HW due 9/27/20

13.3: 3, 9, 11, 13, 15

13.4: 1, 5, 7, 11, 17, 21

13.5: 3, 5, 15, 17, 31

14.1: 1, 3, 7, 21, 23, 33, 35

14.2: 9, 11, 15, 21, 23, 27, 31, 35

13.3

3. $\vec{r}(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \le t \le 4$

$\vec{r}\,'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$\|\vec{r}\,'(t)\| = \sqrt{4 + \frac{1}{t^2} + 4t^2}$

$L = \int_1^4 \sqrt{4 + \frac{1}{t^2} + 4t^2}\, dt$

$= \int_1^4 \sqrt{(2t + \frac{1}{t})^2}\, dt = \int_1^4 2t + \frac{1}{t}\, dt$

$= t^2 + \ln t \Big|_1^4 = (4^2 + \ln 4) - (1^2 + \ln 1)$

$= \boxed{15 + \ln 4}$

9. $r'(u) = \frac{d}{du} \langle u^2, 2u^2, u^3 \rangle$

$= \langle 2u, 4u, 3u^2 \rangle$

$\|r'(u)\| = \sqrt{4u^2 + 16u^2 + 9u^4} = \sqrt{20u^2 + 9u^4}$

$= \sqrt{u^2(20 + 9u^2)} = u\sqrt{20 + 9u^2}$

$\int \|r'(u)\|\, du = \frac{1}{18} \int 18u \sqrt{20 + 9u^2}\, du$

$= \frac{1}{18} \int \sqrt{k}\, dk$

$= \frac{1}{18}\left(\frac{2}{3} k^{\frac{3}{2}}\right) = \frac{1}{27} k^{\frac{3}{2}}$

$= \frac{1}{27}(20 + 9u^2)^{\frac{3}{2}} \Big|_0^t$

$= \boxed{\frac{1}{27}\left((20 + 9t^2)^{\frac{3}{2}} - 20^{\frac{3}{2}}\right)}$

11. $\vec{r}(t) = \langle 2t + 3, 4t - 3, 5 - t \rangle \quad t = 4$

$\vec{r}\,'(t) = \langle 2, 4, -1 \rangle$

$\vec{r}\,'(4) = \langle 2, 4, -1 \rangle$

$\|\vec{r}\,'(4)\| = \sqrt{2^2 + 4^2 + (1)^2} = \boxed{\sqrt{21}}$ speed

13. $\vec{r}(t) = \langle t, \ln t, (\ln t)^2 \rangle \quad t = 1$

$\vec{r}\,'(t) = \langle 1, \frac{1}{t}, \frac{2}{t}(\ln t) \rangle$

$\vec{r}\,'(1) = \langle 1, 1, 2\ln 1 \rangle = \langle 1, 1, 0 \rangle$

$\|\vec{r}\,'(1)\| = \sqrt{1^2 + 1^2 + 0^2} = \boxed{\sqrt{2}}$ speed

15. $\vec{r}(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle \quad t = \frac{\pi}{2}$

$\vec{r}\,'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$\vec{r}\,'(\frac{\pi}{2}) = \langle 3\cos\frac{3\pi}{2}, -4\sin 2\pi, -5\sin\frac{5\pi}{2} \rangle$

$= \langle 0, 0, -5 \rangle$

$\|\vec{r}\,'(\frac{\pi}{2})\| = \sqrt{0^2 + 0^2 + (-5)^2} = \boxed{5}$: speed

13.4

1. $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$

$T(t) = \frac{r'(t)}{\|r'(t)\|} \quad \|r'(t)\| = \sqrt{64t^2 + 81}$

$T(t) = \frac{\langle 8t, 9 \rangle}{\sqrt{64t^2 + 81}}$

$T(1) = \left\langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \right\rangle$

5. $\vec{r}(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$\vec{r}\,'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$\|r'(t)\| = \sqrt{\pi^2(\sin^2 \pi t + \cos^2 \pi t) + 1} = \sqrt{\pi^2 + 1}$

$T(t) = \frac{\langle -\pi \sin \pi t, \pi \cos \pi t, t \rangle}{\sqrt{\pi^2 + 1}}$

$T(1) = \left\langle 0, \frac{-\pi}{\sqrt{\pi^2 + 1}}, \frac{1}{\sqrt{\pi^2 + 1}} \right\rangle$

7. $\vec{r}(t) = \langle 1, e^t, t \rangle$

$\vec{r}\,'(t) = \langle 0, e^t, 1 \rangle$

$\vec{r}\,''(t) = \langle 0, e^t, 0 \rangle$

$K(t) = \frac{\|\vec{r}\,'(t) \times \vec{r}\,''(t)\|}{\|\vec{r}\,'(t)\|^3} = \frac{\|\langle 0, e^t, 1 \rangle \times \langle 0, e^t, 0 \rangle\|}{\|\langle 0, e^t, 1 \rangle\|^3}$

$\langle 0, e^t, 1 \rangle \times \langle 0, e^t, 0 \rangle = 1(-e^t) + 0 + 0$

$K(t) = \frac{\|\langle -e^t, 0, 0 \rangle\|}{(\sqrt{e^{2t} + 1})^3} = \frac{\sqrt{e^{2t}}}{(1 + e^{2t})^{3/2}} = \boxed{\frac{e^t}{(1 + e^{2t})^{3/2}}}$

11. $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle \quad t = -1$

$r'(t) = \langle -\frac{1}{t^2}, \frac{-2}{t^3}, 2t \rangle$

$r'(-1) = \langle -1, 2, -2 \rangle$

$\|r'(-1)\| = \sqrt{1+4+4} = \sqrt{9} = 3$

$r''(t) = \langle \frac{2}{t^3}, \frac{6}{t^4}, 2 \rangle$

$r''(-1) = \langle -2, 6, 2 \rangle$

$$r'(-1) \times r''(-1) = \begin{vmatrix} i & j & k \\ -1 & 2 & -2 \\ -2 & 6 & 2 \end{vmatrix}$$

$= (4-(-12))i - ((-2)-4)j + (-6-(-4))k$

$= 16i + 6j - 2k$

$\|r'(-1) \times r''(-1)\| = \sqrt{16^2 + 6^2 + (-2)^2} = \sqrt{296}$

$K(-1) = \frac{\sqrt{296}}{3^3} = \frac{\sqrt{296}}{27} = \boxed{\frac{2\sqrt{74}}{27}}$

17. $y = t^4 \quad t = 2$

$y'(t) = 4t^3 \quad y''(t) = 12t^2$

$K = \frac{|y''(t)|}{(1+[y'(t)]^2)^{\frac{3}{2}}} = \frac{|12t^2|}{(1+16t^6)^{\frac{3}{2}}}$

$K(2) = \frac{48}{1025^{3/2}} \approx \boxed{0.0015}$

13.5

3. $\vec{r}(t) = \langle t^3, 1-t, 4t^2 \rangle \quad t = 1$

$\vec{r}'(t) = \langle 3t^2, -1, 8t \rangle$

$\vec{r}'(1) = \langle 3, -1, 8 \rangle$

$\vec{r}''(t) = \langle 6t, 0, 8 \rangle$

$\vec{r}''(1) = \langle 6, 0, 8 \rangle$

$v(1) = \|\vec{v}(1)\| = \sqrt{3^2 + (-1)^2 + 8^2} = \sqrt{74}$

5. $\vec{r}(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle \quad \theta = \frac{\pi}{3}$

$\vec{r}'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$\vec{r}'(\frac{\pi}{3}) = \langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \rangle$

$\vec{r}''(\theta) = \langle -\sin\theta, \cos\theta, -9\cos 3\theta \rangle$

$\vec{r}''(\frac{\pi}{3}) = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 9 \rangle$

$v(\frac{\pi}{3}) = \|\vec{v}(\frac{\pi}{3})\| = \sqrt{(\frac{1}{2})^2 + (-\frac{\sqrt{3}}{2})^2 + 0^2} = \boxed{1}$

15. $\vec{a}(t) = \langle t, 4 \rangle \quad \vec{v}(0) = \langle 3, -2 \rangle \quad \vec{r}(0) = \langle 0, 0 \rangle$

$\vec{v}(t) = \int_0^t \vec{a}(u)\,du = \int_0^t \langle u, 4 \rangle\,du = \langle \frac{t^2}{2}, 4t \rangle + \vec{v}(0)$

$= \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$\vec{r}(t) = \int_0^t \vec{v}(u)\,du = \langle \frac{1}{6}t^3 + 3t, 2t^2 - 2t \rangle + \vec{r}(0)$

$= \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

17. $\vec{a}(t) = t\hat{k} \quad \vec{v}(0) = \hat{i} \quad \vec{r}(0) = \hat{j}$

$\vec{v}(t) = \int_0^t \vec{a}(u)\,du = \int_0^t u\hat{k}\,du = \frac{t^2}{2}\hat{k} + \vec{u}(0)$

$\vec{v}(t) = \frac{t^2}{2}\hat{k} + \hat{i}$

$\vec{r}(t) = \int_0^t \vec{v}(u)\,du = \int_0^t (\hat{i} + \frac{1}{2}u^2\hat{k})\,du$

$= t\hat{i} + \hat{j} + \frac{1}{6}t^3\hat{k}$

31. speed is slowing down

14.1

1. $f(x,y) = x + yx^3 \quad (2,2), (-1,4)$

$f(2,2) = 2 + (2)(2)^3 = 2 + 16 = 18$

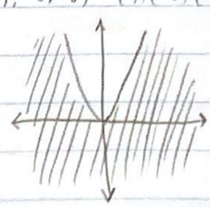
$f(-1,4) = -1 + 4(-1)^3 = -5$

3. $h(x,y,z) = xyz^{-2} \quad (3,8,2), (3,-2,-6)$

$h(3,8,2) = (3)(8)(2)^{-2} = 6$

$h(3,-2,-6) = (3)(-2)(-6)^{-2} = -\frac{1}{6}$

7.



14.2

9. $\lim_{(x,y)\to(2,5)} (g(x,y) - 2f(x,y))$

$= \lim_{(x,y)\to(2,5)} (g(x,y)) - 2\lim_{(x,y)\to(2,5)} (f(x,y))$

$= 7 - 2(3) = \boxed{1}$

11. $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)} = e^{(3)^2} \cdot e^{-7} = e^9 \cdot e^{-7} = \boxed{e^2}$