Multivariable HW
13.3 Due Sept 27th

Rahul Paleja

Section 13.3 - #3, 9, 11, 13, 15:

(3) $r(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \le t \le 4$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$|r'(t)| = \sqrt{2^2 + (\frac{1}{t})^2 + (2t)^2} = \sqrt{\frac{4}{t^2} + \frac{1}{t^2} + \frac{4t^2}{t^2}} = \sqrt{\frac{4t^2+1+4t^4}{t^2}}$

$= \sqrt{\frac{(2t^2+1)^2}{t^2}} = \frac{(2t^2+1)}{t}$

$\int_1^4 \frac{2t^2+1}{t}\,dt = \int_1^4 \left(2t + \frac{1}{t}\right) dt$

$= t^2\Big]_1^4 + \ln t\Big]_1^4$

$= 16 - 1 + \ln(4) - \ln(1)^{\,0} = \boxed{15 + \ln(4)}$

(9) $r(t) = \langle t^2, 2t^2, t^3 \rangle \quad a=0 \qquad r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$|r'(t)| = \sqrt{(2t)^2 + (4t)^2 + (3t^2)^2} = \sqrt{4t^2 + 16t^2 + 9t^4}$

$= \sqrt{20t^2 + 9t^4} = \sqrt{t^2(20+9t^2)} = t\sqrt{20+9t^2}$

arc length:

change bounds to $20 + 20+9t^2$ $\rightarrow \int_0^t t\sqrt{20+9t^2}\,dt$

$u = 20+9t^2$

$\frac{du}{18} = \frac{18t\,dt}{18}$

$= \frac{1}{18}\int_{20}^{20+9t^2} \sqrt{u}\,du = \frac{1}{18}\left(\frac{2u^{3/2}}{3}\right]_{20}^{20+9t^2}$

$= \frac{1}{18}\left(\frac{2(20+9t^2)^{3/2}}{3} - \frac{2(20)^{3/2}}{3}\right)$

length $= \frac{1}{27}\left((20+9t^2)^{3/2} - \sqrt{8000}\right)$

(11) Speed = $|v(t)|$ $r(t) = \langle 2t+3, 4t-3, 5-t \rangle$ $t=4$

$r'(t) = v(t) = \langle 2, 4, -1 \rangle$

$|v(t)| = \sqrt{2^2+4^2+1^2} = \boxed{\sqrt{21} \text{ units} = \text{speed at } t=4}$

(13) $r(t) = \langle t, \ln t, (\ln t)^2 \rangle$ $t=1$

$v(t) = r'(t) = \langle 1, \frac{1}{t}, 2(\ln t) \cdot \frac{1}{t} \rangle$

$|v(t)| = \sqrt{1^2 + (\frac{1}{t})^2 + (\frac{2\ln t}{t})^2}$

$|v(1)| = \sqrt{1^2 + (\frac{1}{1})^2 + (\frac{2\ln(1)}{1})^2}$

$= \sqrt{1^2+1^2+0^2} = \boxed{\sqrt{2} \text{ units}}$

(15) $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle$ $t = \frac{\pi}{2}$

$v(t) = r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$|r'(\frac{\pi}{2})| = \sqrt{(3\cos(\frac{3\pi}{2}))^2 + (-4\sin(2\pi))^2 + (-5\sin(\frac{5\pi}{2}))^2}$

$= \sqrt{0+0+25}$

$= \boxed{5 \text{ units}}$

Section 13.4 HW

Due 9/27

Rahul Paleja

13.4 - # 1, 5, 7, 11, 17, 21:

(1) $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$ $\quad |r'(t)| = \sqrt{(8t)^2 + 81} = \sqrt{64t^2 + 81}$

$T(t) = \left\langle \frac{8t}{\sqrt{64t^2+81}}, \frac{9}{\sqrt{64t^2+81}} \right\rangle$

$T(1) = \left\langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \right\rangle$

(5) $r(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$r'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$|r'(t)| = \sqrt{(-\pi \sin \pi t)^2 + (\pi \cos \pi t)^2 + (1)^2}$

$T(t) = \left\langle \frac{-\pi \sin \pi t}{\sqrt{(-\pi \sin \pi t)^2 + (\pi \cos \pi t)^2 + (1)^2}}, \frac{\pi \cos \pi t}{\sqrt{(-\pi \sin \pi t)^2 + (\pi \cos \pi t)^2 + (1)^2}}, \frac{1}{\sqrt{(-\pi \sin \pi t)^2 + (\pi \cos \pi t)^2 + (1)^2}} \right\rangle$

$\sqrt{\pi^2(\sin^2 \pi t + \cos^2 \pi t) + 1^2}$

$T(1) = \left\langle \frac{-\pi \sin \pi t}{\sqrt{\pi^2+1}}, \frac{\pi \cos \pi t}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \right\rangle$

$= \left\langle 0, \frac{-\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \right\rangle$

(7) $r(t) = \langle 1, e^t, t \rangle$ $\quad r'(t) = \langle 0, e^t, 1 \rangle$ $\quad r''(t) = \langle 0, e^t, 0 \rangle$

$|r'(t) \times r''(t)| = \begin{vmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix} = i(0 - e^t) - j(0-0) + k(0-0)$

$= -e^t i - 0j + 0k$

$= |\langle -e^t, 0, 0 \rangle| = \sqrt{(-e^t)^2} = \sqrt{e^{2t}} = e^t$

$|r'(t)|^3 = \sqrt{(e^t)^2 + 1^2} = \left(\sqrt{e^{2t}+1}\right)^3$

$$\boxed{k(t) = \frac{e^t}{\left(\sqrt{e^{2t}+1}\right)^3}}$$

(11) $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle$ $t=-1$

$r'(t) = \langle \frac{-1}{t^2}, \frac{-2}{t^3}, 2t \rangle$ $r''(t) = \langle \frac{2}{t^3}, \frac{6}{t^4}, 2 \rangle$

$r'(-1) = \langle -1, 2, -2 \rangle$ $r''(-1) = \langle -2, 6, 2 \rangle$

$r'(-1) \times r''(-1)$: $\begin{matrix} i & j & k \\ -1 & 2 & -2 \\ -2 & 6 & 2 \end{matrix}$ $i(16) - j(-6) + k(-2)$

$\langle 16, 6, -2 \rangle$

$|r'(-1) \times r''(-1)| = \sqrt{16^2 + (6)^2 + (-2)^2} = \sqrt{296}$

$|r'(-1)|^3 = \sqrt{(-1)^2 + (2)^2 + (-2)^2} = \sqrt{9} = 3^3$

$K(t) = \frac{\sqrt{296}}{3^3} = \boxed{\frac{\sqrt{296}}{27}}$

☆ (17) $y = t^4$, $t = 2$

Curvature of plane curve/graph of f: $K(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$ ← abs. value

$f'(\cancel{t}) = 4t^3$

$f''(t) = 12t^2$

$K(t) = \frac{|12t^2|}{(1+(4t^3)^2)^{3/2}} = \frac{12(4)}{(1+1024)^{3/2}} = \boxed{\frac{48}{\sqrt{1025}^3}}$

Section 13.4 HW
Due 9/27

Rahul Palegai

(21) $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$ has $k(t) = \operatorname{sech} t$

$r'(t) = \langle 1 - \operatorname{sech}^2 t, -\operatorname{sech} t \tanh t \rangle$
$= \langle \tanh^2 t, -\operatorname{sech} t \tanh t \rangle$
$r''(t) = \langle 2 \tanh t \operatorname{sech}^2 t, -\operatorname{sech}^3 t + \tanh^2 t \operatorname{sech} t \rangle$
$r''(t) = \langle 2 \tanh \operatorname{sech}^2 t, \operatorname{sech} t\, t(1 - 2\operatorname{sech}^2 t) \rangle$

$r'(t) \times r''(t) = \begin{vmatrix} i & j \\ \tanh^2 t & -\operatorname{sech} t \tanh t \\ 2\tanh \operatorname{sech}^2 t & -\operatorname{sech}^3 t + \tanh^2 t \operatorname{sech} t \end{vmatrix}$

$r'(t) \times r''(t) = (\tanh^2 t \operatorname{sech} t - 2\tanh^2 t \operatorname{sech}^3 t + 2\operatorname{sech}^3 \tanh t$

$= \tanh^2 t \operatorname{sech} t$

$K(t) = \dfrac{\tanh^2 t \operatorname{sech} t}{(\tanh t)^{1}} = \boxed{\operatorname{csch} t}$

Section 13.5
Due 9/27

Rahul Pateja

13.5 - #3, 5, 15, 17, 31, 33:

(3) $r(t) = \langle t^3, 1-t, 4t^2 \rangle \quad t=1$
$r'(t) = v(t) = \langle 3t^2, -1, 8t \rangle \quad \boxed{v(1) = \langle 3, -1, 8 \rangle}$

$r''(t) = a(t) = \langle 6t, 0, 8 \rangle \quad \boxed{a(1) = \langle 6, 0, 8 \rangle}$

$|v(t)| = \text{speed} = \sqrt{3^2 + (-1)^2 + (8)^2} = \boxed{\sqrt{74}}$

(5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle \quad \theta = \frac{\pi}{3}$
$r'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$
$v(\frac{\pi}{3}) = r'(\frac{\pi}{3}) = \langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \rangle$

$a(\frac{\pi}{3}) = r''(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$
$r''(\frac{\pi}{3}) = \langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9 \rangle$

$|r'(\frac{\pi}{3})| = \text{speed} = \sqrt{\frac{1}{4} + (-\frac{\sqrt{3}}{2})^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = \boxed{1}$

(15) $a(t) = \langle t, 4 \rangle \quad v(0) = \langle 3, -2 \rangle \quad r(0) = \langle 0, 0 \rangle$
$v(t) = \int a(t)\,dt = \frac{t^2}{2} i + 4t j + C$

$v(0) = 0^2 i + 0 j + C = 3i - 2j$
$\int v(t) = \langle \frac{t^2}{2} + 3 \rangle i + \langle 4t - 2 \rangle j$
$r(t) = (\frac{t^3}{6} + 3t) i + (2t^2 - 2t) j + C$

$r(0) = 0i + 0j + C = 0i + 0j$

$r(t) = (\frac{t^3}{6} + 3t) i + (2t^2 - 2t) j$

$= \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

(17) $a(t) = t\,k \qquad v(0) = i \qquad r(0) = j$

$v(t) = \frac{t^2}{2}k + C$

$v(0) = 0k + C = i$

$v(t) = 1i + \frac{t^2}{2}k$

$r(t) = ti + \frac{t^3}{6}k + C$

$r(0) = 0i + 0k + C = j$

$r(t) = ti + 1j + \frac{t^3}{3}k$

$\boxed{r(t) = \langle t, 1, \frac{t^3}{3} \rangle}$

(31) To answer this, we need to take the

$(\|v\|^2)' = 2\,v' \cdot v = 2\langle 2, 1, -3 \rangle \cdot \langle 12, 20, 20 \rangle$

$= -32$

Negative,

thus speed is negative and decreasing

(33) $r(t) = \langle t, \cos t, \sin t \rangle$

$r'(t) = \langle 1, -\sin t, \cos t \rangle \qquad a_N = 1$

$r''(t) = \langle 0, -\cos t, -\sin t \rangle \qquad a_T = 0$

$a_T = a \cdot T = \frac{a \cdot v}{\|v\|}$

$a_N = \sqrt{\|a\|^2 - |a_T|^2}$

Section 14.1 HW
Due 9/27

Rahul Paleja

14.1 → #1, 3, 7, 21, 23, 33, 35, 37 (optional):

① $f(x,y) = x + yx^3$ $(2,2)$, $(-1,4)$

$f(2,2) = 2 + 2(8) = \boxed{18}$ $f(-1,4) = -1 + 4(-1)^3 = \boxed{-5}$

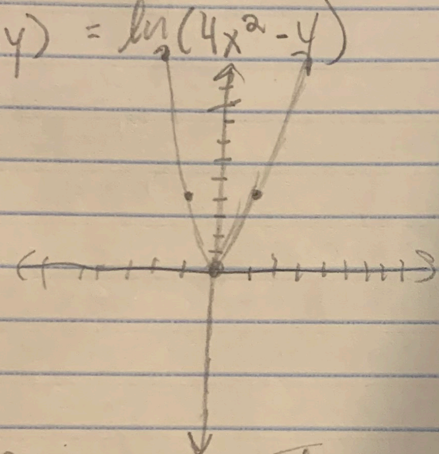
③ $h(x,y,z) = xyz^{-2}$ $(3,8,2)$, $(3,-2,-6)$

$= 3 \cdot 8 \cdot \frac{1}{2^2} = \boxed{6}$

$h(3,-2,-6) = 3 \cdot -2 \cdot \frac{1}{-6^2} = \frac{-6}{36} = \boxed{\frac{-1}{6}}$

⑦ $f(x,y) = \ln(4x^2 - y)$ $4x^2 - y > 0$ $4x^2 > y$

No negatives



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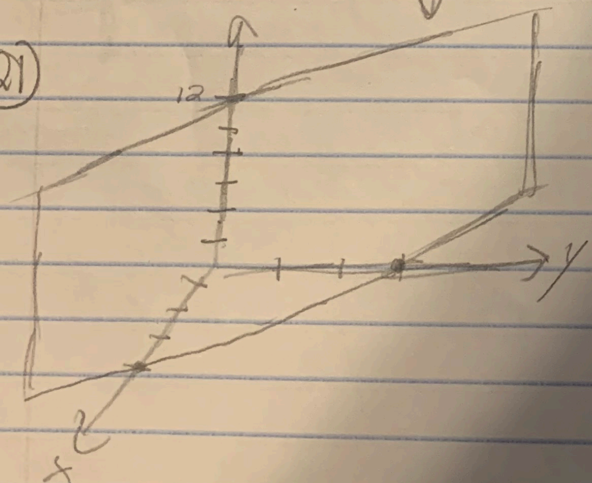
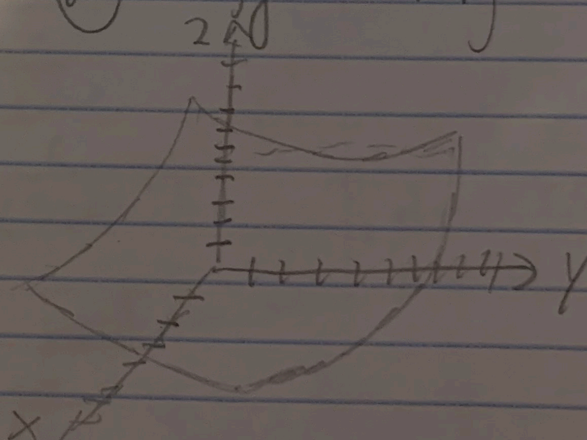
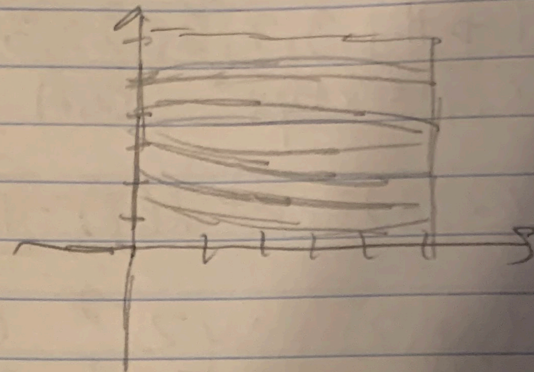


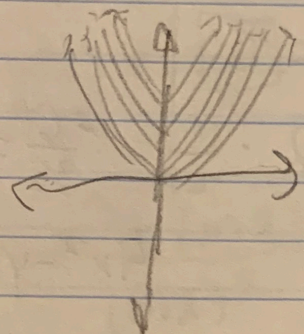
㉓ $f(x,y) = x^2 + 4y^2$



33 $f(x,y)=x^2+4y^2$



35 $f(x,y)=x^2$



Section 14.2 HW
Due 9/27

Rahul Paleja

14.2 → # 9, 11, 15, 21, 23, 27, 31, 35:

(9) $\lim_{(x,y)\to(2,5)} (g(x,y) - 2f(x,y))$

$7 - 2(3) = 7 - 6 = \boxed{1}$

(11) $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)} = e^{9-7} = e^2$

(15) $f(x,y) = \frac{x^3+y^3}{xy^2}$ set $y = mx$ $\lim_{(x,y)\to(0,0)} f(x,y)$

$\frac{x^3+(mx)^3}{(mx)^2} = \frac{x^3+m^3x^3}{x \cdot m^2 x^2} = \frac{\cancel{x^3}(1+m^3)}{\cancel{x^3}\, m^2}$

$= \frac{1+m^3}{m^2}$ limit depends on slope m which is different for different lines so limit does not exist.

(21) $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$ $\frac{0 \cdot 0}{0+0} = \frac{0}{0}$

$y = cx$ $\lim_{(x,y)\to(0,0)} \frac{x \cdot (cx)}{3x^2+2(cx)^2} = \frac{c\cancel{x^2}}{\cancel{x^2}(3+2c^2)}$

$= \lim_{(x,y)\to(0,0)} \frac{c}{(3+2c^2)}$ limit depends on slope c so it doesn't exist

(23) $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$ hold variables $x, y = 0$

so:

$\lim_{z\to 0} \frac{z}{z^2} = \lim_{z\to 0} \frac{1}{z}$

limit does not exist as it approaches 0

(27) $\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}}$ Plug In:

$\frac{(-2)^4\cos(\pi)}{e^{-2+1}} = \frac{16\cdot -1}{e^{-1}} \cdot e = \boxed{-16e}$

Wait — as written: $\frac{(-2)^4 \cos(\pi)}{e^{-2+1}} = \underline{16 \cdot -1 \cdot e} = \boxed{-16e}$

(31) $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}$ Plug In: $\frac{1}{\sqrt{9+16}} = \frac{1}{\sqrt{25}} = \boxed{\frac{1}{5}}$

(35) $\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy)$ Plug In:

$(-3^2)(-2)^3 + 4(-3\cdot -2)$

$= 9\cdot -8 + 24 = -72+24 = \boxed{-48}$