9/19/20 13.3 Arc Length and Speed HW.

13.3 # 3, 9, 11, 13, 15.

3) $r(t) = \langle 2t, \ln(t), t^2 \rangle$ from $0 \le t \le 3$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$\|r'(t)\| = \sqrt{4 + \frac{1}{t^2} + 4t^2}$

$= \frac{1}{t} + 2t$

$\int_0^3 \left(\frac{1}{t} + 2t\right) dt$ $\quad t^{-1}$

$\ln(t) + t^2 \Big|_1^4$

~~$\ln(3t) + 9$~~

$\ln(4) + 16 - 1$

$15 + \ln(4)$

9) $r(t) = \langle t^2, 2t^2, t^3 \rangle$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

~~$\|r'(t)\|$~~ $\|r'(t)\| = \sqrt{4t^2 + 16t^2 + 9t^4}$

$\int_0^t \sqrt{20t^2 + 9t^4}\, dt$

$\int_0^t t\sqrt{9t^2 + 20}\, dt$ let $9t^2 + 20 = u$

$du = 18t\,dt$

$\frac{1}{18} \int_0^t \sqrt{u}\, du$ $\quad u^{\frac{1}{2}}$

$u^{-\frac{1}{2}}$ $\quad -\frac{1}{2} u^{-\frac{1}{2}}$

$\frac{1}{27} (9t^2 + 20)^{\frac{3}{2}} \Big|_0^t$ $\quad \frac{1}{2} u^{\frac{1}{2}}$

$\frac{3}{2} u^{\frac{1}{2}}$

$= \frac{1}{27} (9t^2 + 20)^{\frac{3}{2}}$

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11) $r(t) = \langle 2t+3,\ 4t-3,\ 5-t \rangle$ at $t=4$

$r'(t) = \langle 2,\ 4,\ -1 \rangle$

$\|r'(t)\| = \sqrt{16+4+1} = \sqrt{21}$

at $t=4$, speed $= \sqrt{21}$.

13) $r(t) = \langle t,\ \ln(t),\ (\ln t)^2 \rangle$

$r'(t) = \langle 1,\ \frac{1}{t},\ \frac{2\ln(t)}{t} \rangle$

$\|r'(t)\| = \sqrt{1 + \frac{1}{t^2} + \frac{(2\ln(t))^2}{t^2}}$ at $t=1$

$= \sqrt{1+1} = \sqrt{2}$

15) $r(t) = \langle \sin 3t,\ \cos 4t,\ \cos 5t)$ $t = \frac{\pi}{2}$

$r'(t) = \langle 3\cos 3t,\ -4\sin 4t,\ -5\sin 5t \rangle$

$\|r'(\frac{\pi}{2})\| = \sqrt{9\cos^2 3t + 16\sin^2 \sin 4t + 25\sin^2 \sin 5t}$

$= 5$

9/20/20 13.4 Curvature

13.4 # 1, 5, 7, 11, 17, 21

1) $r(t) = \langle 4t^2, 9t \rangle$.

$r'(t) = \langle 8t, 9 \rangle$ magnitude $= \sqrt{64t^2+81}$

$T(t) = \dfrac{\langle 8t, 9 \rangle}{\sqrt{64t^2+81}}$.

$T(1) = \dfrac{\langle 8, 9 \rangle}{\sqrt{145}} = \langle \dfrac{8}{\sqrt{145}}, \dfrac{9}{\sqrt{145}} \rangle$.

5) $r(t) = \langle \cos(\pi t), \sin(\pi t), t \rangle$

$r'(t) = \langle -\pi \sin(\pi t), \pi \cos(\pi t), 1 \rangle$

$T(t) = \dfrac{\langle -\pi \sin(\pi t), \pi \cos(\pi t, 1 \rangle}{\sqrt{\pi^2+1}}$

$T(1) = \langle 0, \dfrac{-\pi}{\sqrt{\pi^2+1}}, \dfrac{1}{\sqrt{\pi^2+1}} \rangle$.

7) $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

t^{-1} $\quad$ t^{-2}

7 continued)

$r'(t) \times r''(t)$

$$\begin{array}{ccc} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{array}$$

$= \boxed{-e^t\, i}$

$$K(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

$$K(t) = \frac{e^t}{(\sqrt{e^{2t}+1})^3}$$

11) $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle$ at $t = -1$

$r'(t) = \langle -t^{-2}, -2t^{-3}, 2t \rangle$, at $t=-1$ $\langle -1, 2, -2 \rangle$

$r''(t) = \langle 2t^{-3}, 6t^{-4}, 2 \rangle$ at $t=-1$ $\langle -2, 6, 2 \rangle$

$$r'(-1) \times r''(-1) = \begin{array}{ccc} i & j & k \\ -1 & 2 & -2 \\ -2 & 6 & 2 \end{array}$$

$16i + 6j + -2k$

Magnitude: $\sqrt{296}$

$\|r'(1)\|^3 = (\sqrt{1+4+4})^3 = 3^3$

$K(2) = \frac{\sqrt{296}}{27} = \frac{2\sqrt{74}}{27}$

17) $y = t^4$ at $t = 2$

$r'(t) = \langle 0, 4t^3, 0 \rangle$

$r''(t) = \langle 0, 12t^2, 0 \rangle$

$r(t) = \langle 0, t^4, 0 \rangle$

$\cancel{r'(t) = \langle 0, 4t^3, 0 \rangle}$

$\cancel{r''(t) = \langle 0, 12t^2, 0 \rangle}$

$\cancel{\text{at } t=2,\ r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & 32 & 0 \\ 0 & 48 & 0 \end{vmatrix}}$

$K(2) = \frac{12(2)^2}{(1+(4(2)^3)^2)^{\frac{3}{2}}} = \frac{48}{(1+16(2)^6)^{\frac{3}{2}}}$

$= \frac{48}{1025^{\frac{3}{2}}} =$

21) $r(t) = \langle t - \tanh t,\ \operatorname{sech} t \rangle$

$r'(t) = \langle \tanh^2 t,\ -\operatorname{sech}(t)\tanh(t) \rangle,$

$r''(t) = \langle 2\tanh t\, \operatorname{sech}^2 t,\ \operatorname{sech} t\,(1 - 2\operatorname{sech}^2 t) \rangle$

$$\begin{pmatrix} i & j & k \\ \tanh^2 t & -\operatorname{sech} t \tanh t & 0 \\ 2\tanh t\, \operatorname{sech}^2 t & \operatorname{sech} t & 0 \end{pmatrix}$$

$\det = \langle 0,\ 0,\ \tanh^2 \operatorname{sech} t - 2\tan^2 t\,\operatorname{sech}^3 t + \ldots$

$\rightarrow \operatorname{sech}^3 t \tan^2 t \rangle$

$= \langle 0,\ 0,\ \tanh^2 t \operatorname{sech} t \rangle.$

$$K(t) \quad \frac{\tanh^2 t \operatorname{sech} t}{\sqrt{\tan^4 t + t\operatorname{sech}^2 t \tan^2 t}^{\,3}}$$

$$\frac{\tanh^2 t \operatorname{sech} t}{(\tanh t)^3}$$

$$= \frac{\operatorname{sech} t}{\tanh t} = \frac{1}{\cosh t} \cdot \frac{\cosh t}{\sinh t}$$

$K(t) = \operatorname{csch} t.$

9/20/20 13.5 Motion in 3D Space.

13.5 # 3, 5, 15, 17, 31, 33

3) $r(t) = \langle t^3, 1-t, 4t^2 \rangle$

$r'(t) = \langle 3t^2, -1, 8t \rangle$

Velocity $= r'(1) = \langle 3, -1, 8 \rangle$

$r''(t) = \langle 6t, 0, 8 \rangle$

Acceleration $= \langle 6, 0, 8 \rangle$

Speed $= \|v\| = \sqrt{9+1+64} = \sqrt{74}$.

5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$ at $\theta = \frac{\pi}{3}$

$r'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$r''(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

Velocity $= r'(\frac{\pi}{3}) = \langle 1/2, -\sqrt{3}/2, 0 \rangle$

Acceleration $= r''(\frac{\pi}{3}) = \langle -\sqrt{3}/2, -1/2, 9 \rangle$

Speed $= \|v\| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

15) $a(t) = \langle t, 4 \rangle$

$v(t) = \int a(t)\,dt = \langle \frac{t^2}{2}, 4t \rangle + C$

$v(0) = \langle 3, -2 \rangle$

$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

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15 cont) $r(t) = \int v(t)\,dt$.

$\int \langle \frac{t^2}{2}+3, 4t-2 \rangle \,dt$

$= \langle \frac{t^3}{6}+3t, 2t^2-2t \rangle + C$

$r(0) = \langle 0, 0 \rangle$

thus, $r(t) = \langle \frac{t^3}{6}+3t, 2t^2-2t \rangle$ (position)

$r'(t) = \langle \frac{t^2}{2}+3, 4t-2 \rangle$ (velocity.

17) $a(t) = tk$ $v(0) = \langle 1, 0, 0 \rangle$, $r(0) = \langle 0, 1,$

$v(t) = \int a(t)\,dt$

$= \int \langle 0, 0, 1 \rangle \,dt$

$= \langle 0, 0, t \rangle + C$

at $v(0)$ $\langle 0,0,0 \rangle + C = \langle 1, 0, 0 \rangle$ $C = \langle 1, 0, 0$

$r(t) = \int v(t)\,dt$

$\int \langle 1, 0, 0 \rangle \,dt$

$\langle t, 0, 0 \rangle + C$

at $r(0)$ $\langle 0,0,0 \rangle + C = \langle 0, 1, 0 \rangle$

$v(t) = \langle 1, 0, 0 \rangle$

$r(t) = \langle 0, 1, 0 \rangle$.

17) $a(t) = tk, = \langle 0, 0, t \rangle$. $v(0) = \langle 1, 0, 0 \rangle$

$r(0) = \langle 0, 1, 0 \rangle$

$\int a(t)\,dt = \langle 0, 0, t^2/2 \rangle$

$v(0) = \langle 0, 0, 0 \rangle + C = \langle 1, 0, 0 \rangle$

$v(t) = \langle 1, 0, \frac{t^2}{2} \rangle$

$\int v(t)\,dt = \langle t, 0, \frac{t^3}{6} \rangle + C$

$r(0) = \langle 0, 1, 0 \rangle = \langle 0, 0, 0 \rangle + C$

$r(t) = \langle t, 1, \frac{t^3}{6} \rangle$

31) $v = \langle 12, 20, 20 \rangle$

$a = \langle 2, 1, -3 \rangle$

$\frac{v \cdot a}{\|v\| \|a\|} = \cos\theta$

$\approx \frac{-16}{(30)(3.74)} = \cos\theta = -0.1425$

$\theta = 98.19°$

Because the acceleration vector is "opposing" the velocity vector, the particle speed must be decreasing.

33) $r(t) = \langle t, \cos t, \sin t \rangle$

$r'(t) = \langle 1, -\sin t, \cos t \rangle$

$r''(t) = \langle 0, -\cos t, -\sin t \rangle$

$$a_T = \frac{a \cdot v}{\|v\|} = \frac{0 + \sin t \cos t - \sin t \cos t}{\sqrt{2}}$$

$a_N = a \cdot N$

$= \sqrt{\|a\| - |a_T|^2}$

$= \sqrt{1} = 1$

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9/21/20 14.1

14.1 # 1, 3, 7, 21, 23, 33, 35, 37.

1) $f(x,y) = x^3 y + x$

$f(2,2) = 2^3(2) + 2 = 18$

$f(-1,4) = (-1)^4 \cdot 4 + -1 = 3$

3). $h(x,y,z) = xyz^{-2}$

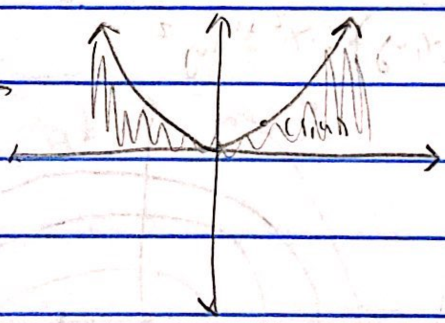
$h(3,8,2) = (3)(8) \cdot \frac{1}{4} = 6$ $\quad -6 \cdot \left(-\frac{1}{36}\right)$

$h(3,-2,-6) = -6 \cdot \left(+\frac{1}{36}\right) = -1/6$

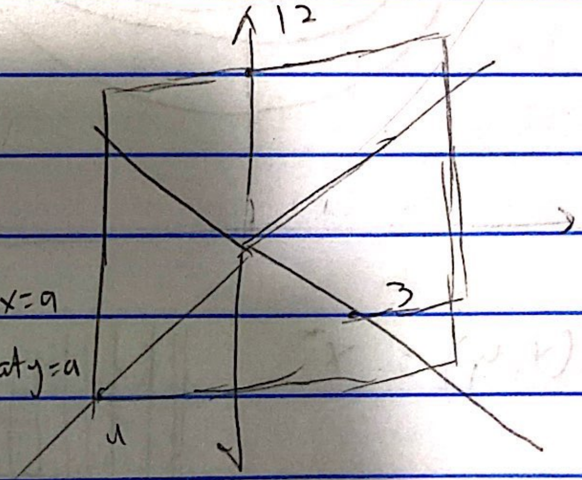
7) $f(x,y) = \ln(4x^2 - y)$

$4x^2 - y > 0$

$4x^2 > y$



1) $f(x,y) = 12 - 3x - 4y$



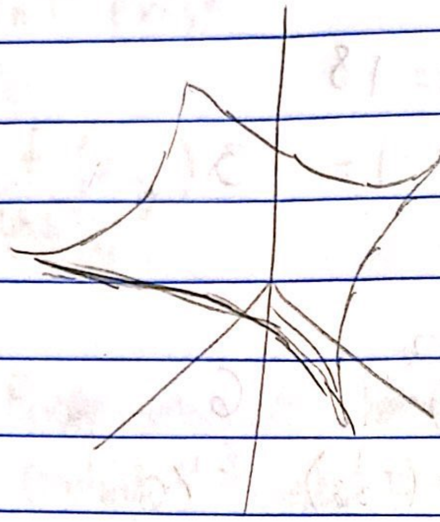
$3x + 4y = 12 - c$

$z = (12 - 3a) - 4y$ at $x = a$

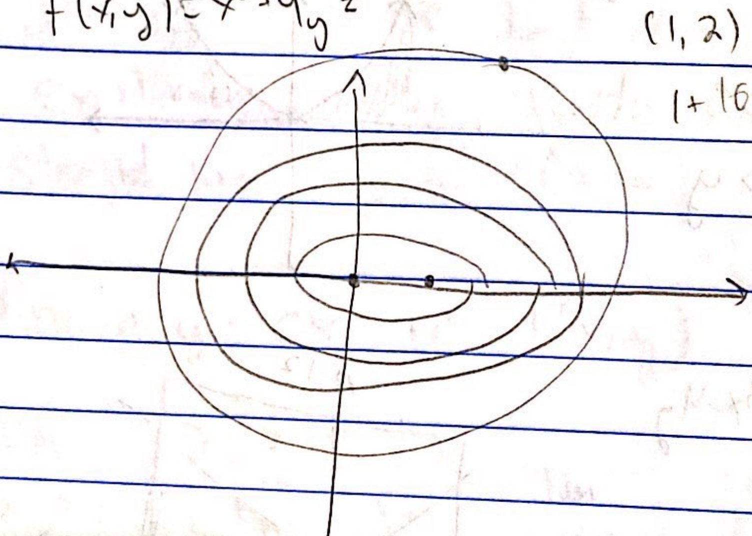
$z = -3x + 12(4 - a)$ at $y = a$

23) at $x=a$, $z=a^2+4y^2$

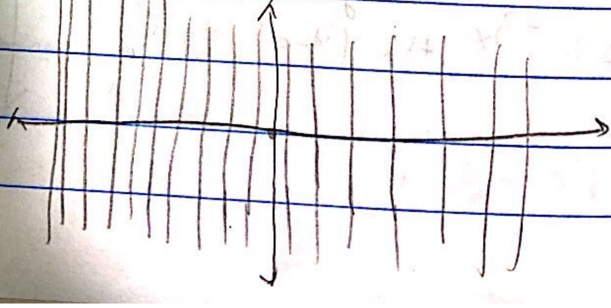
at $y=a$, $z=x^2+4a^2$



33) $f(x,y)=x^2+4y^2$



35) $f(x,y)=x^2$



37) ~~yes~~

When $m=6$

$f(0,0)= a(0)+b(0)+c=6$

$c=6$

$f(-3,0)=-3(a)+0+6=2 \quad a=2$

$b=6$

$m=6, \quad f(x,y)=2x+6y+6.$

When $m=3$

$f(0,0)\ c=3.$

$f(-3,0)=-3(a)+0+3 \quad a=1$

$b=-3$

$f(x,y)= x+3y+3$ at $m=3.$

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9/22/20 14.2 → Limits and Continuity in Several Variables

14.2 # 5, 15, 21, 23, 27, 31, 35

5) $\lim_{(x,y)\to(\frac{\pi}{4},0)} \tan x \cos y = 1 \cdot 1 = 1$

15) $f(x,y) = \dfrac{x^3 + y^3}{xy^2}$ $y = mx$

$$\frac{x^3 + x^3m^3}{x \cdot m^2x^2} = \frac{x^3 + x^3m^3}{x^3m^2}$$

$$= \frac{1+m^3}{m^2}$$ } The limit is dependent on m. Since the limit changes depending on m, there is no limit.

21) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{3x^2+2y^2}$

let $y = mx$

$$\frac{x^2m}{3x^2+2x^2m^2} = \frac{m}{3+2m^2}$$

→ Limit depends on m → DNE

23) $$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$$

Let's look along z axis $(x=0, y=0)$

$$\to \lim_{(x,y,z)\to(0,0,0)} \frac{z}{z^2} = \frac{1}{z} = \text{DNE}.$$

27) $$\lim_{(z,w)\to(-2,\pi)} \frac{z^4(\cos(\pi w))}{e^{z+w}} = \underline{-16e}$$

31) $$\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{5}$$

35) $$\lim_{(x,y)\to(-3,-2)} x^2y^3+4xy = 9(-8)+24$$

$$= -72+24$$

$$= -48$$