Sai Embar
Section 23

13.3 - 14.2 (Sept 27th)

13.3: #3, 9, 11, 13, 15

13.4: #1, 5, 7, 11, 17, 21

13.5: #3, 5, 15, 17, 31, 33

14.1: #1, 3, 7, 21, 23, 33, 35

14.2: #9, 11, 15, 21, 23, 27, 31, 35

13.3: # 3, 9, 11, 13, 15

3) $\frac{dx}{dt} = 2$, $\frac{dy}{dt} = \frac{1}{t}$, $\frac{dz}{dt} = 2t$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$|r'(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

$= \sqrt{2^2 + \left(\frac{1}{t}\right)^2 + (2t)^2}$

$= \sqrt{\frac{(2t^2+1)^2}{t}}$

$= \frac{2t^2+1}{t} = 2t + \frac{1}{t}$

$L = \int_1^4 \left(2t + \frac{1}{t}\right) dt$

$= \left[t^2 + \ln(t)\right]_1^4 = \left[4^2 + \ln(4)\right] - \left[1^2 + \ln(1)\right]$

$= 16 + \ln(4) - 1 = \boxed{15 + \ln(4)}$

9) $r(u) = \langle u^2, 2u^2, u^3 \rangle$

$r'(u) = \langle 2u, 4u, 3u^2 \rangle$

$\|r'(u)\| = \sqrt{(2u)^2 + (4u)^2 + (3u^2)^2} = \sqrt{4u^2 + 16u^2 + 9u^4} = u\sqrt{9u^2 + 20}$

$s(t) = \int_0^t \|r'(u)\| du = \int_0^t u\sqrt{9u^2+20}\ du$

let $9u^2 + 20 = k^2$ | $u = 0 \to k = \sqrt{20}$

$18u\,du = 2k\,dk$ | $u = t \to k = \sqrt{9t^2+20}$

$u\,du = \frac{k}{9} dk$

$s(t) = \int_{\sqrt{20}}^{\sqrt{9t^2+20}} \frac{1}{9}(k^2)\,dk$

$= \boxed{\frac{1}{27}\left[(9t^2+20)^{3/2} - (20)^{3/2}\right]}$

11) $\frac{dr}{dt} = \frac{d}{dt}\left[(2t+3)i + (4t-3)j + (5-t)k\right]$

$= 2i + 4j - 1k$

$\left|\frac{dr}{dt}\right| = \sqrt{2^2+4^2+5^2} = \sqrt{4+16+1^2} = \boxed{\sqrt{21}}$

13) $\frac{dr}{dt} = \frac{d}{dt}\left[(t)i + (\ln(t))j + ((\ln t)^2)k\right]$

$= \langle 1, \frac{1}{t}, \frac{2\ln t}{t} \rangle$

$\left|\frac{dr}{dt}\right| = \sqrt{1^2 + (\frac{1}{t})^2 + (\frac{2\ln(t)}{t})^2}$

$\text{speed}_{t=1} = \sqrt{1^2 + (\frac{1}{1})^2 + (\frac{2\ln 1}{1})^2}$

$= \sqrt{1+1+0^2} = \sqrt{2}$ $\boxed{\text{Speed} = \sqrt{2} \text{ unit}}$

15) $\frac{dr}{dt} = \frac{d}{dt}\left[(\sin(3t))i + (\cos(4t))j + (\cos(5t))k\right]$

$= \langle 3\cos(3t) - 4\sin(4t) - 5\sin(5t) \rangle$

$\|v'(t)\| = \sqrt{(3\cos(3t))^2 + (-4\sin(4t)^2)^2 + (-5\sin(5t))^2}$

$= \sqrt{9\cos^2(3t) + 16\sin^2(4t) + 25\sin^2(5t)}$

$t = \pi/2$, $\sqrt{9(0)^2 + 16(0)^2 + 25(1)^2} = \sqrt{0+0+25} = \sqrt{25} = \boxed{5 = \text{speed}}$

13.4: # 1, 5, 7, 11, 17, 21

1) $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \frac{d}{dt} r(t) = \langle 8t, 9 \rangle$

$|r'(t)| = \sqrt{64t^2+81}$

$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{1}{\sqrt{64t^2+81}} \langle 8t, 9 \rangle$,

$T(1) = \langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \rangle$

5) $r'(t) = \langle -\pi\sin(\pi t), \pi\cos(\pi t), 1 \rangle$

$|r'(t)| = \sqrt{(-\pi\sin(\pi t))^2 + (\pi\cos(\pi t))^2 + (1)^2}$

$= \sqrt{\pi^2\sin^2(\pi t) + \pi^2\cos^2(\pi t) + 1} = \sqrt{\pi^2+1}$

$T(t) = \frac{1}{\sqrt{\pi^2+1}}(-\pi\sin\pi t, \pi\cos\pi t, 1)$, $T(1) = \langle 0, \frac{-\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \rangle$

7) $k = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

$r(t) = \langle 1, e^t, t \rangle = 1\hat{i} + e^t j + tk$

$r'(t) = 0i + e^t j + tk$, $r''(t) = 0i + e^t j + 0\hat{k}$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix}$$

$$= i(-e^t) = -1e^t\,\hat{i} + 0\hat{j} + 0\hat{k}$$

$$|r'(t) * r''(t)| = \sqrt{(-1e^t)^2} = e^t$$

$$|r'(t)| = \sqrt{(e^t)^2 + 1y} = (e^{2t} + y)^{1/2}$$

$$\boxed{k = \frac{1e^t}{(e^{2t}+y)^{3/2}}}$$

11) $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle, t = -1$

$r'(t) = \langle -t^{-2}, -2t^{-3}, 2t \rangle$

$r''(t) = \langle 2t^{-3}, 6t^{-4}, 2 \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -t^{-2} & -2t^{-3} & 2t \\ 2t^{-3} & 6t^{-4} & 2 \end{vmatrix}$$

$$\boxed{k(t) = \frac{2\sqrt{74}}{27}}$$

17. $y = t^4, t = 2$ $\quad y' = 4t^3$

$y'' = 12t^2$

$$k(t) = \frac{12t^2}{(1+16t^6)^{3/2}}$$

$$\boxed{k(t) = \frac{48}{(1025)^{3/2}}}$$

21) $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$ $\quad k(t) = \operatorname{sech}(t)$

$r'(t) = \langle 1 - \operatorname{sech}^2 t, -\operatorname{sech} t \tanh t \rangle$

$r''(t) = \langle 2\tanh t \operatorname{sech}^2 t, \operatorname{sech} t(1 - 2\operatorname{sech}^2 t) \rangle$

$$k(t) = \frac{\tanh^2 t \operatorname{sech} t}{(\sqrt{\tanh^4 t + \operatorname{sech}^2 t \tanh^2 t})^3} \quad k(t) = \frac{\tanh^2 t \operatorname{sech} t}{(\tanh t)^3} \quad \boxed{\operatorname{csch} t}$$

13.5: # 3, 5, 5, 17, 31, 33

3) $v(t) = \langle 3t^2, -1, 8t \rangle$ $v(1) = \langle 3, -1, 8 \rangle$

$a(t) = \langle 6t, 0, 8 \rangle$ $a(1) = \langle 6, 0, 8 \rangle$

5) $v(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$. $v(\pi/3) = \langle \frac{1}{2}, -\sqrt{3}/2, 0 \rangle$

$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$ $a(\pi/3) = \langle -\sqrt{3}/2, -1/2, 9 \rangle$

15) $\int \langle t, 4 \rangle = v(t) = \langle \frac{t^2}{2}, 4t \rangle t$

$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$r(t) = \int v(t) = \langle t^3/6 + 3t, 2t^2 - 2t \rangle$

17) $a(t) = tk$ $v(0) = i$ $r(t) = j$

$v(t) = \frac{t^2}{2} k + v_0$ $v_0 = i$ $v(t) = \frac{t^2}{2} k + i$

$r(t) = ti + j + t^{3/6} k$

31) $(\|v\|^2)' = (v \cdot v)' = 2v' \cdot v = 2a v = 2\langle 7, 1, -3 \rangle \cdot \langle 17, 20, 20 \rangle$

$= 2(24 + 20 - 60) = -32$

Decreasing

33) $r(t) = \langle t, \cos t, \sin t \rangle$

$a_T = a \cdot T = \frac{a \cdot u}{\|u\|}$, $a_n = a \cdot N = \sqrt{\|a\|^2 - |a_T|^2}$

$v(t) = r'(t) = \langle 1, -\sin(t), \cos(t) \rangle$

$a(t) = v'(t) = \langle 0, -\cos(t), -\sin(t) \rangle$

$a \cdot v = 0 + \sin t \cos t - \sin t \cos t = 0$

$\|v\| = \sqrt{1^2 + \sin^2(t) + \cos^2(t)} = \sqrt{2}$ $a_T = 0$

$\|a\|^2 = \left(\sqrt{0 + \cos^2 t + \sin^2 t}\right)^2 = 1$

$a_N = \sqrt{1 - 0} = 1$

$\boxed{a_N = 1}$

14.1: # 1, 3, 7, 21, 23, 33, 35, 37

1) $f(x,y) = x + yx^3$ $(2,2), (-1,4)$

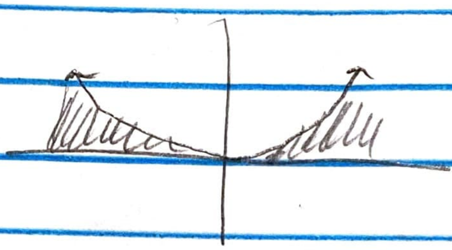
$f(2,2) = 18$

$f(-1,4) = -5$

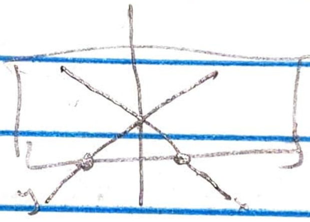
3) $h(x,y,z) = xyz^{-2}$ $(3,8,2), (3,-2,-6)$ $h(3,8,2) = 6$

$h(3,-2,-6) = -1/6$

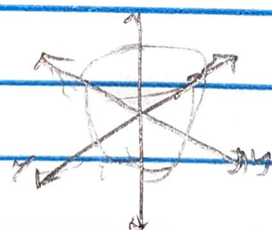
7) $f(x,y) = \ln(4x^2 - y)$



21) $f(x,y) = 12 - 3x - 4y$

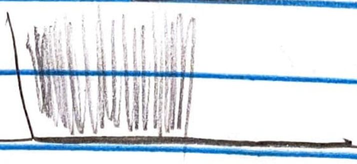


23) $f(x,y) = x^2 + 4y^2$



33) $f(x,y) = x^2 + y^2$

35) $f(x,y) = x^2$



14.2: # 9, 11, 15, 21, 23, 27, 31, 35

9) $\lim_{(x,y)\to(2,5)} (g(x,y) - 2f(x,y)) = 7 - 2(3) = 1$

11) $e^{f(x,y)^2 - g(x,y)} = e^{9-2} = e^7$

(5) $f(x,y) = \frac{x^3+y^3}{xy^2}$ $\lim_{x \to 0} \frac{x^3+m^3x^3}{m^2x^3} = \frac{1+m^3}{m^2}$, DNE

21) $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$ $\lim_{x\to 0} \frac{mx^2}{3p^2+2m^2x^2} = \frac{m}{3+2m^2}$ DNE

23) $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$

$\lim_{x\to 0} \frac{x}{x^2}$ $\lim_{x\to 0} \frac{1}{x}$ $= \boxed{\text{DNE}}$

27) $\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \boxed{-16e}$

31) $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{9+16}} = \boxed{\frac{1}{5}}$

35. $\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy) = 9\cdot 8+24 = 72+24 = \boxed{96}$