homework 3

13.3 3.) $r(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \le t \le 4$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$L \int_a^b ||r'(t)|| dt$

$\int_1^4 3 + \ln t \rightarrow 3(4) + \ln(4) + 3(1) + \ln(1) =$

$12 + \ln(4) + 3 + \cancel{\ln(1)}\ 0$

$15 + \ln(4)$

9.) $r(t) = \langle t^2, 2t^2, t^3 \rangle \ ; \ a = 0$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$s = \int_a^t \sqrt{(f'(u))^2 + (g'(u))^2 + (h'(u))^2}\ du$

$s = \int_a^t \sqrt{(2t)^2 + (4t)^2 + (3t^2)^2}\ du$

$s = \int_a^t \sqrt{4t^2 + 16t^2 + 9t^4}\ du$

$s = \int_a^t \sqrt{t^2(4 + 16 + 9t^2)}\ du$

$\sqrt{t^2(20 + 9t^2)}\ du$

$s = \int_a^t t\sqrt{20 + 9t^2}\ du$

$u = 9x^2 + 20$

$du = 18x$

$dx = \frac{1}{18x}$

$\frac{1}{18}\int \sqrt{u}\ du \rightarrow \frac{1}{18}\int u^{\frac{1}{2}}\ du \rightarrow \frac{\cancel{2/3}}{\cancel{18}} \quad 18 \cdot u^{3/2} = \frac{u^{3/2}}{27}$

$\frac{(9t^2+20)^{3/2}}{27} \Big|_0^t \rightarrow \frac{(9t^2+20)^{3/2}}{27} - \frac{(9(0)^2+20)^{3/2}}{27} = \boxed{\frac{(9t^2+20)^{3/2}}{27} - \frac{(20)^{3/2}}{27}}$

11.) $r(t) = \langle 2t+3, 4t-3, 5-t \rangle \quad t = 4$

$r(t) = \langle 3, -3, 5 \rangle + t\langle 2, 4, -1 \rangle$

$r'(t) = \langle 2, 4, -1 \rangle$

$||r'(t)|| = \sqrt{2^2 + 4^2 + (-1)^2} = \sqrt{4+16+1} = \boxed{\sqrt{21}}$

13.) $r(t) = \langle t, \ln t, (\ln t)^2 \rangle \quad t = 1 \qquad 2\ln t \cdot \frac{1}{t} \rightarrow \frac{2\ln t}{t}$

$r'(t) = \langle 1, \frac{1}{t}, \frac{2\ln t}{t} \rangle$

$\rightarrow 4\ln(1) = 0$

$||r'(t)|| = \sqrt{1^2 + (\frac{1}{t})^2 + (\frac{2\ln t}{t})^2} = \sqrt{1 + \frac{1}{t^2} + \frac{4\ln^2 t}{t}} \rightarrow \sqrt{1 + 1 + 0} = \boxed{\sqrt{2}}$

15.) $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle \quad t = \frac{\pi}{2}$

$r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$||r'(t)|| = \sqrt{(3\cos 3t)^2 + (-4\sin 4t)^2 + (-5\sin 5t)^2} = \sqrt{(9\cos^2 3t) + (16\sin 4t) + (25\sin 5t)} =$

$\sqrt{9\cos^2 \frac{3\pi}{2} + 16\sin^2 2\pi + 25\sin \frac{5\pi}{2}} =$

$\sqrt{0 + 0 + 25(1)} = \sqrt{25} = \boxed{5}$

13.4

1.) $r(t) = \langle 4t^2, 9t \rangle$

$\boxed{r'(t) = \langle 8t, 9 \rangle}$

$||r'(t)|| = \sqrt{(8t)^2 + 9^2} = \sqrt{64t^2 + 81}$

$\boxed{T(t) = \frac{\langle 8t, 9 \rangle}{\sqrt{64t^2+81}}} \rightarrow \boxed{T(1) = \left\langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \right\rangle}$

5.) $r(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$\rightarrow r'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$= \sqrt{(-\pi \sin \pi 1)^2 + (\pi \cos \pi 1)^2 + 1^2}$

$= \sqrt{0 + \pi^2 + 1} = \sqrt{\pi + 1}$

$\boxed{T(t) = \frac{\langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle}{\sqrt{\pi^2+1}}} \rightarrow \boxed{T(1) = \left\langle 0, -\frac{\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \right\rangle}$

7.) $r(t) = \langle 1, e^t, t \rangle$

$K = \frac{||T'(t)||}{||r'(t)||} \rightarrow r'(t) = \langle 0, e^t, 0 \rangle \rightarrow ||r'(t)|| = \sqrt{0^2 + (e^t)^2 + 0^2} = \sqrt{e^{2t}} = e^t$

$T(t) = \frac{\langle 0, e^t, 0 \rangle}{(1 + e^{2t})^{3/2}} \rightarrow \boxed{K = \frac{e^t}{(1+e^{2t})^{3/2}}}$

11.) $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle \quad t = -1$

$r'(t) = \langle -\frac{1}{t^2}, -\frac{2}{t^3}, 2t \rangle \quad ||r'(t)|| = \sqrt{(-\frac{1}{t^2})^2 + (\frac{2}{t^3})^2 + (2t)^2} = \sqrt{1+4+4} = \sqrt{5}$

$K = \frac{||T'(t)||}{||r'(t)||} = \boxed{\frac{2\sqrt{14}}{\sqrt{27}}}$

17.) $y = t^4$ $t = 2$

$$K = \frac{\|f''(x)\|}{(1+[f'(x)]^2)^{3/2}}$$

$y' = 4t^3$ $12^2 t^4 = 144 \cdot 2^4 = 2304$

$y'' = 12t^2$ $1 + (4t^3)^2 = (1+16t^6)^{3/2}$

$(1+16(2))^{3/2} = (33)^{3/2}$ $\Rightarrow$ $\boxed{0.0015}$

21.) $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle \rightarrow K(t) = \operatorname{sech} t$

$r'(t) = \langle 1 - \operatorname{sech}^2 t, -\operatorname{sech} t \tanh t \rangle$

$r''(t) = \langle 2 \tanh t \operatorname{sech}^2 t, \operatorname{sech} t (\tanh^2 t - \operatorname{sech}^2 t) \rangle$

$r''(t) = \langle 2 \tanh t \operatorname{sech}^2 t, \operatorname{sech} t (1 - 2\operatorname{sech}^2 t) \rangle$

$r'(t) \times r''(t) = (\tanh^2 t \operatorname{sec} t (1 - 2\operatorname{sech}^2 t) + (\operatorname{sech} t \tanh t)(2 \operatorname{sech}^2 t \tanh t)) k$

$r'(t) \times r''(t) = \tanh^2 t \operatorname{sech} t$

$$K(t) = \frac{\|\tanh^2 \operatorname{sech} t\, k\|}{\|\langle \tanh^2 t, -\operatorname{sech} t \tanh t \rangle\|^3} = \frac{\tanh^2 t \operatorname{sech} t}{(\sqrt{\tanh^4 t + \operatorname{sech} t^2 t \tanh^2 t})^3}$$

$$K(t) = \frac{\tanh^2 t \operatorname{sech} t}{(\tanh t)^3} \rightarrow \frac{\operatorname{sech} t}{\tanh t} = \frac{1}{\sinh t} = \boxed{K(t) = \operatorname{csch} t}$$

13.5

3.) $r(t) = \langle t^3, 1 - t, 4t^2 \rangle$ $t = 1$

$v(t) = \langle 3t^2, -1, 8t \rangle \rightarrow v(1) = \boxed{\langle 3, -1, 8 \rangle}$ $\|v\| = \sqrt{3^2+1^2+8^2} = \boxed{\sqrt{74}}$

$a(t) = \langle 6t, 0, 8 \rangle \rightarrow a(1) = \boxed{\langle 6, 0, 8 \rangle}$

5.) $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$ $\theta = \frac{\pi}{3}$

$v'(\theta) = \langle \cos\theta, -\sin\theta, 3 \sin 3\theta \rangle \rightarrow v(\frac{\pi}{3}) = \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, -3\sin\frac{\pi}{3}(3) \rangle = \boxed{\langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \rangle}$

$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$ $a(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, -\cos\theta\frac{\pi}{3}, -9\cos\frac{\pi}{3}(3) \rangle = \boxed{\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9 \rangle}$

$|v| = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 + 0} = \sqrt{0.25 + 0.75} = \boxed{1}$

15.) $a(t) = \langle t, 4 \rangle$ $v(0) = \langle \frac{1}{3}, -2 \rangle$ $r(0) = \langle 0, 0 \rangle$

$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$r(t) = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

17.) $a(t) = tk$, $v(0) = i$, $r(0) = j$

$v(t) = i + \frac{t^2}{2} k$

$r(t) = ti + j + \frac{t^3}{6} k$

31.) $v = \langle 12, 20, 20 \rangle$
$a = \langle 2, 1, -3 \rangle$
speed is decreasing

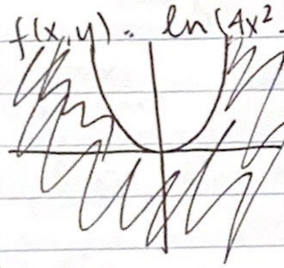
33.) $r(t) = \langle t^2, t^3 \rangle$
$a_T = 0$
$a_N = 1T$

14.1

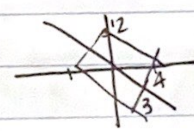
1.) $f(x,y) = x + yx^3 \quad (2,2), (-1,4)$
$f(2,2) = 2 + (2)(2)^3 = 18$
$f(-1,4) = (-1) + (4)(-1)^3 = -5$

3.) $h(x,y,z) = xyz^{-2} \quad (3,8,2), (3,-2,-6)$
$h(3,8,2) = 3 \cdot 8 \cdot 2^{-2} = 24 \cdot \frac{1}{4} = 6$
$h(3,-2,-6) = 3 \cdot -2 \cdot -6^{-2} = -6 \cdot \frac{1}{36} = -\frac{1}{6}$

7.) $f(x,y) = \ln(4x^2 - y)$

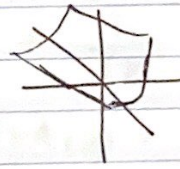


21.) $f(x,y) = 12 - 3x - 4y$



hor: $3x + 4y = 12 - c$
vert: $z = (12 - 3a) - 4y \quad z = -3x + (12 - 4a)$

23.) $f(x,y) = x^2 + 4y^2$



hor: $c > 0$
vert x: $z = a^2 + 4y^2$
vert y: $z = x^2 + 4a^2$

33.) $f(x,y) = x^2 + 4y^2$
not sure how to draw

35.) $f(x,y) = x^2$
also not sure how to draw

37.) $m = 6 \rightarrow$ linear if ~~...~~ $m = 3$
$m = 6 \rightarrow f(x,y) = 2x + 4y + 6$
$m = 3 \rightarrow f(x,y) = x + 3y + 3$

9, 11, 15, 21, 23, 27, 31, 35

14.2

9.) $\lim_{(x,y)\to(2,5)} f(x,y) = 3 \qquad \lim_{(x,y)\to(2,5)} g(x,y) = 7$

$(g(x,y) - 2f(x,y))$

$7 - 2(3) = 1$

11.) $e^{f(x,y)^2 - g(x,y)}$

$e^{9-7} = e^2$

15.) $f(x,y) = \dfrac{x^3+y^3}{xy^2} \to \dfrac{1+m^3}{m^2}$ while $m \neq 0$

21.) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{3x^2+2y^2} =$ limit DNE

$\lim_{x\to 0} \dfrac{mx^2}{3x^2+2m^2x^2} = \dfrac{m}{2m^2+3}$

23.) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{x+y+z}{x^2+y^2+z^2} \to \lim_{(x,y,z)\to(0,0,0)} \qquad \dfrac{1}{x} = \infty$

27.) $\lim_{(z,w)\to(-2,1)} \dfrac{z^4\cos(\pi w)}{e^{z+w}} = \dfrac{(-2)^4 \cos(1)\pi}{e^{-1}} = -16e$

31.) $\lim_{(x,y)\to(3,4)} \dfrac{1}{\sqrt{x^2+y^2}} \to \cancel{\dfrac{1}{\sqrt{9+16}}} = \cancel{\dfrac{1}{\sqrt{25}}} \cancel{\dfrac{1}{5}}$

$\dfrac{1}{\sqrt{9+16}} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

35.) $\lim_{(x,y)\to(-3,2)} (x^2y^3+4xy) = ((-3)^2(-2)^3 + 4(-3)(2)) = -48$