt{2}$

15. $r(t)=\langle \sin 3t, \cos 4t, \cos 5t\rangle$, $t=\frac{\pi}{2}$

$v(t)=r'(t)=\langle 3\cos 3t, -4\sin 4t, -5\sin 5t\rangle$

$|v(\frac{\pi}{2})|=\sqrt{(3\cos 3t)^2+(4\sin 4t)^2+(5\sin 5t)^2}$

$=\sqrt{9\times 0+16\times 0+25\times 1}$

$=\sqrt{25}=5$

13.4

1. $r(t) = \langle 8t, 9 \rangle$

$|r'(t)| = \sqrt{64t^2+81}$

$T(t) = \frac{\langle 8t, 9 \rangle}{\sqrt{64t^2+81}}$

$T(1) = \langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \rangle$

5. $r(t) = \langle \cos\pi t, \sin\pi t, t \rangle$

$r'(t) = \langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle$

$|r'(t)| = \sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1}$

$T(t) = \frac{\langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle}{\sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1}}$

$T(1) = \langle 0, \frac{-\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \rangle$

7. $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$r'(t) \times r''(t) = -e^t\, i$

$|r'(t) \times r''(t)| = \sqrt{(-e^t)^2} = \sqrt{(e^t)^2} = e^t$

$|r'(t)| = \sqrt{(e^t)^2+1} = \sqrt{e^{2t}+1}$

$k(t) = \frac{e^t}{(\sqrt{e^{2t}+1})^3}$

11. $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle$, $t=1$

$r'(t) = \langle -t^{-2}, -2t^{-3}, 2t \rangle$

$r''(t) = \langle 2t^{-3}, 6t^{-4}, 2 \rangle$

$r'(t) \times r''(t) = (4t^{-3} - 12t^{-3})i - (-2t^{-2} - 4t^{-2})j + (-6t^{-6} - 4t^{-3})k$

$\cancel{|r'(t) \times r''(t)| = \sqrt{r'(t) \times r''(t)}}$

$|r'(t)| = \sqrt{(t^{-2})^2 + (2t^{-3})^2 + 4t^2}$

$|r'(t) \times r''(t)| = \sqrt{(-4t^{-3} - 12t^{-3})^2 + (2t^{-2} - 4t^{-2})^2 + (-6t^{-6} - 4t^{-2})^2}$

$k(t) = \frac{|r'(t) \times r''(t)|}{(|r'(t)|)^3} = \frac{2\sqrt{14}}{27}$

17. $y = t^4$, $t = 2$

$r'(t) = 4t^3$

$r''(t) = 12t^2$

$r'(t) \times r''(t) = 48t^5$

$|r'(t) \times r''(t)| = \sqrt{(48t^5)^2}$

$|r'(t)| = \sqrt{(4t^3)^2}$

$k(t) = 0.046875$ X

$k(2) = \frac{48\sqrt{14}}{2057}$

21. no idea

don't know how to

What's the derivate of secht.

$r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$

$k(t) = \operatorname{sech} t$

13.5

3. $r(t) = \langle t^3, 1-t, 4t^2 \rangle,\ t=1$

$v(t) = r'(t) = \langle 3t^2, -1, 8t \rangle$

$v(1) = \langle 3, -1, 8 \rangle$

$a(t) = v'(t) = \langle 6t, 0, 8 \rangle$

$a(1) = \langle 6, 0, 8 \rangle$

speed $= |v(t)| = \sqrt{9t^4 + 1 + 64t^2}$

$|v(1)| = \sqrt{9+1+64}$

$= \sqrt{74}$

5. $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$ $\theta = \frac{\pi}{3}$

$v(\theta) = r'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$v(\frac{\pi}{3}) = \langle \frac{1}{2}, \frac{\sqrt{3}}{-2}, 0 \rangle$

$a(\theta) = v'(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$a(\frac{\pi}{3}) = \langle \frac{\sqrt{3}}{-2}, -\frac{1}{2}, 9 \rangle$

Speed $|v(t)| = \sqrt{\cos^2\theta + \sin^2\theta + 9\sin^2 3\theta}$

$|v(\frac{\pi}{3})| = \sqrt{\frac{1}{4} + \frac{3}{4} + 0}$

$= \sqrt{1} = 1$

15. $a(t) = \langle t, 4 \rangle$

$v(t) = \int a(t)\,dt = \langle \frac{t^2}{2}, 4t \rangle$

$v(0) = \langle 3, -2 \rangle$

$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$r(t) = \int v(t)\,dt = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

$r(0) = \langle 0, 0 \rangle$

17. $a(t) = tk = \langle 0, 0, t \rangle$

$v(t) = \int a(t)\,dt = \langle 0, 0, \frac{t^2}{2} \rangle$

$v(0) = \langle 1, 0, 0 \rangle$

$v(t) = \langle 1, 0, \frac{t^2}{2} \rangle = i + \frac{t^2}{2}k$

$r(t) = \int v(t)\,dt = \langle t, 0, \frac{t^3}{6} \rangle$

$r(0) = j = \langle 0, 1, 0 \rangle$

$r(t) = \langle t, 1, \frac{t^3}{6} \rangle = j + \frac{t^3}{6}k + ti$

31. $v = \langle 12, 20, 20 \rangle$

$a = \langle 2, 1, -3 \rangle$

on the z plane,

v and a are not in the same direction, so the particle is ~~speeding do~~ slowing down.

33.

$r(t) = \langle t, \cos t, \sin t \rangle$

$a_T = 0$

$a_N = 1$

扫描全能王 创建

14.1

1. $f(x,y)=x+yx^3$, $(2,2)$, $(-1,4)$

$(2,2) = 2+2\times 8 = 18$

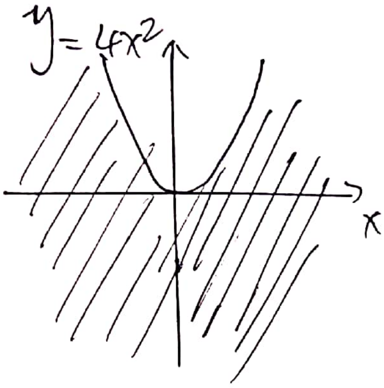
$(-1,4) = -1+4\times(-1) = -5$

3. $h(x,y,z)=xyz^{-2}$ $(3,8,2)$ $(3,-2,-6)$

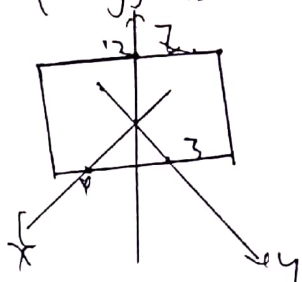
$(3,8,2) = 3\times 8\times \frac{1}{4} = 6$

$(3,-2,-6) = 3\times(-2)\times\frac{1}{36} = -\frac{1}{6}$

7. $f(x,y)=\ln(4x^2-y)$



21. $f(x,y)=12-3x-4y$

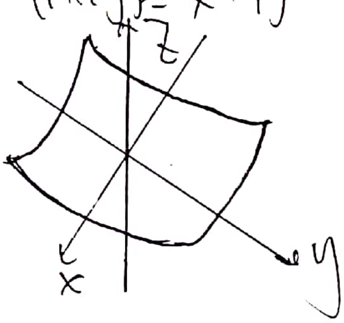


Horizontal trace: $3x+4y=12-c$ $z=c$

Vertical trace: $z=(12-3a)-4y$ $x=a$

$z=\cancel{t}-3x+(12-4a)$ $y=a$

23. $f(x,y)=x^2+4y^2$

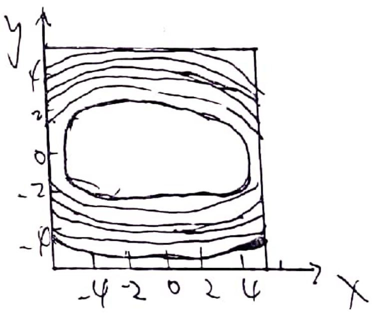


horizontal traces $c>0$

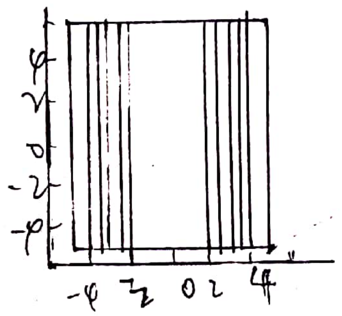
vertical traces: $z=a^2+4y^2$, $x=a$

$z=x^2+4a^2$, $y=a$

33. $f(x,y)=x^2+4y^2$



35. $f(x,y)=x^2$



14.2

9. $\lim_{(x,y)\to(2,5)} (g(x,y)-2f(x,y))$

$= 7 - 2\times 3$ $\quad \lim_{(x,y)\to(2,5)} f(x,y)=3$

$= 7-6$

$= 1$ $\quad \lim_{(x,y)\to(2,5)} g(x,y)=7$

11. $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)}$

$= e^{3^2-7}$

$= e^{9-7}$

$= e^2$

15. $\lim_{(x,y)\to(0,0)} f(x,y) = \dfrac{x^3+y^3}{xy^2}$

$= \lim_{(x,y)\to(0,0)} \dfrac{x^3+m^3x^3}{xm^2x^2}$

$= \lim_{(x,y)\to(0,0)} \dfrac{x^3(1+m^3)}{x^3m^2}$

$= \lim_{(x,y)\to(0,0)} \dfrac{1+m^3}{m^2}$

The limit is $\dfrac{1+m^3}{m^2}$

is not exist.

21. $\lim_{(x,y)\to(0,0)} \dfrac{xy}{3x^2+2y^2}$ $\quad y=cx$

plug-in $y=cx$

$\lim_{(x,y)\to(0,0)} \dfrac{x\cdot cx}{3x^2+2c^2x^2}$

$= \lim_{(x,y)\to(0,0)} \dfrac{cx^2}{x^2(3+2c^2)}$

$= \lim_{(x,y)\to(0,0)} \dfrac{c}{3+2c^2}$

the limit does not exist.

23. $\lim_{(x,y,z)\to(0,0,0)} \dfrac{x+y+z}{x^2+y^2+z^2}$

let it appoach towards origin along the x-coordinate axis $x\to 0$

$y=z=0$ $\quad \lim_{(x,y,z)\to(0,0,0)} = \dfrac{x+y+z}{x^2+y^2+z^2}$

$= \lim_{x\to 0} \dfrac{x}{x^2} = \lim_{x\to 0} \dfrac{1}{x}$

$= \infty$

27. $\lim_{(z,w)\to(-2,1)} \dfrac{z^4\cos(\pi w)}{e^{z+w}}$

$= \dfrac{2^4\cos\pi}{e^{-2+1}} = \dfrac{16\times(-1)}{e^{-1}}$

$= -16e$

31. $\lim_{(x,y)\to(3,4)} \dfrac{1}{\sqrt{x^2+y^2}}$

$= \dfrac{1}{\sqrt{9+16}}$

$= \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

35. ~~$\lim_{(x,y)\to(1,-3)} e^{x-y}\ln(x-y)$~~

$\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy)$

$= 9\cdot(-8)+4\times 6$

$= -72+24$

$= -48$