13.3: 3, 9, 11, 13, 15
13.4: 1, 5, 7, 11, 17, 21
13.5: 3, 5, 15, 17, 31, 33

Chapter 13 HW

Orion Kress-Sanfilippo

13.3

3) $r(t) = (2t, \ln(t), t^2) \quad 1 \le t \le 4$

$$S = \int_1^4 \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

$$= \int_1^4 \sqrt{(2)^2 + \left(\frac{1}{t}\right)^2 + (2t)^2}\, dt$$

$$= \int_1^4 \sqrt{4t^2 + 4 + \frac{1}{t^2}}\, dt$$

$$= \int_1^4 \sqrt{\left(\frac{1}{t^2}\right)(4t^4 + 4t^2 + 1)}\, dt = \int_1^4 \frac{1}{t}(2t^2+1)\, dt$$

$$= t^2 + |\ln(t)| \Big|_1^4 = \boxed{15 + \ln(4)}$$

9) $r(t) = (t^2, 2t^2, t^3)$

$$s(t) = \int_0^t \sqrt{(4t^2 + 16t^2 + 9t^4)} = \int_0^t \sqrt{(t)^2(9t^2 + 20)}$$

$$= \int_0^t t\left(\sqrt{9t^2 + 20}\right) = \left(\frac{3}{2}\right)\int_0^t 2t\left(\sqrt{t^2 + \frac{20}{9}}\right) dt$$

$$u = t^2 + \frac{20}{9} \quad du = 2t\,dt$$

$$s(t) = \frac{3}{2}\int_0^t \sqrt{u}\, du = \frac{3}{2}\, \frac{u^{3/2}}{\frac{3}{2}} = \boxed{\sqrt{t^3}}$$

13.3 Cont.

11) $r(t) = \langle 2t+3,\ 4t-3,\ 5-t \rangle \quad t=4$

speed $= |v(t)| = |r'(t)| = \sqrt{(2)^2+(4)^2+(-1)^2} = \boxed{\sqrt{21}}$

13) $r(t) = \langle t,\ \ln(t),\ (\ln(t))^2 \rangle \quad t=1$

$r'(t) = \langle 1,\ \frac{1}{t},\ 2\ln(t)\cdot\frac{1}{t} \rangle$

$|v(t)| = \sqrt{(1) + \left(\frac{1}{t}\right)^2 + \left(\frac{2\ln(t)}{t}\right)}$

$|v(1)| = \sqrt{1 + 1 + 0} = \boxed{\sqrt{2}}$

15) $r(t) = \langle \sin(3t),\ \cos(4t),\ \cos(5t) \rangle \quad @\ t = \frac{\pi}{2}$

$r'(t) = \langle 3\cos(3t),\ -4\sin(4t),\ -5\sin(5t) \rangle$

$|v(\frac{\pi}{2})| = \sqrt{(3\cos(\frac{3\pi}{2}))^2 + (-4\sin(2\pi))^2 + (-5\sin(\frac{5\pi}{2}))^2}$

$= \boxed{5}$

13.4 HW

1) $r(t) = \langle 4t^2,\ 9t \rangle;\ r'(t) = \langle 8t,\ 9 \rangle;$

$T(t) = \langle \frac{8t}{\sqrt{64t^2+81}},\ \frac{9}{\sqrt{64t^2+81}} \rangle;\ T(1) = \langle \frac{8}{\sqrt{145}},\ \frac{9}{\sqrt{145}} \rangle$

5) $r(t) = \langle \cos(\pi t),\ \sin(\pi t),\ t \rangle;\ r'(t) = \langle -\pi\sin(\pi t),\ \pi\cos(\pi t),\ 1 \rangle$

$T(t) = \langle \frac{-\pi(\sin(\pi t))}{\sqrt{\pi^2+1}},\ \frac{+\pi\cos(\pi t)}{\sqrt{\pi^2+1}},\ \frac{1}{\sqrt{\pi^2+1}} \rangle;\ T(1) = \left(0,\ \frac{-\pi}{\sqrt{\pi^2+1}},\ \frac{1}{\sqrt{\pi^2+1}}\right)$

13.4 Cont

~~9) $r(t) = \langle 4t+1, 4t-3, 2t \rangle$ $\quad k(t) = \dfrac{|r'(t) \times r''(t)|}{(|r'(t)|)^3}$~~

~~$r'(t) = \langle 4, 4, 2 \rangle$~~

~~$r''(t) = \langle 0, 0, 0 \rangle$~~ Disregard

~~$|r'(t) \times r''(t)| = \left|\det \begin{bmatrix} i & j & k \\ 4 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}\right| = \sqrt{0^2+0^2+0^2} = 0$~~

~~$\boxed{k(t) = 0}$~~

7) $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$ $\quad \left(|r'(t)| = \sqrt{e^{2t}+1}\right)^3 = (e^{2t}+1)^{3/2}$

$r''(t) = \langle 0, e^t, 0 \rangle$

$|r'(t) \times r''(t)| = \left|\det \begin{bmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{bmatrix}\right| = \sqrt{(e^t)^2 + 0 + 0}$

$$\boxed{k(t) = \frac{e^t}{(e^{2t}+1)^{3/2}}}$$

11) $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle ; \boxed{t = -1}$

$r'(t) = \langle \frac{-1}{t^2}, -\frac{2}{t^3}, 2t \rangle$ $\quad |r'(t)| = \sqrt{\frac{1}{t^4} + \frac{4}{t^6} + 4t^2}$

$r''(t) = \langle \frac{+2}{t^3}, \frac{6}{t^4}, 2 \rangle$ $\quad \boxed{(|r'(1)|)^3 = (\sqrt{1+4+4}) = 3}$

$|r'(t) \times r''(t)| = \left|\det \begin{bmatrix} i & j & k \\ -\frac{1}{t^2} & -\frac{2}{t^3} & 2t \\ \frac{2}{t^3} & \frac{6}{t^4} & 2 \end{bmatrix}\right| = \left|\left(\frac{-4}{t^3} + \frac{12}{t^3}\right)\hat{i} - \left(\frac{-2}{t^2} + \frac{4}{t^2}\right)\hat{j}\right.$

$\left. + \left(\frac{-6}{t^6} - \frac{4}{t^6}\right)\hat{k}\right|$

$= \sqrt{\left(\frac{8}{t^3}\right)^2 + \left(\frac{-2}{t^2}\right)^2 + \left(\frac{-10}{t^6}\right)^2} = \sqrt{\left(\frac{1}{t^4}\right)\left(\frac{64}{t^2} + 4 + \frac{100}{t^8}\right)}$

13.4 Cont.

11) $\frac{|r'(1) \times r''(1)|}{|r'(1)|^3} = \frac{\sqrt{(64+4+100}}{3} = \boxed{\frac{\sqrt{168}}{3} = k(1)}$

17) $f(t) = y = t^4$ @ $t = 2$ $\quad k(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$

$k(t) = \frac{|12t^2|}{(1+(4t^3)^2)^{3/2}} \quad k(2) = \frac{|12(4)|}{(1+(32)^2)^{3/2}} = \frac{64}{(1025)^{3/2}}$

$= \boxed{\frac{64}{125(41)^{3/2}}} \boxed{\approx 1.95E\text{-}3}$

21) $r(t) = \langle t - \tanh(t), \operatorname{sech}(t) \rangle$

$r'(t) = \langle \cancel{1} + (\cancel{1} + \tanh^2(x)), -\operatorname{sech}(x)\tanh(x) \rangle$

$r''(t) = \langle 2(\tanh(x)) \cdot (1 - \tanh^2(x)), + \operatorname{sech}(x)\tanh^2(x) + (-\operatorname{sech}(x)(1-\tanh^2(x))) \rangle$

$= \langle 2(\tanh(x) - \tanh^3(x)), \operatorname{sech}(x)\tanh^2(x) + (\operatorname{sech}(x)(\tanh^2(x) - 1)) \rangle$

$|r'(t)| = \sqrt{\tanh^4(x) + \operatorname{sech}^2(x)\tanh^2(x)}$

$= \boxed{\tanh} \cancel{\sqrt{\tanh^2(x) + \operatorname{sech}^2(x)}} = 1$

$(|r'(t)|)^3 = \tanh^3(x)$

$|r'(t) \times r''(t)| = \left| \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \tanh^2(x) & -\operatorname{sech}(x)\tanh(x) & 0 \\ 2\tanh(x)(1-\tanh^2(x)) \; [\operatorname{sech}^2(x)] & (\operatorname{sech}(x)(1-\operatorname{sech}^2(x)) - \operatorname{sech}^3(x)) & 0 \end{bmatrix} \right|$

Not worth the effort, sorry

13.5 HW

3) $r(t) = \langle t^3, 1-t, 4t^2 \rangle$ @ $t=1$

$v(t) = r'(t) = \langle 3t^2, -1, 8t \rangle$ @ $t=1$

$a(t) = r''(t) = \langle 6t, 0, 8 \rangle$

$|v(t)| = \sqrt{9t^4 + 1 + 64t^2} \overset{@t=1}{=} \sqrt{74}$

5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos(3\theta) \rangle$ $\theta = \frac{\pi}{3}$

$v(\theta) = \langle \cos(\theta), -\sin(\theta), -3\sin(3\theta) \rangle$

$a(\theta) = \langle -\sin\theta, -\cos(\theta), +9\cos(3\theta) \rangle$

$\boxed{|v(\frac{\pi}{3})| = \sqrt{(1) + 9\sin(\pi)} = 1}$

15) $a(t) = t\hat{i} + 4\hat{j}$ $v(0) = \langle 3, -2 \rangle$ $r(0) = \langle 0, 0 \rangle$

$v(t) = \int a(t)\,dt = \int (t\hat{i} + 4\hat{j})\,dt = \frac{t^2}{2}\hat{i} + 4t\hat{j} + C$

$v(0) = \frac{0}{2}\hat{i} + 0\hat{j} + \boxed{C = 3\hat{i} - 2\hat{j}}$

$v(t) = (\frac{t^2}{2} + 3)\hat{i} + (4t - 2)\hat{j}$

$r(t) = \int v(t)\,dt = \int ((\frac{t^2}{2} + 3)\hat{i} + (4t - 2)\hat{j})\,dt$

$\boxed{r(t) = (\frac{t^3}{6} + 3t)\hat{i} + (\frac{t^2}{2} - 2t)\hat{j}}$ ~~$(+C)$~~

$r(0) = 0\hat{i} + 0\hat{j} + C = \langle 0, 0 \rangle$

13.5 Cont.

17) $a(t) = \langle 0, 0, t \rangle \quad v(0) = \hat{\imath} \quad r(0) = \hat{\jmath}$

$v(t) = \frac{t^2}{2}\hat{k} + C = 1\hat{\imath} + \frac{t^2}{2}\hat{k}$

$r(t) = t\hat{\imath} + \frac{t^3}{6}k + C = \langle t, 1, \frac{t^3}{6} \rangle$

31) $v(\alpha) = \langle 12, 20, 20 \rangle \quad a(\alpha) = \langle 2, 1, -3 \rangle$

$\theta_{v\text{-}a} = \cos^{-1}\left(\frac{v \cdot a}{|v||a|}\right) = \cos^{-1}\left(\frac{24 + 20 - 60}{(30.7)(3.74)}\right)$

$\theta = 98.0 > 90$

$\therefore$ Slowing Down ?

33) $r(t) = \langle t, \cos(t), \sin(t) \rangle$

$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|} \quad a_N = K|v|^2 = \frac{|r' \times r''|}{|r'|^2}$

$r'(t) = \langle 1, -\sin(t), \cos(t) \rangle$

$r''(t) = \langle 0, -\cos(t), -\sin(t) \rangle$

$a_T = \frac{1 + \sin(t)\cos(t) - \sin(t)\cos(t)}{\sqrt{1 + (1)}} = \frac{1}{\sqrt{2}}$

$a_N = \frac{\sqrt{(\sin^2(t) + \cos^2(t))^2 + (+(+\sin(t))^2 + (-\cos(t))^2}}{\sqrt{2}}$

14.1: 1, 3, 7, 21, 23, 33, 35, ~~X~~
14.2: 9, 11, 15, 21, 23, 27, 31, 35

Chapter 14 HW

14.1

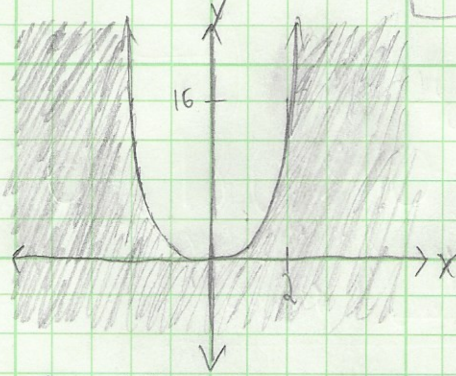
1) $f(x, y) = x + yx^3$ @ $(2, 2), (-1, 4)$

$f(2, 2) = 18$ $\quad f(-1, 4) = -5$

3) $h(x, y, z) = xyz^{-2}$ @ $(3, 8, 2), (3, -2, -6)$

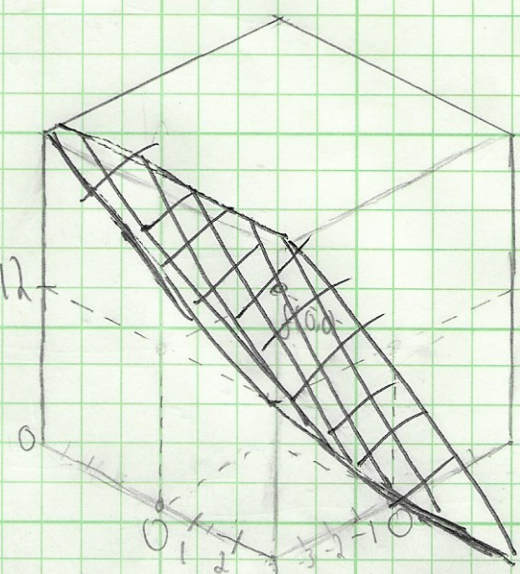
$h(3, 8, 2) = 6$ $\quad h(3, -2, -6) = \frac{-1}{6}$

7) $f(x, y) = \ln(4x^2 - y)$ $\quad \boxed{4x^2 - y > 0}$



21) $f(x, y) = 12 - 3x - 4y$

A plane w/ a
z-intercept of 12,
x-slope of -3,
y-slope of -4

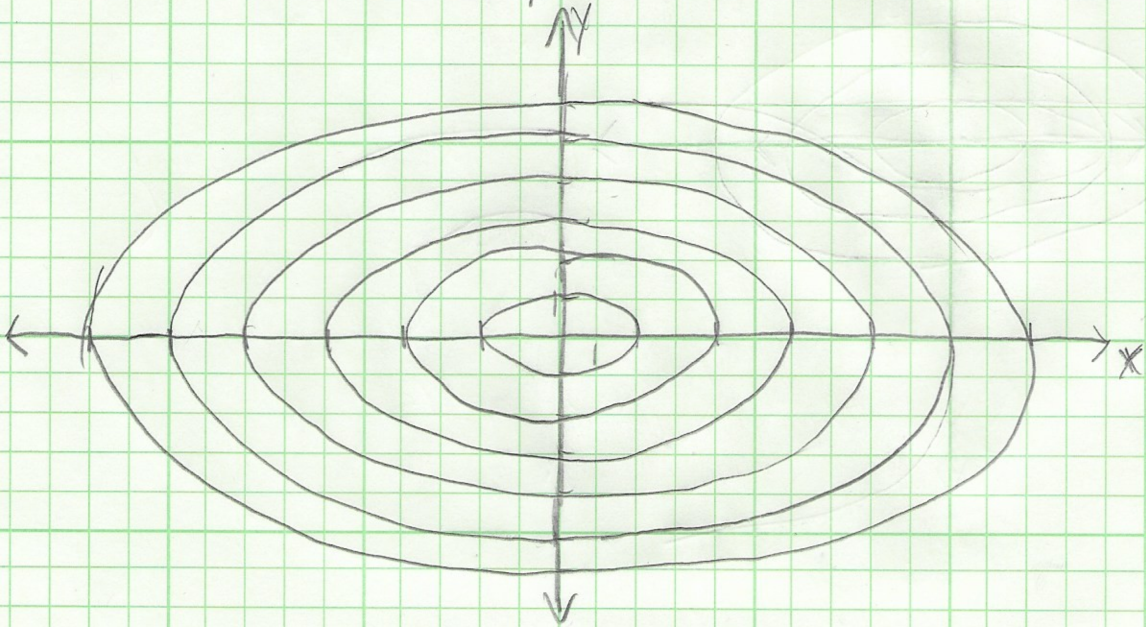


Dear professor: I am here to learn how to use multi-dimensional funx. If you just want a 3D graph, plug it into Maple!!

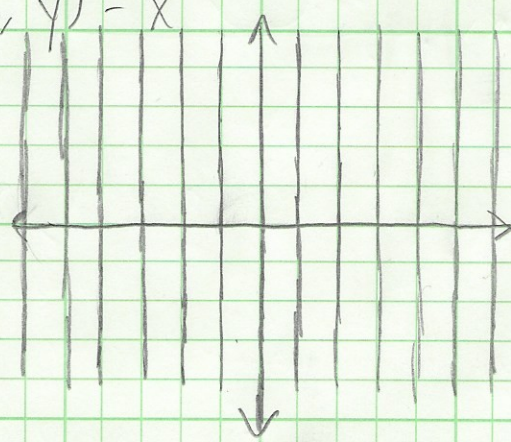
How the hell do I draw a plane in 3D space by hand???

14.1 Cont

33) $f(x, y) = x^2 + 4y^2$



35) $f(x, y) = x^2$



14.2 HW

7) $\lim_{(x,y)\to(2,5)} f(x, y) = 3 \quad \lim_{(x,y)\to(2,5)} g(x, y) = 7$

9) $\lim_{(x,y)\to(2,5)} \left(g(x, y) - 2(f(x, y))\right) = \boxed{1}$

11) $\lim_{(x,y)\to(2,5)} en\left((f(x, y))^2 - g(x, y)\right) = \boxed{e^2}$

14.2 HW

15) $f(x,y) = \dfrac{x^3+y^3}{xy^2}$ $y = mx$

$$\lim_{(x,y)\to(0,0)} = \frac{x^3(1+m^3)}{x^3(m^2)} = \frac{1+m^3}{m^2} = \boxed{m + \frac{1}{m^2}}$$

Depends on slope $\underline{m}$, $\therefore$ $\boxed{\text{DNE}}$

21) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$ $y = mx \Rightarrow \displaystyle\lim_{(x,y)\to(0,0)} \frac{mx^2}{(3+2m)x^2}$

$$= \frac{m}{3+2m} \therefore \boxed{\text{DNE}}$$

23) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$ $y=0$, $z=0$ $\rightarrow$ $\boxed{\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x}{x^2} = \frac{1}{0}}$

$\therefore$ DNE

27) $\displaystyle\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \frac{16\cos(\pi)}{e^{-1}} = \boxed{-16e}$

31) $\displaystyle\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \boxed{\frac{1}{5}}$

35) $\displaystyle\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy) = 9(-8)+4(+6) = \boxed{-48}$