Homework

Ex 13.3

3) $r(t) = \langle 2t, \ln(t), t^2 \rangle, 1 \le t \le 4$

$r(t) = 2t\hat{i} + \ln(t)\hat{j} + t^2\hat{k}$

$r'(t) = 2\hat{i} + \frac{1}{t}\hat{j} + 2t\hat{k}$

$|r'(t)| = \sqrt{4 + \frac{1}{t^2} + 4t^2}$

$= \int_1^4 \sqrt{4t^2 + 1 + 4t^4} = \int_1^4 2t + \frac{1}{t}$

$= [t^2 + \ln(t)]_1^4$

$= 16 - 1 + \ln(4) = 15 + \ln(4)$

9) $r(t) = \langle t^2, 2t^2, t^3 \rangle, a = 0$

$r'(t) = 2t\hat{i} + 4t\hat{j} + 3t^2\hat{k}$

$= \sqrt{4t^2 + 16t^2 + 9t^4}$

$= \sqrt{20t^2 + 9t^4}$

$= t\sqrt{20 + 9t^2}$

$= \int_0^{t_1} t\sqrt{20 + 9t^2}$

$\frac{1}{2}\int_0^{t_1} \sqrt{20 + 9u}\, du.$

$\frac{1}{2}\int_0^{t_1} 9\, dv$

$20 + 9u = v$

$9\, du = dv$

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$\frac{1}{2}[9v]_0^{t_1}$

$\frac{9}{2}[20+9t^2]_0^{t_1}$

$[90+\frac{81}{2}t^2]_0^{t_1}$

$\cancel{90}+\frac{81}{2}t_1^2-\cancel{90}$

$s=\frac{81}{2}t_1^2$

$\frac{2s}{81}=t_1^2$

$t_1=\frac{\sqrt{2s}}{9}$

$r(t)=\langle t^2, 2t^2, t^3\rangle$

$=\frac{2s}{81}\hat{i}+\frac{4s}{81}\hat{j}+\frac{2s\sqrt{2s}}{729}\hat{k}$

11) $r(t)=(2t+3)\hat{i}+(4t-3)\hat{j}+(5-t)\hat{k}$

$r'(t)=5\hat{i}+1\hat{j}+4\hat{k}$

$|r'(t)|=\sqrt{25+1+16}$
$=\sqrt{46}$

13) $r(t)=\langle t, \ln(t), (\ln t)^2\rangle \quad t=1$

$r'(t)=1\hat{i}+\frac{1}{t}\hat{j}+\frac{2\ln(t)}{t}\hat{k}$

$|r'(t)|=\sqrt{1^2+\frac{1}{t^2}+\frac{4}{t^2}(\ln t)^2}$

$=\sqrt{1+1}$
$=\sqrt{2}$

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14. 15. $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle \quad t = \frac{\pi}{2}$

$r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$|r'(t)| = \sqrt{9 + 16 + 25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

Ex 13.4:

1. $r(t) = \langle 4t^2, 9t \rangle \qquad T(t)$ and $T(1)$

$r'(t) = \langle 8t, 9 \rangle$

$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{8t\hat{i} + 9\hat{j}}{\sqrt{64t^2 + 81}}$

$= \frac{8\hat{i} + 9\hat{j}}{\sqrt{145}}$

5. $r(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$r'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$|r'(t)| = \pi^2 \sin^2 \pi t + \pi^2 \cos \pi t + 1$

$= \pi^2 + 1$

$T(t) = \frac{-\pi \sin \pi t\,\hat{i} + \pi \cos \pi t\,\hat{j} + 1\hat{k}}{\pi^2 + 1}$

$T(1) = \frac{-\pi \hat{j} + 1k}{\pi^2 + 1}$

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7. $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$|r'(t)| = \sqrt{e^{2t}+1}$

$r''(t) = \langle 0, e^t, 0 \rangle$

$r'(t) \times r''(t) = -e^t i$

$|r'(t) \times r''(t)| = \sqrt{e^{2t}} = e^t$

$$K = \frac{e^t}{\sqrt{e^{2t}+1}}$$

11) $r(t) = \left\langle \frac{1}{t}, \frac{1}{t^2}, t^2 \right\rangle, t=-1$

$r'(t) = \left\langle -\frac{1}{t^2}, \frac{-2}{t^3}, 2t \right\rangle = \langle -1, 2, -2 \rangle$

$= \sqrt{1+4+4}$

$= \sqrt{9} = 3$

$r''(t) = \left\langle \frac{2}{t^3}, \frac{6}{t^4}, 2 \right\rangle$

$r'(t) \times r''(t) = \frac{-2}{t^6}(8t^3\hat{i} - 3t^4\hat{j} + k)$

$= -2(-8\hat{i} - 3\hat{j} + k)$

$= \sqrt{4+36+256}$

$= \sqrt{296}$

$= \frac{\sqrt{296}}{3} = 5.735$

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(17) ?? $y = t^4$ , $t = 2$

$y' = 4t^3 \hat{j}$

$y'' = 12t^2 \hat{j}$

$= 0$

21. $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$

$r'(t) = \langle 1 - \frac{e^x - e^{-x}}{e^x + e^{-x}}, \frac{2}{e^x + e^{-x}} \rangle$

$$\boxed{\frac{\cancel{e^x} + e^{-x} - \cancel{e^x} + e^{-x}}{e^x + e^{-x}}}$$

$$\langle \frac{2e^{-x}}{e^x + e^{-x}}, \frac{2}{e^x + e^{-x}} \rangle$$

$r''(t) =$

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Ex 13.5

(3) $r(t) = \langle t^3, 1-t, 4t^2 \rangle, \; t=1$
$r'(t) = \langle 3t^2, -1, 8t \rangle$

$r''(t) = \langle 6t, 0, 8 \rangle$

$|r'(t)| = v = \sqrt{9t^4 + 1 + 64t^2}$

$= \sqrt{9+1+64}$
$= \sqrt{74}$

$|r''(t)| = a = \sqrt{36t^2 + 64}$
$= \sqrt{100} = 10$

(5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle, \; \theta = \frac{\pi}{3}$

$r'(\theta) = \langle \cos(\theta), -\sin(\theta), -3\sin(3\theta) \rangle$

$r''(\theta) = \langle -\sin\theta, -\cos(\theta), -9\cos(3\theta) \rangle$

$v = |r'(\theta)| = \sqrt{1 + 9\sin^2(3\theta)}$
$= 1$

$acc = |r''(\theta)| = \sqrt{1 + 81\cos^2(3\theta)}$

$= \sqrt{82}$

(15) $a(t) = t\hat{i} + 4\hat{j}$
$v(t) = \frac{t^2}{2}\hat{i} + 4t\hat{j} + C$

$v(0) = C$
$\therefore C = 3\hat{i} - 2\hat{j}$

$v(t) = \left(\frac{t^2}{2} + 3\right)\hat{i} + (4t - 2)\hat{j}$

$r(t) = \left(\frac{t^3}{6} + 3t\right)\hat{i} + (2t^2 - 2t)\hat{j}$

17. $a(t) = t\hat{k}$

$v(t) = \frac{t^2}{2}\hat{k} + 1\hat{i}$

$r(t) = t\hat{i} + \frac{t^3}{6}\hat{k} + \hat{i}$

$r(t) = (t+1)\hat{i} + \frac{t^3}{6}\hat{k}$

31. $v = \langle 12, 20, 20 \rangle$

$a = \langle 2, 1, -3 \rangle$

since v and a dont depend on t, it is constant

Ex 14.1

(1) $f(x,y) = x + yx^3$ , $(2,2), (-1,4)$

$f(2,2) = 2 + 2^4$

$= 18$

$f(-1,4) = -1 - 4$

$= -5$

(3) $h(x,y,z) = xyz^{-2}$

$= \frac{xy}{z^2}$

$h(3,8,2) = \frac{3 \times 8^2}{4} = 6$

$h(3,-2,-6) = \frac{3 \times -2}{36} = -\frac{1}{6}$

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(7) $f(x,y) = \ln(4x^2 - y)$
$(1,4), (-1,4)$

$(4,16) \quad (-4,16)$

$(-x, 4x) \; (x, 4x) \; f(x,y)$
$(0,0)$

(21) $f(x,y) = 12 - 3x - 4y$
$f(x,y) = x^2 + 4y^2$

ex 14.2

(9) $\lim_{(x,y)\to(3,5)} = g(x,y) - 2(f(x,y))$
$= 7 - 2(3)$
$= 1$

(11) $\lim_{x,y\to(2,5)} e^{9-7} = e^2$

(15) $f(x,y) = \dfrac{x^3+y^3}{xy^2}$

$\lim_{x\to0} \dfrac{x^3+m^3x^3}{x(m^2x)^3}$

$\lim_{x\to0} \dfrac{\cancel{x^3}(1+m^3)}{m^2\cancel{x^3}}$

$= \dfrac{1+m^3}{m^2}$

$\therefore$ it depends on m, the limit doesnt exist

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(21) $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$

Let $y = cx$

$\lim_{x\to 0} \frac{cx^2}{3x^2+2c^2x^2}$

$\lim_{x\to 0} \frac{c}{3+2c^2} = \frac{c}{3+2c^2}$, the limit does not exit.

(23) $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$

$= \frac{1.5}{0.75}$

$= 2$

At $(0.1, 0.1, 0.1) = \frac{0.3}{0.03}$

$= 10$

At $(0.01, 0.01, 0.01) = 100$

(27) $\lim_{(z,w)\to(-2,1)} \frac{z^4 \cos(\pi w)}{e^{z+w}}$

At $(-2,1) =$ $16(-1)e$

$= -16e$

(31) $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}$

$= \frac{1}{5}$

(35) $\lim_{(x,y)\to(3,-2)} (x^2y^3 + 4xy)$

$9(-8) + 4(6)$

$= -48$