Krithika Patrachari

13.3: 3, 9, 11, 13, 15

3) $r(t) = \langle 2t, \ln t, t^2 \rangle$, $1 \le t \le 4$

$r'(t) = \langle 2, 1/t, 2t \rangle$

$|r'(t)| = \sqrt{4 + 1/t^2 + 4t^2}$

$\int_1^4 (4 + \frac{1}{t^2} + 4t^2)^{1/2}$

$L = 15 + \ln(4) - \cancel{\ln(1)}^{0}$

$L = 15 + \ln(4)$

9) $r(t) = \langle t^2, 2t^2, t^3 \rangle$, $a = 0$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$|r'(t)| = \sqrt{4t^2 + 16t^2 + 9t^4} = \sqrt{20t^2 + 9t^4}$

$s(t) = \int_0^t \sqrt{20t^2 + 9t^4}$

$= \frac{1}{27}\left((20 + t^2 + 9t^4)^{3/2} - 20^{3/2}\right)$

11) $r(t) = \langle 2t+3, 4t-3, 5-t \rangle$, $t = 4$

$r'(t) = \langle 2, 4, -1 \rangle$

$|r'(t)| = \sqrt{4 + 16 + 1} = \sqrt{21}$

13) $r(t) = \langle t, \ln t, (\ln t)^2 \rangle$, $t = 1$

$r'(t) = \langle 1, \frac{1}{t}, 2\ln t \cdot \frac{1}{t} \rangle$

$|r'(t)| = \sqrt{1^2 + \frac{1}{t^2} + (2\ln t \cdot \frac{1}{t})^2}$

$|r'(1)| = \sqrt{1 + 1 + 0} = \sqrt{2}$

15) $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle$, $t = \frac{\pi}{2}$

$r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$|r'(t)| = \sqrt{(3\cos 3t)^2 + (-4\sin 4t)^2 + (-5\sin 5t)^2}$

$|r'(\frac{\pi}{2})| = \sqrt{(0)^2 + (0)^2 + (-5)^2} = 5$

13.4 : 1, 5, 7, 11, 17, 21

1) $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle \rightarrow |r'(t)| = \sqrt{64t^2+81}$

$T(t) = \dfrac{\langle 8t, 9 \rangle}{\sqrt{64t^2+81}}$

$T(1) = \left\langle \dfrac{8}{\sqrt{145}}, \dfrac{9}{\sqrt{145}} \right\rangle$

5) $r(t) = \langle 3+4t, 3-5t, 9t \rangle$

$r'(t) = \langle 4, -5, 9 \rangle$

$|r'(t)| = \sqrt{16+25+81} = \sqrt{122}$

$T(t) =$
$T(1) = \left\langle \dfrac{4}{\sqrt{122}}, \dfrac{-5}{\sqrt{122}}, \dfrac{9}{\sqrt{122}} \right\rangle$

7) $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$|r'(t)| = \sqrt{1+e^{2t}}$

$r'(t) \times r''(t)$

$$\begin{vmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix} = (-e^t)i$$

$\langle -e^t, 0, 0 \rangle$

$|r'(t) \times r''(t)| = \sqrt{(-e^t)^2} = e^t$

$k(t) = \dfrac{e^t}{(\sqrt{1+e^{2t}})^3}$

11) $r(t) = \langle 1/t, 1/t^2, t^2 \rangle$, $t = -1$

$r'(t) = \langle -t^{-2}, -2t^{-3}, 2t \rangle$

$r'(-1) = \langle -1, 2, -2 \rangle$

$r''(t) = \langle 2t^{-3}, 6t^{-4}, 2 \rangle$

$r''(-1) = \langle -2, 6, 2 \rangle$

$|r'(-1)| = \sqrt{1+4+4}$
$= \sqrt{9} = 3$

$r'(-1) \times r''(-1)$

$$\begin{vmatrix} i & j & k \\ -1 & 2 & -2 \\ -2 & 6 & 2 \end{vmatrix} = (-4+12)i - (-2+4)j + (-6+4)k$$

$= 8i - 2j - 2k$

$\langle 8, -2, -2 \rangle$

$\sqrt{64+4+4}$
$= \sqrt{72}$

$k = \dfrac{|r'(1) \times r''(1)|}{|r'(1)|^3}$

$= \dfrac{2\sqrt{74}}{27}$

17) $y = t^4$, $t = 2$

$f' = 4t^3 \rightarrow f'(2) = 32$

$f'' = 12t^2 \rightarrow f''(2) = 48$

$|f''| = 48$

$k(2) = \dfrac{|f''(2)|}{(1+f'(2)^2)^{3/2}} = \dfrac{48}{(1+32^2)^{3/2}} = \dfrac{48}{(1025)^{3/2}}$

21) show that tractrix $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$ has the curvature function $k(t) = \operatorname{sech} t$

$r'(t) = \langle 1 - \sec^2 ht, \sec\tan ht \rangle$

$r''(t) = \langle -\sec^2\tan^2(ht), \sec^3\tan ht \rangle$

? Answer not in book?

13.5 : 3, 5, 15, 17, 31, 33

3) $r(t) = \langle t^3, 1-t, 4t^2 \rangle$, $t=1$

$v = r'(t) = \langle 3t^2, -1, 8t \rangle = \langle 3, -1, 8 \rangle$

$a = r''(t) = \langle 6t, 0, 8 \rangle = \langle 6, 0, 8 \rangle$

$s = |v| = |r'(t)| = \sqrt{9+1+64} = \sqrt{74}$

5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$, $\theta = \frac{\pi}{3}$

$v = r'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$= \langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \rangle$

$a = r''(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$= \langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, -9 \rangle$

$s = |r'\theta| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

15) $a(t) = \langle t, 4 \rangle$, $v(0) = \langle 3, -2 \rangle$
$r(0) = \langle 0, 0 \rangle$

$v(t) = \int \langle t, 4 \rangle = \langle \frac{1}{2}t^2 + c, 4t + c \rangle$

$\langle \frac{1}{2}t^2 + 3, 4t - 2 \rangle$

$r(t) = \int \langle \frac{1}{2}t^2 + 3, 4t - 2 \rangle = \langle \frac{1}{6}t^3 + 3t, 2t^2 - 2t \rangle$

17) $a(t) = tk$ $v(0) = i$ $r(0) = j$

$\langle 0, 0, t \rangle$ $\langle 1, 0, 0 \rangle$ $\langle 0, 1, 0 \rangle$

$v = \int \langle 0, 0, t \rangle = \langle c, c, \frac{1}{2}t^2 \rangle$

$= \langle 1, 0, \frac{1}{2}t^2 \rangle$

$r = \int \langle 1, 0, \frac{1}{2}t^2 \rangle = \langle t, c, \frac{1}{6}t^3 \rangle$

$= \langle t, 1, \frac{1}{6}t^3 \rangle$

31) At a certain moment, a particle moving along a path has a velocity $v = \langle 12, 20, 20 \rangle$ and an acceleration $a = \langle 2, 1, -3 \rangle$

The particle is slowing down since the velocity and acceleration are opposite signs in the z axis.

33) $r(t) = \langle t, \cos t, \sin t \rangle$

$r'(t) = \langle 1, -\sin t, \cos t \rangle$

$r''(t) = \langle 0, -\cos t, -\sin t \rangle$

$|r'(t)| = \sqrt{1 + \sin^2 t + \cos^2 t} = \sqrt{2}$

$r' \times r''$

$$\begin{vmatrix} i & j & k \\ 1 & -\sin & \cos \\ 0 & -\cos & -\sin \end{vmatrix} = (\sin^2 t + \cos^2 t)i - (-\sin t)j + (-\cos t)k$$

$= \langle 1, \sin t, -\cos t \rangle$

$|r' \times r''| = \sqrt{1 + \sin^2 t + \cos^2 t} = \sqrt{2}$

$a_T = \frac{r' \cdot r''}{|r'|} = \frac{0 + \sin\cos t - \sin\cos t}{\sqrt{2}} = 0$

$a_N = \frac{|r' \times r''|}{|r'|} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

14.1 : 1, 3, 7, 21, 23, 33, 35

1) $f(x,y) = x + yx^3$, $(2,2)(-1,4)$

$f(2,2) = 2 + 2(8) = 18$

$f(-1,4) = -1 + 4(-1) = -5$

3) $h(x,y,z) = xyz^{-2}$, $(3,8,2)(3,-2,-6)$

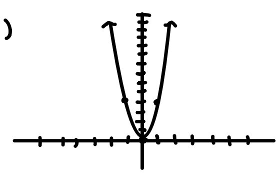
$h(3,8,2) = 3(8)(2^{-2}) = 6$

$h(3,-2,-6) = 3(-2)(-6^{-2}) = \frac{-3}{18} = -\frac{1}{6}$

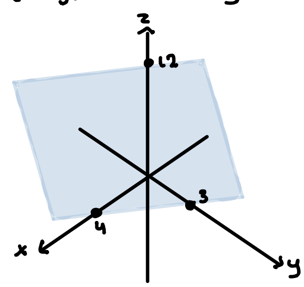
7) $f(x,y) = \ln(4x^2 - y)$

$4x^2 - y \geq 0$

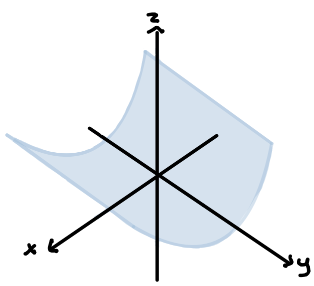
$4x^2 = y$



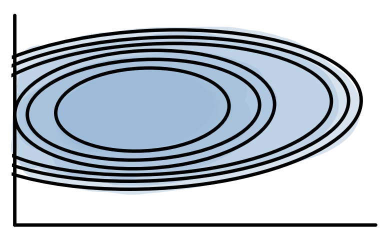
21) $f(x,y) = 12 - 3x - 4y$



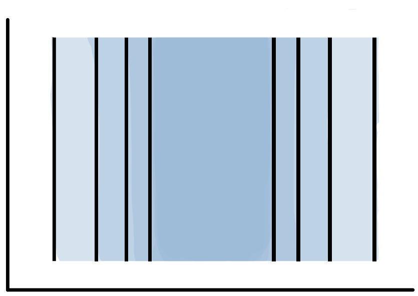
23) $f(x,y) = x^2 + 4y^2$



33) $f(x,y) = x^2 + 4y^2$



35) $f(x,y) = x^2$



14.2 : 9, 11, 15, 21, 23, 27, 31, 35

For 9 & 11: $\lim_{(x,y)\to(2,5)} f(x,y) = 3$ & $\lim_{(x,y)\to(2,5)} g(x,y) = 7$

9) $\lim_{(x,y)\to(2,5)} (g(x,y) - 2f(x,y))$

$= 7 - 2(3) = 7 - 6 = 1$

11) $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)}$

$e^{3^2-7} = e^{9-7} = e^2$

15) $f(x,y) = \dfrac{x^3+y^3}{xy^2}$ $\quad y = mx$

$f(x,mx) = \dfrac{x^3+(mx)^3}{x(mx)^2}$

$= \dfrac{\cancel{x^3}(1+m^3)}{\cancel{x^3}(m)} = \dfrac{1+m^3}{m}$

for all $m \neq 0$

21) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{3x^2+2y^2} = \dfrac{0(0)}{0+0} = 0$

$y = cx$

$\lim_{x\to0} \dfrac{x(cx)}{3x^2+2(cx)^2} = \lim_{x\to0} \dfrac{x^2}{x^2(3+2c)}$

$= \lim_{x\to0} \dfrac{c}{3+2}$ $\quad x = r\cos\theta$, $y = r\sin\theta$

$= \lim_{r\to0} \dfrac{r\cos\theta(\sin\theta)}{3r^2\cos^2\theta + 2r^2\sin^2\theta}$

$= \lim_{r\to0} \dfrac{\cos\theta\sin\theta}{r^2(3\cos\theta^2 + 2\sin^2\theta)} = \dfrac{\cos\sin}{0}$

The limit does not exist

23) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{x+y+z}{x^2+y^2+z^2}$ $\quad \dfrac{0+0+0}{0+0+0} = \dfrac{0}{0}$

$y = cx$

$\lim_{x\to0} \dfrac{x+cx+z}{x^2+(cx)^2+z} = \dfrac{z}{z}$

$\dfrac{1}{x} = \infty$

27) $\lim_{(z,w)\to(-2,1)} \dfrac{z^4\cos(\pi w)}{e^{z+w}}$

$= \dfrac{(-2)^4\cos(\pi)}{e^{-2+1}} = \dfrac{16(-1)}{e^{-1}} = -16e$

31) $\lim_{(x,y)\to(3,4)} \dfrac{1}{\sqrt{x^2+y^2}} = \dfrac{1}{\sqrt{9+16}} = \dfrac{1}{5}$

35) $\lim_{(x,y)\to(-3,-2)} (x^2y^3 + 4xy)$

$= (9(-8) + 4(-3)(-2))$

$= (-72 + 24) = -48$