①

home works

13.3

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section22
9.27.2020.

3. $r(t) = \langle 2t, \ln t, t^2 \rangle$, $1 \le t \le 4$.

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

length: $\int_1^4 |r'(t)|\, dt$

$|r'(t)| = \sqrt{4 + \frac{1}{t^2} + 4t^2} = \sqrt{\frac{4t^2+1+4t^4}{t^2}} = \sqrt{\frac{(2t^2+1)^2}{t^2}} = \frac{2t^2+1}{t} = 2t + \frac{1}{t}$

~~$L = \int_1^4 (4 + \frac{1}{t^2} + 4t^2)^{\frac{1}{2}} dt$~~

$L = \int_1^4 \left(\frac{1}{t} + 2t\right) dt$

$= \ln t + t^2 \Big|_1^4$

$= \ln 4 + 16 - \ln 1 - 1$

$= \ln 4 + 15 - 0$

$= \ln 4 + 15$.

$\boxed{L = \ln 4 + 15.}$

9. $r(t) = \langle t^2, 2t^2, t^3 \rangle$, $a = 0$ ∴ $s(t) = \int_a^t \| r'(u) \| du$.

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$ ∴

$|r'(t)| = \sqrt{4t^2 + 16t^2 + 9t^4} = \sqrt{9t^4 + 20t^2} =$ ~~$\sqrt{t^2(9t^2+20)}$~~ $(9t^4 + 20t^2)^{\frac{1}{2}}$

$s(t) = \int_0^t (9t^4 + 20t^2)^{\frac{1}{2}} dt$

$= \int_0^t t(9t^2 + 20)^{\frac{1}{2}} dt$

$u = 9t^2 + 20$ $\quad = \int_0^t \frac{1}{18} u^{\frac{1}{2}} du$

$du = 18t\, dt$ $\quad = \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_0^t$

$\frac{1}{18} du = t\, dt$ $\quad = \frac{1}{27} u^{\frac{3}{2}} \Big|_0^t$

$= \frac{1}{27} (9t^2 + 20)^{\frac{3}{2}} \Big|_0^t = \frac{1}{27} [(9t^2+20)^{\frac{3}{2}} - 20^{\frac{3}{2}}]$.

$\boxed{s(t) = \frac{1}{27} [(9t^2+20)^{\frac{3}{2}} - 20^{\frac{3}{2}}].}$

11. $r(t) = \langle 2t+3, 4t-3, 5-t \rangle$, $t=4$ ②

$v(t) = r'(t) = \langle 2, 4, -1 \rangle$

$v(4) = \sqrt{2^2+4^2+(-1)^2} = \sqrt{4+16+1} = \sqrt{21}$

speed equal $\sqrt{21}$ when $t=4$.

13. $r(t) = \langle t, \ln t, (\ln t)^2 \rangle$, $t=1$

$v(t) = r'(t) = \langle 1, \frac{1}{t}, 2\ln t \cdot \frac{1}{t} \rangle$

$v(1) = \langle 1, 1, 2\ln 1 \cdot 1 \rangle$

$= \langle 1, 1, 0 \rangle$

$|v(1)| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

when $t=1$, speed is $\sqrt{2}$.

15. $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle$, $t = \frac{\pi}{2}$.

$v(t) = r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$t=\frac{\pi}{2}$:

$v(5) = \langle 3\cos\frac{3}{2}\pi, -4\sin 2\pi, -5\sin\frac{5}{2}\pi \rangle$

$= \langle 0, 0, -5 \rangle$

$v(5) = \sqrt{(-5)^2} = 5$

when $t = \frac{\pi}{2}$, speed $= 5$.

homeworks
13.4

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section 22

1. $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$

$T(t) = \frac{\langle 8t, 9 \rangle}{\sqrt{64t^2+81}}$

$T(1) = \frac{\langle 8, 9 \rangle}{\sqrt{145}}$

5. $r(t) = \langle \cos\pi t, \sin\pi t, t \rangle$

$r'(t) = \langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle$

$T(t) = \frac{\langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle}{\sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1}} = \frac{1}{\sqrt{\pi^2+1}} \langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle$

$T(1) = \frac{\langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle}{\sqrt{0 + \pi^2 + 1}}$

$= \frac{\langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle}{\sqrt{\pi^2+1}}$

$= \frac{\langle 0, \cancel{\pi\cos\pi} -\pi, 1 \rangle}{\sqrt{\pi^2+1}}$

7. $r(t) = \langle 1, e^t, t \rangle$ Calculate curvature function $k(t)$.

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$r'(t) \times r''(t) = (0 - e^t)i - (0-0)j + (0-0)k$

$\begin{vmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix} = -e^t i$

$|r'(t) \times r''(t)| = e^t$

$|r'(t)| = \sqrt{0^2 + e^{2t} + 1} = \sqrt{e^{2t}+1}$

$k(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

$= \frac{e^t}{(\sqrt{e^{2t}+1})^3}$

11、 $r(t)=\langle \frac{1}{t}, \frac{1}{t^2}, t^2\rangle$, $t=-1$. evaluate the curvature at the given point.

$r'(t)=\langle -t^{-2}, -2t^{-3}, 2t\rangle = \langle -1, +2, -2\rangle$

$r''(t)=\langle 2t^{-3}, 6t^{-4}, 2\rangle = \langle -2, 6, 2\rangle$

$r'(t)\times r''(t) = (4-(-12))i-(-2-4)j+(-6-(-4))k$

$\begin{vmatrix} i & j & k \\ -1 & 2 & -2 \\ -2 & 6 & 2 \end{vmatrix} = 16i+6j-2k$

$|r'(t)\times r''(t)| = \sqrt{16^2+6^2+(-2)^2} = 2\sqrt{74}$

$|r'(t)| = \sqrt{(-1)^2+2^2+(-2)^2} = \sqrt{9} = 3$

$k(t) = \frac{|r'(t)\times r''(t)|}{|r'(t)|^3} = \frac{2\sqrt{74}}{27}$

17、 $y=t^4$, $t=2$

$y'=4t^3$ $\langle 2, 32\rangle$

$y''=12t^2$ $\langle 2, 48\rangle$

~~$k=\frac{y'\times y''}{}$~~

$y'\times y'' = 32$

$\begin{vmatrix} i & j \\ 2 & 32 \\ 2 & 48 \end{vmatrix}$

$y'=\sqrt{4+32^2} = 2\sqrt{257}$

$k(t) = \frac{\cancel{32}\,4}{\cancel{8}(\sqrt{257})^3} = \frac{4}{(\sqrt{257})^3}$

~~$k(t)$~~ $=k(2)\approx 0.0015$.

21、 show that the tractrix $r(t)=\langle t-\tanh t, \operatorname{sech} t\rangle$ has the curvature function $k(t)=\operatorname{sech} t$.

~~$r'(t)=\langle 1-\operatorname{sech} t, \operatorname{sech} t \tanh t$~~

①

homeworks 13.5

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section 22.

3. $r(t)=\langle t^3, 1-t, 4t^2\rangle$, $t=1$

$v(t)=r'(t)=\langle 3t^2, -1, 8t\rangle$

$a(t)=v'(t)=r''(t)=\langle 6t, 0, 8\rangle = \langle 6,0,8\rangle$

$v(1)=\langle 3,-1,8\rangle$

speed $=\sqrt{9+1+64}=\sqrt{74}$

5. $r(\theta)=\langle \sin\theta, \cos\theta, \cos 3\theta\rangle$, $\theta=\frac{\pi}{3}$

$v(\theta)=r'(\theta)=\langle \cos\theta, -\sin\theta, -3\sin 3\theta\rangle$

$a(\theta)=v'(\theta)=r''(\theta)=\langle -\sin\theta, -\cos\theta, -9\cos 3\theta\rangle = \langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9\rangle$

$v(\frac{\pi}{3})=\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\rangle$

speed $=\sqrt{\frac{1}{4}+\frac{3}{4}+0}=\sqrt{1}=1$

15. $a(t)=\langle t,4\rangle$ $v(0)=\langle 3,-2\rangle$ $r(0)=\langle 0,0\rangle$

$a(t)=t\mathbf{i}+4\mathbf{j}$

$v(t)=\int a(t)=\frac{1}{2}t^2\mathbf{i}+4t\mathbf{j}+C$

$v(0)=0+0+C=3\mathbf{i}-2\mathbf{j}$

$v(t)=\frac{1}{2}t^2\mathbf{i}+4t\mathbf{j}+3\mathbf{i}-2\mathbf{j}$

$=(\frac{1}{2}t^2+3)\mathbf{i}+(4t-2)\mathbf{j}$

$r(t)=\int v(t)=(\frac{1}{6}t^3+3t)\mathbf{i}+(2t^2-2t)\mathbf{j}+C$

$r(0)=0\mathbf{i}+0\mathbf{j}+C=0\mathbf{i}+0\mathbf{j}$

$r(t)=(\frac{1}{6}t^3+3t)\mathbf{i}+(2t^2-2t)\mathbf{j}$

17、 $a(t)=tk$, $v(0)=i$, $r(0)=j$ 2

$v(t)=\int a(t)=\frac{1}{2}t^2k+C$

$v(0)=C=i$

$v(t)=i+\frac{1}{2}t^2k$

$r(t)=\int v(t)=ti+\frac{1}{6}t^3k+C$

$r(0)=C=j$

$r(t)=ti+j+\frac{1}{6}t^3k$

31、 $r=\langle 12,20,20\rangle$ $a=\langle 2,1,-3\rangle$

In i and j direction, the particle speeding up.

In k direction, the particle speeding down.

The speed is decreasing

$r(t)=\langle t,\cos t,\sin t\rangle$

33、 $a_N=\frac{|r'\times r''|}{|r'|}=\frac{\sqrt{2}}{\sqrt{2}}=1$

$a_T=\frac{r'\cdot r''}{|r'|}=\frac{0}{\sqrt{2}}=0$

$r'(t)=\langle 1,-\sin t,\cos t\rangle$ $r'\cdot r''=\sin t\cos t-\sin t\cos t$
$=0$

$r''(t)=\langle 0,-\cos t,-\sin t\rangle$

$r'\times r''=\cancel{(\sin^2 t)i}(\sin^2t+\cos^2t)i-(-\sin t\cdot 0)j+(-\cos t-0)k$

$=i+\sin t\,j-\cos t\,k$

$|r'\times r''|=\sqrt{1+\sin^2t+\cos^2t}=\sqrt{2}$

$|r'(t)|=\sqrt{1+\sin^2t+\cos^2t}=\sqrt{2}$

①

homeworks

14.1

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section 22.

1. $f(x,y) = \cancel{x} + yx^3$, $(2,2)$, $(-1,4)$

$f(2,2) = 2+16 = 18$

$f(-1,4) = -1+4\times(-1)$

$= -1-4$

$= -5.$

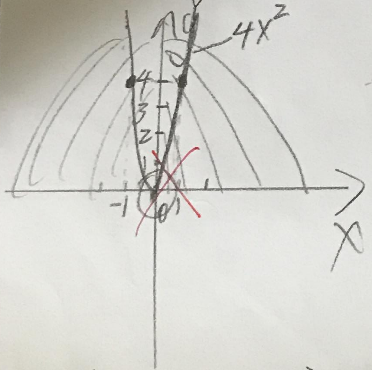
3. $h(x,y,z) = xyz^{-2}$, $(3,8,2)$, $(3,-2,-6)$

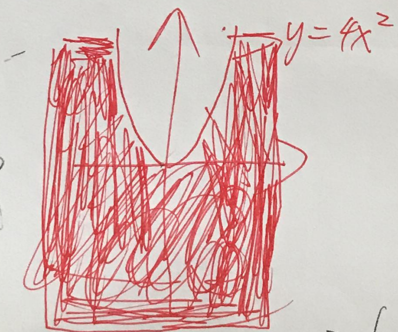
$h(3,8,2) = 24 \times \frac{1}{4} = 6$

$h(3,-2,-6) = \cancel{3\times(-2)}^{-6} \times \frac{1}{\cancel{36}_{-6}} = \frac{1}{-6} = -\frac{1}{6}.$

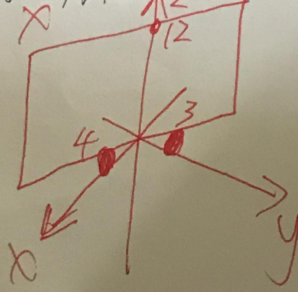
7. $f(x,y) = \ln(4x^2-y)$

$4x^2 - y > 0$

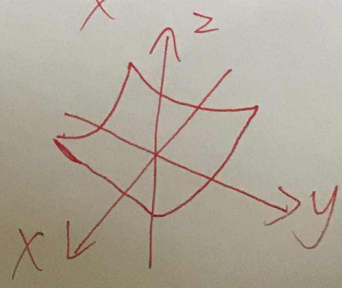




21. $f(x,y) = 12-3x-4y$

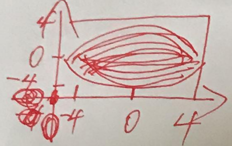


23. $f(x,y) = x^2+4y^2$

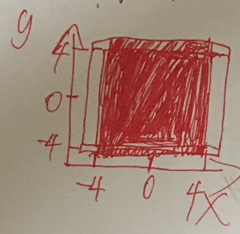


②

33、 $f(x,y) = x^2+4y^2$



36、 $f(x,y)=x^2$



37、

$m=6$; $f(x,y)=2x+6y+6$

$m=3$; $f(x,y)=x+3y+3$.

1

homeworks 14.2

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section 22

5. ~~I can't~~ I didn't find question.

$\lim_{(x,y)\to(\frac{\pi}{4},0)} \tan x \cos y = 1$

15. Let $f(x,y) = \frac{x^3+y^3}{xy^2}$, set $y=mx$ and show that the resulting limit depends on m, and therefore the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

① $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{xy^2} = \frac{0}{0}$

② $y=mx$

$\lim_{x\to0} \frac{x^3+(mx)^3}{x\cdot(mx)^2} = \lim_{x\to0} \frac{x^3(m^3+1)}{m^2x^3} = \frac{m^3+1}{m^2}$

Ans: the limit does not exist.

The limit is $\frac{1+m^3}{m^2}$ for all $m\neq0$.

21. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$

① $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = \frac{0}{0}$

② $y=cx$

$\lim_{x\to0} \frac{x\cdot(cx)}{3x^2+2(cx)^2} = \lim_{x\to0} \frac{cx^2}{3x^2+2c^2x^2} = \lim_{x\to0} \frac{cx^2}{x^2(3+2c^2)} = \frac{c}{3+2c^2}$

The limit does not exist.

23. $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$

① $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2} = \frac{0}{0}$

② $y=cx \quad z=dx$

$\lim_{x\to0} \frac{x+cx+dx}{x^2+(cx)^2+(dx)^2}$

$= \lim_{x\to0} \frac{x(1+c+d)}{x^2(1+c^2+d^2)}$

$= \lim_{x\to0} \frac{(1+c+d)}{x(1+c^2+d^2)}$

$= \infty$

The limit does not exist.

② 27,

$$\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}}$$

① $\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \frac{16\cos\pi}{e^{-1}} = -16e$

The limit exist and equal $-16e$.

31,

$$\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}$$

① $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{9+16}} = \frac{1}{5}$

The limit exist and equal $\frac{1}{5}$.

35, $\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy)$

① $\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy) = \lim_{(x,y)\to(-3,-2)} (9\times(-8)+4\times(-3)\times(-2))$

$= -72+24$

$= -48$

The limit exist and equal -48.