Calc 251 HW: 13.3

3. $r(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \le t \le 4$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle \Rightarrow |r'(t)| = \sqrt{4 + \frac{1}{t^2} + 4t^2}$

$|r'(t)| = \sqrt{\frac{4t^2 + 1 + 4t^4}{t^2}} = \sqrt{\frac{(2t^2+1)^2}{t^2}} = \frac{2t+1}{t}$

$s(t) = \int^4 |r'(t)|\, dt$

$s(t) = \int^4 (2t + \frac{1}{t})\, dt$

$s(t) = t^2\Big|_1^4 + \ln(t)\Big|_1^4$

$s(t) = 16 - 1 + \ln(4) - \ln(1)$

$\boxed{s(t) = 15 + \ln(4)}$

9. $r(t) = \langle t^2, 2t^2, t^3 \rangle, a = 0$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle \Rightarrow r'(t) = \sqrt{4t^2 + 16t^2 + 9t^4}$

$|r'(t)| = t\sqrt{20 + 9t^2}$

$s(t) = \int_0^t |r'(t)|\, dt \qquad u = 9t^2,\ du = 18t\, dt$

$s(t) = \int_0^t t\sqrt{20 + 9t^2}\, dt$

$s(t) = \int_0^t \frac{1}{18}\sqrt{20 + u}\, du$

$s(t) = \frac{1}{18} \frac{(20+u)^{3/2}}{3/2}\Big|_0^t$

$s(t) = \frac{1}{27}(20 + 9t^2)^{3/2}\Big|_0^t$

$\boxed{s(t) = \frac{1}{27}\left[(20 + 9t^2)^{3/2} - (20)^{3/2}\right]}$

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11. $r(t) = \langle 2t+3, 4t-3, 5-t \rangle, t=4$

$r'(t) = \langle 2, 4, -1 \rangle$

$|r'(t)| = \sqrt{2^2+4^2+1^2} = \sqrt{4+16+1} = \sqrt{21}$

$\boxed{|r'(4)| = \sqrt{21}}$

13. $r(t) = \langle t, \ln t, (\ln t)^2 \rangle; t=1$

$r'(t) = \langle 1, \frac{1}{t}, \frac{2}{t}(\ln t) \rangle$

$r'(1) = \langle 1, 1, 0 \rangle$

$|r'(1)| = \sqrt{1^2+1^2+0} = \sqrt{2}$

$\boxed{|r'(1)| = \sqrt{2}}$

15. $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle \quad t = \frac{\pi}{2}$

$r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$r'(\frac{\pi}{2}) = \langle 0, 0, -5 \rangle$

$|r'(\frac{\pi}{2})| = \sqrt{25}$

$\boxed{|r'(\frac{\pi}{2})| = 5}$

Calc 251 HW: 13.4

1. $r(t) = \langle 4t^2, 9t \rangle \quad T(t) = \frac{r'(t)}{\|r'(t)\|}$

$\boxed{r'(t) = \langle 8t, 9 \rangle}$

$\boxed{T(t) = \frac{\langle 8t, 9 \rangle}{\sqrt{64t^2+81}}}$

$64 + 81 = 145$

$\boxed{T(1) = \frac{\langle 8, 9 \rangle}{\sqrt{145}}}$

5. $r(t) = \langle \cos\pi t, \sin\pi t, t \rangle$

$$\boxed{r'(t) = \langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle}$$

$$\boxed{T(t) = \frac{\langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle}{\sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1}}}$$

$$T(1) = \frac{\langle -\pi\sin\pi, \pi\cos\pi, 1 \rangle}{\sqrt{\pi^2(\sin\pi^2 + \cos\pi^2) + 1}} = \frac{\langle -\pi\sin\pi, \pi\cos\pi, 1 \rangle}{\sqrt{\pi^2 + 1}}$$

$$\boxed{T(1) = \left\langle 0, \frac{\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \right\rangle}$$

7. $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$r'(t) \times r''(t)$

$$\begin{vmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix} = \begin{vmatrix} e^t & 1 \\ e^t & 0 \end{vmatrix} i - \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} j + \begin{vmatrix} 0 & e^t \\ 0 & e^t \end{vmatrix} k$$

$$(0 - e^t)i - (0)j + (0)k$$

$$r'(t) \times r''(t) = -e^t j$$

$$|r'(t) \times r''(t)| = \sqrt{(-e^t)^2} = e^t$$

$$|r'(t)|^3 = (e^{2t}+1)^{3/2}$$

$$\boxed{k(t) = \frac{e^t}{(e^{2t}+1)^{3/2}}}$$

$t^{-2}$

$t^{-2-1}$

11. $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle, t=-1$

$r'(t) = \langle -t^{-2}, -2t^{-3}, 2t \rangle$

$r''(t) = \langle 2t^{-3}, 6t^{-4}, 2 \rangle$

$r'(-1) = \langle -\frac{1}{1}, -\frac{2}{(-1)^3}, 2(-1) \rangle$

$r'(-1) = \langle -1, 2, -2 \rangle$

$r''(-1) = \langle -2, 6, 2 \rangle$

$r'(-1) \times r''(-1)$

$$\begin{vmatrix} i & j & k \\ -1 & 2 & -2 \\ -2 & 6 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -2 \\ 6 & 2 \end{vmatrix} i - \begin{vmatrix} -1 & -2 \\ -2 & 2 \end{vmatrix} j + \begin{vmatrix} -1 & 2 \\ -2 & 6 \end{vmatrix} k$$

$(4+12)i - (-2-4)j + (-6+4)k$

$16i + 6j - 2k$

$|r'(-1) \times r''(-1)| = \sqrt{16^2 + 6^2 + (-2)^2} = \sqrt{256+36+4}$

$= 2\sqrt{74}$

$|r'(-1)| = \left(\sqrt{1^2+2^2+2^2}\right)^3 = 27$

$$\boxed{k(t) = \frac{2\sqrt{74}}{27}}$$

17. $y = t^4, t=2$ $\quad y' = 4t^3, y'' = 12t^2$

$k(t) = \frac{|y''|}{(1+y'^2)^{3/2}} = \frac{|12t^2|}{(1+(4t^3)^2)^{3/2}}$

$k(2) = \frac{48}{(1+1024)^{3/2}} = \frac{48}{1025^{3/2}}$

$$\boxed{k(2) = \frac{48}{1025^{3/2}}}$$

21. $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$, $\kappa(t) = \operatorname{sech} t$

$r'(t) = \langle 1 - \operatorname{sech}^2 t, -\operatorname{sech} t \tanh t \rangle$

$r''(t) = \langle 2\operatorname{sech}^2 t \tanh t, \operatorname{sech} t \tanh^2 t - \operatorname{sech}^3 t \rangle$

$r'(t) = \langle \tanh^2 t, -\operatorname{sech} t \tan t \rangle$

$r''(t) = \langle 2\operatorname{sech}^2 t \tanh t, \operatorname{sech} t(1 - 2\operatorname{sech}^2 t) \rangle$

$r'(t) \times r''(t) = \tanh^2 t \operatorname{sech} t$

$|r'(t)|^3 = \left(\sqrt{\tanh t^4 + \operatorname{sech}^2 t \tanh^2 t}\right)^3$

$\kappa(t) = \dfrac{\tanh^2 t \operatorname{sech} t}{\left(\sqrt{\tanh t^4 + \operatorname{sech}^2 t \tanh^2 t}\right)^3}$

$\kappa(t) = \dfrac{\tanh^2 t \operatorname{sech} t}{(\tanh t)^3}$

$\kappa(t) = \dfrac{\operatorname{sech} t}{\tanh t}$

$\boxed{\kappa(t) = \operatorname{csch} t}$

Calc 251 HW: 13.5

3. $r(t) = \langle t^3, 1-t, 4t^2 \rangle$, $t = 1$

$r'(t) = \langle 3t^2, 1, 8t \rangle$

$r''(t) = \langle 6t, 0, 8 \rangle$

$$\boxed{\begin{aligned} v(1) &= \langle 3, 1, 8 \rangle \\ a(1) &= \langle 6, 0, 8 \rangle \\ |v(1)| &= \sqrt{74} \end{aligned}}$$

5. $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$, $\theta = \frac{\pi}{3}$

$r'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$r''(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$$\boxed{\begin{aligned} v(\tfrac{\pi}{3}) &= \langle \tfrac{1}{2}, -\tfrac{\sqrt{3}}{2}, 0 \rangle \\ \alpha(\tfrac{\pi}{3}) &= \langle -\tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}, 9 \rangle \\ |v(\tfrac{\pi}{3})| &= \sqrt{(\tfrac{1}{2})^2 + (-\tfrac{\sqrt{3}}{2})^2} = \sqrt{\tfrac{1}{4} + \tfrac{3}{4}} = 1 \end{aligned}}$$

15. $a(t) = \langle t, 4 \rangle$, $v(0) = \langle 3, -2 \rangle$, $r(0) = \langle 0, 0 \rangle$

$\int a(t)\,dt = \langle \frac{t^2}{2}, 4t \rangle \rightarrow \langle \frac{t^2}{2} + 3, 4t - 2 \rangle = v(t)$

$\int v(t)\,dt = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle = r(t)$

$\frac{1}{2}$ $\frac{t^3}{3}$

$$\boxed{\begin{aligned} v(t) &= \langle \tfrac{t^2}{6} + 3, 4t - 2 \rangle \\ r(t) &= \langle \tfrac{t^3}{6} + 3t, 2t^2 - 2t \rangle \end{aligned}}$$

17. $a(t) = tk$, $v(0) = i$, $r(0) = j$

$v(t) = \frac{t^2}{2}k + i$

$r(t) = \frac{t^3}{3}k + ti + j$

$$\boxed{\begin{aligned} v(t) &= \tfrac{t^2}{2}k + i \\ r(t) &= \tfrac{t^3}{3}k + ti + j \end{aligned}}$$

31. $2(a \cdot v) = 2\langle 2, 1, -3\rangle \cdot \langle 12, 20, 20\rangle$

$= 2(24 + 20 - 60)$

$2(a \cdot v) = -32$

$\boxed{\text{Particle is slowing down}}$

33. $r(t) = \langle t, \cos t, \sin t\rangle$

$v(t) = \langle 1, -\sin t, \cos t\rangle$

$a(t) = \langle 0, -\cos t, -\sin t\rangle$

$a_T = a \cdot T = \dfrac{a \cdot v}{\|v\|}$

$a \cdot v = \langle 0, -\cos t, -\sin t\rangle \cdot \langle 1, -\sin t, \cos t\rangle$

$a \cdot v = 0 + \sin t \cos t - \sin t \cos t = 0$

$\|v\| = \sqrt{1 + \sin^2 t + \cos^2 t} = \sqrt{2}$

$\boxed{a_T = 0}$

$a_N = \sqrt{\|a\|^2 - |a_T|^2}$

$a_N = \sqrt{1 - 0} = 1$

$\boxed{a_N = 1}$

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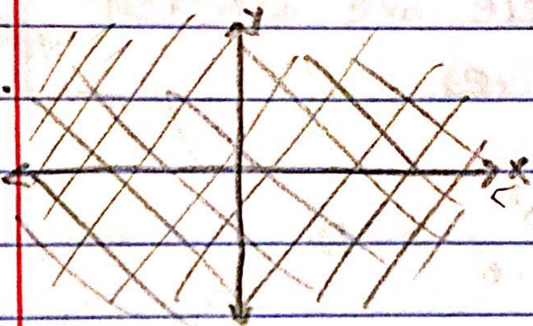
Calc 251 HW: 14.1

1. $f(x,y) = x + yx^3 \quad (2,2) \quad (-1,4)$

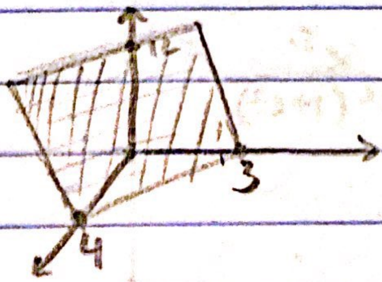
$$\boxed{\begin{aligned} f(2,2) &= 2 + 2(2)^3 = 18 \\ f(-1,4) &= -1 + 4(-1)^3 = -5 \end{aligned}}$$

3. $h(x,y,z) = xyz^{-2} \quad (3,8,2) \quad (3,-2,-6)$

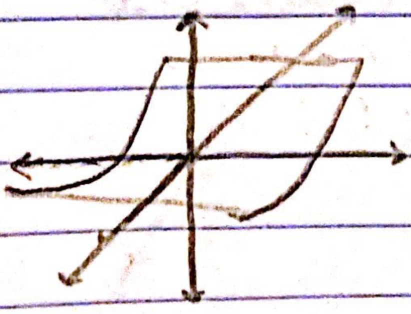
$$\boxed{\begin{aligned} h(3,8,2) &= 3(8)(2)^{-2} = 6 \\ h(3,-2,-6) &= 3(-2)(-6)^{-2} = \tfrac{1}{6} \end{aligned}}$$

7. 


All of $R^2$

21. 


$f(x,y) = 12 - 3x - 4y$

23. 

$f(x,y) = x^2 + 4y^2$

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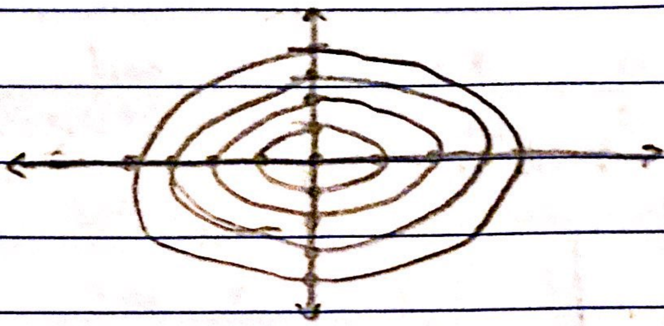
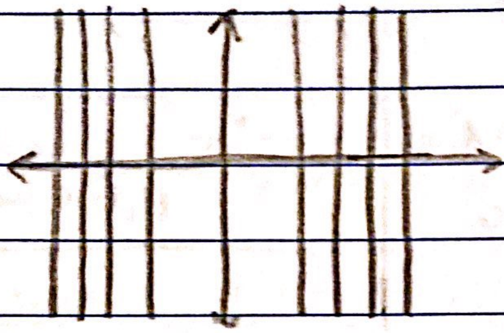
33.



35.



37. $f(x,y) = qx + ry + s = c$

$f(0,0) = q(0) + r(0) + s = 6 \rightarrow s = 6$

@ $c=0$ curve passes $(-3,0)$ and $(0,-1)$

$f(-3,0) = q(-3) + r(0) + 6 = 0 \rightarrow q = 2$

$f(0,-1) = q(0) + r(-1) + 6 = 0 \rightarrow r = 6$

$\boxed{\text{For } m=6,\ 2x + 6x + 6}$

$f(0,0) = q(0) + r(0) + s = 3 \rightarrow s = 3$

@ $c=0$ curve passes $(-3,0)$ and $(0,-1)$

$f(-3,0) = q(-3) + r(0) + 3 = 0 \rightarrow q = 1$

$f(0,-1) = q(0) + r(-1) + 3 = 0 \rightarrow r = 3$

$\boxed{\text{For } m=3,\ x + 3y + 3}$

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Calc 251 HW: 14.2

5. $\lim_{(x,y)\to(\frac{\pi}{4},0)} \tan x \cos y = \tan(\frac{\pi}{4})\cos(0)$

$$\boxed{\lim_{(x,y)\to(\frac{\pi}{4},0)} \tan x \cos y = 1}$$

15. $f(x,y) = \frac{x^3+y^3}{xy^2}$, $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{xy^2} = \frac{0+0}{0}$$

$$\lim_{x\to 0} \frac{x^3+(mx)^3}{x(mx)^2} = \lim_{x\to 0} \frac{x^3(1+m^3)}{x^3m^2}$$

$$\lim_{x\to 0} \frac{1+m^3}{m^2} = \lim_{x\to 0} \frac{1}{m^2}+m$$

$\boxed{\text{Limit DNE because there are diff limits for diff lines}}$

21. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} \to \frac{0(0)}{0+0}$

$y = mx$

$$\lim_{x\to 0} \frac{x(mx)}{3x^2+2(mx)^2} = \lim_{x\to 0} \frac{\cancel{x^2}m}{3\cancel{x^2}+2\cancel{x^2}m}$$

$$\lim_{x\to 0} \frac{m}{3+2m} = \frac{m}{3}+\frac{1}{2}$$

$\boxed{\text{Limit DNE}}$

$x^{-1}$

$-1x^2$

23. $\lim\limits_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$ $\frac{0+0+0}{0+0+0}=0$

set $y,z=0$ look at path along x-axis

$\lim\limits_{x\to 0}\frac{x}{x^2}=\lim\limits_{x\to 0}\frac{1}{x}$ $\boxed{\text{Limit DNE}}$

27. $\lim\limits_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}}=\frac{(-2)^4\cdot\cos\pi}{e^{-2+1}}=\frac{-16}{e^{-1}}$

$$\boxed{\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}}=-16e}$$

31. $\lim\limits_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}=\frac{1}{\sqrt{9+16}}=\frac{1}{5}$

$$\boxed{\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}=\frac{1}{5}}$$

35. $\lim\limits_{(x,y)\to(-3,-2)} (x^2y^3+4xy)=((-3)^2(-2)^3+4(-3)(-2))$ 72 +24 96

$=(-72+24)=-48$

$$\boxed{\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy)=-48}$$