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$*T(t) = \frac{r'(t)}{\|r'(t)\|}$

13.3 Homework Exercises: 3, 9, 11, 13, 15

(3) $r(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \le t \le 4$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$|r'(t)| = \sqrt{4 + \frac{1}{t^2} + 4t^2}$

$s = \int_1^4 \sqrt{4 + \frac{1}{t^2} + 4t^2}\, dt = \ln(4) + 15$

(9) $r(t) = \langle t^2, 2t^2, t^3 \rangle, \quad a = 0$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$s = \int_0^t \sqrt{4t^2 + 16t^2 + 9t^4}\, dt$

$s = \sqrt{20t^2 + 9t^4}\, dt = \frac{1}{27}(20t^2 + 9t^4)^{\frac{2}{3}} \Big|_0^t$

$s = \frac{1}{27}((20 + 9t^2)^{\frac{2}{3}} - 20^{\frac{3}{2}})$

(11) $r(t) = \langle 2t + 3, 4t - 3, 5 - t \rangle, \quad t = 4$

$r'(t) = \langle 2, 4, -1 \rangle = v(t)$

$|v(4)| = \sqrt{2^2 + 4^2, -1^2} = \sqrt{21}$

(13) $r(t) = \langle t, \ln t, (\ln t)^2 \rangle, \quad t = 1$

$r'(t) = \langle 1, \frac{1}{t}, 2\frac{1}{t} \rangle = v(t)$

$|v(t)| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

$\uparrow t = 1$

(15) $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle, \quad t = \frac{\pi}{2}$

?

$r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle \quad t = \frac{\pi}{2}$

$|v(\frac{\pi}{2})| = \sqrt{9\cos^2\frac{3\pi}{2} + 16\sin^2 2\pi + 25\sin^2\frac{5\pi}{2}} = 5$

13.4 Homework Exercises: 1, 5, 7, 11, 17, 21

(1) $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle \quad \|r'(t)\| = \sqrt{64t^2 + 81}$

$T(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{1}{\sqrt{64t^2 + 81}} \langle 8t, 9 \rangle$

$T(1) = \langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \rangle$

(5) $r(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$r'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$T(t) = \frac{1}{\sqrt{\pi^2(\sin^2 \pi t + \cos^2 \pi t) + 1}} \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$T(1) = \langle 0, \frac{-\pi}{\sqrt{\pi^2 + 1}}, \frac{1}{\sqrt{\pi^2 + 1}} \rangle$

(13.4 continued)

(7) $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$\kappa(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$

$\|r'(t) \times r''(t)\| = -e^t i + j = \langle -e^t, 0, 0 \rangle = e^t$

$\|r'(t)\| = (\sqrt{1 + e^{2t}})^3$

$\therefore \kappa(t) = \frac{e^t}{(\sqrt{1 + e^{2t}})^3}$

(11) $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle, \quad t = -1$

$r'(t) = \langle -\frac{1}{t^2}, -\frac{2}{t^3}, 2t \rangle$

$r''(t) = \langle \frac{1}{t^3}, \frac{6}{t^4}, 2 \rangle$

$\|r'(t) \times r''(t)\| = \langle -\frac{16}{t^3}, (-\frac{2}{t^2} + \frac{2}{t^6}), \frac{-4}{t^2} \rangle$

$= \langle 16, 0, -4 \rangle$

$\kappa(s) = \frac{\sqrt{16^2 + 16}}{3^3} = \frac{2\sqrt{74}}{27}$

(17) $y = t^4, \quad t = 2$

?

$r(t) = \langle 0, t^4, 0 \rangle$

$r'(t) = \langle 0, 4t^3, 0 \rangle$

$r''(t) = \langle 0, 12t^2, 0 \rangle$

(21) $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$

?

$r'(t) = \langle 1 - \operatorname{sech}^2(x), -\operatorname{sech}(x)\tanh(x) \rangle$

$r''(t) = \langle 2\operatorname{sech}^2(x)\tanh(x), \operatorname{sech}(x)\tanh^2(x) - \operatorname{sech}^3(x) \rangle$

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Chapter 13.5 Homework Exercises

#'s 3, 5, 15, 17, 31, 33

(3) $r(t) = \langle t^3, 1-t, 4t^2 \rangle \quad t=1$

$r'(t) = v(t) = \langle 3t^2, -1, 8t \rangle$

$r'(1) = \langle 3, -1, 8 \rangle = \sqrt{74}$

$r''(t) = a(t) = \langle 6t, 0, 8 \rangle$

$a(1) = \langle 6, 0, 8 \rangle$

(5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle \quad \theta = \frac{\pi}{3}$

$r'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$r'(\frac{\pi}{3}) = \langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \rangle = 1$

$r''(\frac{\pi}{3}) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$= \langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9 \rangle$

(15) $a(t) = \langle t, 4 \rangle$, $v(0) = \langle 3, -2 \rangle$, $r(0) = \langle 0, 0 \rangle$ → $v(t) = \langle \frac{t^2}{2} + c, 4t + c \rangle$

$\frac{t^2}{2} + c = 3 \rightarrow c = 3$

$4t + c = -2 \rightarrow c = -2$

$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$r(t) = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

(17) *** ¿

Solve for v(t) and r(t)

initial $\left[a(t) = tk,\ v(0) = i,\ r(0) = j \right]$ → $v(t) = i + \frac{t^2}{2}k$

$r(t) = ti + j + \frac{t^3}{6}k$

(31) $v = \langle 12, 20, 20 \rangle$

$a = \langle 2, 1, -3 \rangle$

The particle's speed is decreasing since a(t) and v(t) have opposite signs.

(33) $r(t) = \langle t, \cos t, \sin t \rangle$

a_T vs. a_N

$= 0 \qquad = 1$

$v(t) = \langle 1, -\sin t, \cos t \rangle$

$a(t) = \langle 0, -\cos t, -\sin t \rangle$

?

$a_T = 0$

$a_N = 1$

14.2: 9, 11, 15, 21, 23, 27, 31, 35

(9) $\lim_{(x,y)\to(2,5)} f(x,y) = 3 \qquad \lim_{(x,y)\to(2,5)} g(x,y) = 7$

$\lim_{(x,y)\to(2,5)} (g(x,y) - 2f(x,y))$

$= 7 - 2(3) = 1$

(11) $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)}$

$= \lim_{(x,y)\to(2,5)} e^2 = e^2$

(15) $f(x,y) = \frac{2x^2 + 3y^2}{xy}$

$\lim_{(x,y)\to(0,0)} \frac{2x^2 + 3(mx)^2}{x(mx)} = \frac{2x^2 + 3m^2x^2}{x^2 m}$

$= \lim_{(x,y)\to(0,0)} \frac{\cancel{x^2}(2+3m^2)}{\cancel{x^2}m} = \frac{2+3m^2}{m}$

$= \frac{1+m^3}{m^2}$ for all m ≠

∴ The limit DNE since you approach the point (0,0) on different lines.

(21) $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2 + 2y^2} \rightarrow = \frac{c}{2+3c^2}$ DNE

* (23) $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$

(showing DNE by approaching the origin along one or more coordinate axis)

↓ along the x-coordinate axis $(y = z = 0)$

$= \lim_{(x,y,z)\to(0,0,0)} \frac{1}{x} = \infty$

(27) $\lim_{(z,w)\to(-2,1)} \frac{z^4 \cos(\pi w)}{e^{z+w}} = \frac{-16}{e^{-2+1}} = -16e$

?

(31) $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{5}$

(35) $\lim_{(x,y)\to(-3,-2)} (x^2y^3 + 4xy) = -48$