13.3

3) Compute arc length:

$\vec{r}(t) = \langle 2t, \ln t, t^2 \rangle$, $1 \le t \le 4$

Arc length $= \int_1^4 \sqrt{2^2 + (1/t)^2 + (2t)^2}\, dt = \int_1^4 \sqrt{4 + 1/t^2 + 4t^2} = \int_1^4 \sqrt{\frac{4t^2+1+4t^4}{t^2}}\, dt =$

$\int_1^4 \frac{\sqrt{4t^4+4t^2+1}}{t}\, dt = 15 + \ln(2)$

9) Compute arc length function:

$r(t) = \langle t^2, 2t^2, t^3 \rangle$, $a = 3$

$s(t) = \int_3^t \sqrt{4u^2 + 16u^2 + 9u^4}\, du = \int_3^t \sqrt{20u^2 + 9u^4}\, du = \int_3^t u\sqrt{20 + 9u^2}\, du = \int_3^t u\sqrt{20}\sqrt{1 + \frac{9}{20}u^2}\, du$

let $u = \sqrt{\frac{20}{9}} \tan\theta$ $\quad du = \sqrt{\frac{20}{9}} \sec^2\theta\, d\theta$ $\longrightarrow$

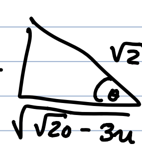
$\int_3^t \frac{20}{3} \tan\theta \sec\theta \left(\frac{\sqrt{20}}{9} \sec^2\theta\, d\theta \right) = \frac{20\sqrt{20}}{27} \int_3^6 \tan\theta \sec^3\theta\, d\theta$

$\int_3^t \tan\theta \sec\theta \sec^2\theta\, d\theta = \sec^3\theta - 2\int \tan \sec^3\theta$

$f(x) = \sec^2\theta$ $\qquad g(x) = \sec\theta$

$f'(x) = 2\sec^2\theta \tan\theta\, d\theta$ $\qquad g'(x) = \sec\theta \tan\theta\, d\theta$

$\boxed{\frac{\sec^3\theta}{2}}$



$s(t) = \frac{\sqrt{20}}{2\sqrt{\sqrt{20} - 3u}} \Big|_a^t \left[\left(\frac{20\sqrt{20}}{27} \right) \right]$

$u\sqrt{\frac{9}{20}} = \frac{3u}{\sqrt{20}}$

11) Find the speed

$r(t) = \langle 2t+3, 4t-3, 5-t \rangle$, $t = 4$

$v(t) = \langle 2, 4, -1 \rangle$

$\|v(t)\| = \sqrt{2^2+4^2+1^2} = \sqrt{21}$

13) Find the speed

$r(t) = \langle t, \ln(t), (\ln(t))^2 \rangle$, $t = 1$

$v(t) = \langle 1, 1/t, 2\ln(t)(1/t) \rangle$

$v(1) = \langle 1, 1, 0 \rangle$

$||v(1)|| = \sqrt{2}$

15) $r(t) = \langle \sin 3t, \cos(4t), \cos 5t \rangle, t = \pi/2$

$v(t) = \langle 3\cos(3t), -4\sin(4t), -5\sin(5t) \rangle$

$v(\pi/2) = \langle 0, 0, -5 \rangle$

$||v(\pi/2)|| = 5$

13.4

1) Calculate $r'(t)$, $T(t)$, and evalute $T(1)$.

$r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$

$T(t) = \frac{r'(t)}{||r'(t)||} = \left\langle \frac{8t}{\sqrt{64t^2+81}}, \frac{9}{\sqrt{64t^2+81}} \right\rangle$

$T(1) = \left\langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \right\rangle$

5) Calculate $r'(t)$, $T(t)$, and evalute $T(1)$.

$r(t) = \langle \cos(\pi t), \sin(\pi t), t \rangle$

$r'(t) = \langle -\pi\sin(\pi t), \pi\cos(\pi t), 1 \rangle$

$T(t) = \frac{r'(t)}{||r'(t)||} = \left\langle \frac{-\pi\sin(\pi t)}{\sqrt{(-\pi\sin(\pi t))^2 + (\pi\cos(\pi t))^2 + 1}}, \frac{\pi\cos(\pi t)}{\sqrt{(-\pi\sin(\pi t))^2 + (\pi\cos(\pi t))^2 + 1}}, \frac{1}{\sqrt{(-\pi\sin(\pi t))^2 + (\pi\cos(\pi t))^2 + 1}} \right\rangle$

$T(1) = \left\langle 0, \frac{-\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \right\rangle$

7) Calculate curvature function

$r(t) = \langle 1, e^t, t \rangle$ $\quad K(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3} = \frac{e^t}{(\sqrt{e^{2t}+1})^3}$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$$\begin{bmatrix} 0 \\ e^t \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ e^t \\ 0 \end{bmatrix} = \begin{bmatrix} 0 - e^t \\ 0 - 0 \\ 0 - 0 \end{bmatrix} = ||\langle e^t, 0, 0 \rangle|| = e^t$$

11) Evaluate curvature at given point

$r(t) = \langle 1/t, 1/t^2, t^2 \rangle, t = 1$ $\quad K(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3}$

$r'(t) = \langle -t^{-2}, -2t^{-3}, 2t \rangle$

$r''(t) = \langle 2t^{-3}, 6t^{-4}, 2 \rangle$

$r'(1) = \langle -1, -2, 2 \rangle$

$r''(1) = \langle 2, 6, 2 \rangle$

$$\begin{bmatrix}-1\\-2\\2\end{bmatrix} \times \begin{bmatrix}2\\6\\2\end{bmatrix} = \begin{bmatrix}-4-12\\4-(-2)\\-6-(-4)\end{bmatrix} = \begin{bmatrix}-16\\6\\-2\end{bmatrix} \rightarrow \left\| \begin{bmatrix}-16\\6\\-2\end{bmatrix} \right\| = \sqrt{16^2+6^2+2^2} = \sqrt{296}$$

$$= 14$$

$$K(1) = \frac{14}{3}$$

17) Find curvature on plane curve

$$y=t^4,\ t=2 \Big\}\ \begin{matrix} y'=4t^3 \\ y'\big|_{t=2}=32 \end{matrix} \Big\}\ \begin{matrix} y''=12t^2 \\ y''\big|_{t=2}=48 \end{matrix} \qquad K(t) = \frac{|f''(x)|}{(1+f'(x)^2)^{3/2}} = \frac{48}{(1+32^2)^{3/2}} \approx 0.0015$$

21) Show that the tractrix $r(t) = \langle t - \tanh t, \operatorname{sech} t\rangle$ has the function $\kappa(t) = \operatorname{sech} t$

$r(t) = \langle t - \tanh t, \operatorname{sech} t\rangle$

$r'(t) = \langle 1 - \operatorname{sech}^2 t, -\tanh \operatorname{sech}\rangle = \langle \tanh^2(t), -\tanh(t)\operatorname{sech}(t)\rangle$

$r''(t) = \langle 2\tanh(t)\operatorname{sech}^2(t), \operatorname{sech}(t)(1-2\operatorname{sech}^2(t))\rangle$ $\qquad h(t)\rangle$

$r'(t) \times r''(t) = \tanh^2(t)\operatorname{sech}(t)(1-2\operatorname{sech}^2(t)) + (\operatorname{sech}(t)\tanh(t))(2\operatorname{sech}^2(t)\tanh(t))$

$= \tanh^2(t)\operatorname{sech}(t)\ \hat{k}$

$\|r'(t) \times r''(t)\| = \tanh^2(t)\operatorname{sech}(t)$ $\qquad \sinh^2 \cancel{+} \cosh^2 = 1$ $\qquad \cancel{\tanh^2(t) = \operatorname{sech}^2(t)-1}$

$\|r'(t)\|^3 = (\tanh^4(t) + \operatorname{sech}^2(t)\tanh^2(t))^{3/2}$ $\qquad \cancel{\tanh^2(t)+1 = \operatorname{sech}^2}(t)$ $\qquad \tanh^2 + \operatorname{sech}^2 = 1$

$$\kappa(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = \frac{\tanh^2 t \operatorname{sech} t}{(\tanh^4(t) + \operatorname{sech}^2(t)\tanh^2(t))^{3/2}} = \frac{\tanh^2(t)\operatorname{sech}(t)}{\left(\sqrt{\tanh^2(t)(\tanh^2(t)+\operatorname{sech}^2(t))}\right)^3} =$$

$$\frac{\tanh^2(t)\operatorname{sech}(t)}{(\tanh(t)\sqrt{1})^3} = \frac{\tanh^2 \operatorname{sech}(t)}{\tanh^3} = \frac{\operatorname{sech}(t)}{\tanh(t)} = \operatorname{csch}(t)\ ?$$

14.1

1) $f(x,y) = x + yx^3$, $(2,2), (-1,4)$

$f(2,2) = 2 + 2(2^3) = 18$
$f(-1,4) = -1 - 4 = -5$

3) $h(x,y,z) = xyz^{-2}$, $(3,8,2), (3,-2,-6)$

$h(3,8,2) = 3 \cdot 8 \cdot 2^{-2} = 6$
$h(3,-2,-6) = 3 \cdot -2 \cdot (6)^{-2} = -1/6$

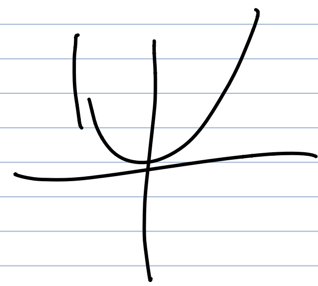
7) sketch domain of function

$f(x,y) = \ln(4x^2 - y)$

$4x^2 - y \geq 0$

$4x^2 \geq y$

$y = 4x^2$

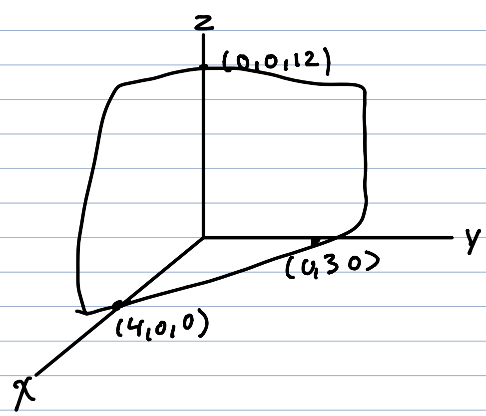


21) sketch the graph and draw several vertical and horizontal traces

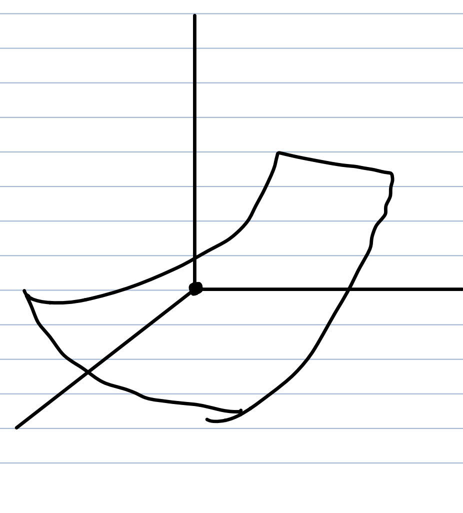
$f(x,y) = 12 - 3x - 4y$

$f(0,0) = 12$

$f(x,y) = 0$
f



23) $f(x,y) = x^2 + 4y^2$



13.5

3) Calculate velocity and accel

$r(t) = \langle t^3, 1-t, 4t^2 \rangle$, $t=1$
$v(t) = \langle 3t^2, -1, 8t \rangle$
$v(1) = \langle 3, -1, 8 \rangle$
$a(t) = \langle 6t, 0, 8 \rangle$
$a(1) = \langle 6, 0, 8 \rangle$

5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos(3\theta) \rangle$, $\theta = \pi/3$
$v(\theta) = \langle \cos\theta, -\sin\theta, -3\sin(3\theta) \rangle$
$v(\pi/3) = \langle \frac{1}{2}, -\sqrt{3}/2, 0 \rangle$
$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos(3\theta) \rangle$
$a(1) = \langle -\sqrt{3}/2, -1/2, 9 \rangle$

15) Find $r(t)$ & $v(t)$

$a(t) = \langle t, 4 \rangle$
$v(t) = \langle t^2 + C_1, 4t + C_2 \rangle$
$v(0) = \langle 3, -2 \rangle$
$v(t) = \langle t^2 + 3, 4t - 2 \rangle$
$r(t) = \langle t^3/3 + 3t + C_1, 2t^2 - 2t + C_2 \rangle$
$r(0) = \langle 0, 0 \rangle$
$r(t) = \langle t^3/3 + 3t, 2t^2 - 2t \rangle$

17) $a(t) = t\hat{k}$
$v(t) = (t^2/2 + C_1)k$
$v(0) = i$
$v(t) = i + t^2/2\, k$
$r(t) = t\hat{i} + (t^3/6 + C_1)k$
$r(0) = j$
$r(t) = t\hat{i} + \hat{j} + t^3/6\,\hat{k}$

?31) $v(t) = \langle 12, 20, 20 \rangle$ The particle is slowing down because of z component
$a(t) = \langle 2, 1, -3 \rangle$

33) Find the coefficients of a_T and a_N

$r(t) = \langle t, \cos(t), \sin(t) \rangle$
$v(t) = \langle 1, -\sin(t), \cos(t) \rangle$
$a(t) = \langle 1, -\cos(t), -\sin(t) \rangle$

$$a_T = \frac{a(t) \cdot v(t)}{\|\vec{v}\|} = \frac{1 \cdot 1 + \cancel{\sin(t)\cos(t)} - \cancel{\sin(t)\cos(t)}}{\sqrt{1^2 + \sin^2(t) + \cos^2(t)}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$a_N = \sqrt{\|a(t)\|^2 - a_T^2} = \sqrt{(1 + \cos^2 + \sin^2)^2 - (1/\sqrt{2})^2} = \sqrt{4 - 1/2} = \sqrt{\frac{7}{2}}$$

14.2 9, 11, 15, 21, 23, 27, 31, 35

$\lim_{(x,y)\to(2,5)} f(x,y)=3$, $\lim_{(x,y)\to(2,5)} g(x,y)=7$

9) $\lim_{(x,y)\to(2,5)} (g(x,y)-2f(x,y)) = 3-2(7) = -11$

11) $\lim_{(x,y)\to(2,5)} \frac{f(x,y)}{f(x,y)+g(x,y)} = \frac{7}{3+7} = \frac{7}{10}$

15) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{xy^2}$; let $y=mx$; $\lim_{(x,y)\to(0,0)} = \frac{x^3+x^3m^3}{x^2m}$; limit DNE because its dependent on m

21) $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$; $y=mx$; $\lim_{(x,y)\to(0,0)} \frac{mx^2}{3x^2+2m^2x^2}$ limit DNE

23) $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$

$\lim_{x\to0} f(x,0,0) = \frac{x}{x^2} = \frac{1}{x}$ DNE

$\lim_{y\to0} f(0,y,0) = \frac{y}{y^2} = \frac{1}{y}$ DNE

$\lim_{z\to0} f(0,0,z) = \frac{z}{z^2} = \frac{1}{z}$ DNE

27) $\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \frac{(-2)^4\cos(\pi)}{e^{-2+1}} = \frac{-16}{-e} = 16e$

31) $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{3^2+4^2}} = \frac{1}{5}$

35) $\lim_{(x,y)\to(-3,-2)} (x^2+y^3+4xy) = (-3)^2+(-2)^3+4(-3)(-2) = 25$