13.3

3. $r(t)=\{2t, \ln t, t^2\}$, $1\le t\le 4$

$r'(t)=\{2, \frac{1}{t}, 2t\}$, $1\le t\le 4$

$|r'(t) = \sqrt{(2t)^2+2^2+(\frac{1}{t})^2} = 2t+\frac{1}{t}$

$\int_1^4 (2t+\frac{1}{t})dt = t^2+\ln t\Big|_1^5$

$= 25+\ln 5-1-\ln 1$

$= 24+\ln 5$

9. $r(t)=\{t^2, 2t^2, t^3\}$, $a=0$

$r'(t)=\{2t, 4t, 3t^2\}$

$\|r'(t)\| = \sqrt{(2t)^2+(4t)^2+(3t^2)^2} = \sqrt{4t^2+16t^2+9t^4}$

$= \sqrt{20t^2+9t^4}$

$s(t) = \int_0^t \sqrt{20t^2+9t^4}\, dt$

$= \frac{1}{27}\left((20+9t^2)^{3/2}-20^{3/2}\right)$

11. $r(t)=(2t+3, 4t-3, 5-t)$, $t=4$

$v(t)=r'(t)=(2, 4, -1)$

$\|r'(t)\| = \sqrt{2^2+4^2+(-1)^2} = \sqrt{21}$

13. $r(t)=(t, \ln t, (\ln t)^2)$, $t=1$

$r'(t)=(1, \frac{1}{t}, \frac{2\ln x}{x})$

$r'(1)=(1, 1, \cancel{2\ln 1}0)$

$\|r'(1)\| = \sqrt{1^2+1^2+\cancel{(2\ln 1)^2}} = \sqrt{2}$

15. $r(t)=(\sin 3t, \cos 4t, \cos 5t)$, $t=\frac{\pi}{2}$

$r'(t)=(3\cos 3t, -4\sin 4t, -5\sin 5t)$

$r'(\frac{\pi}{2}) = (0, 0, -5)$

$\|r'(\frac{\pi}{2})\| = \sqrt{(-5)^2} = 5$

13.4

1. $r(t) = (4t^2, 9t)$

$r'(t) = (8t, 9)$

$\|r'(t)\| = \sqrt{64t^2+81}$

$T(t) = \frac{1}{\sqrt{64t^2+81}}(8t, 9)$

$T(1) = \left(\frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}}\right)$

5. $r(t) = (\cos\pi t, \sin\pi t, t)$

$r'(t) = (-\pi\sin\pi t, \pi\cos\pi t, 1)$

$\|r'(t)\| = \sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1^2} = \sqrt{\pi^2+1}$

$T(t) = \frac{1}{\sqrt{\pi^2+1}}(-\pi\sin\pi t, \pi\cos\pi t, 1)$

$T(1) = \left(0, \frac{-\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}}\right)$

7. $r(t) = (1, e^t, t)$

$r'(t) = (0, e^t, 1)$

$r''(t) = (0, e^t, 0)$

$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix} = (-e^t, 0, 0)$

$\|r'(t) \times r''(t)\| = e^t$

$\|r'(t)\|^3 = (\sqrt{(e^t)^2+1})^3$

$k(t) = \frac{e^t}{(\sqrt{e^{2t}+1})^3}$

11. $r(t) = \left(\frac{1}{t}, \frac{1}{t^2}, t^2\right)$, $t = -1$

$r'(t) = \left(-\frac{1}{t^2}, -\frac{2}{t^3}, 2t\right)$

$r''(t) = \left(\frac{2}{t^3}, \frac{6}{t^4}, 2\right)$

$r' \times r'' = \left(-\frac{16}{t^3}, \frac{6}{t^2}, -\frac{2}{t^6}\right)$ $t = -1 \Rightarrow (16, 6, -2)$

$\|r'(t) \times r''(t)\| = \sqrt{16^2+6^2+(-2)^2} = \sqrt{296} = 2\sqrt{74}$

$\|r'(-1)\|^3 = (\sqrt{(-1)^2+2^2+(-2)^2})^3 = 27$

$k(-1) = \frac{2\sqrt{74}}{27}$

17. $y = t^4$, $t = 2$

$f'(t) = 4t^3$

$f''(t) = 12t^2$

$k(t) = \frac{12t^2}{(1+(4t^3)^2)^{3/2}}$

$= \frac{12t^2}{(1+16t^6)^{3/2}}$

$k(2) = \frac{12 \cdot 4}{(1+16 \cdot 64)^{3/2}}$

$= \frac{48}{(1025)^{3/2}}$

21. $r(t) = (t - \tanh t, \operatorname{sech} t)$

$r'(t) = (1 - \operatorname{sech}^2(t), -\operatorname{sech}(t)\tanh(t$

$r''(t) = (2\operatorname{sech}^2(t)\tanh(t), \operatorname{sech}(t)(\tanh^2(t) - s$

15. 13.5

3. $r(t) = (t^3, 1-t, 4t^2)$, $t=1$

$v(t) = r'(t) = (3t^2, -t, 8t)$

$v(1) = (3, -1, 8)$

$a(t) = r''(t) = (6t, -1, 8)$

$a(1) = (6, -1, 8)$

$\|v(1)\| = \sqrt{3^2+(-1)^2+8^2} = \sqrt{74}$.

5. $r(\theta) = (\sin\theta, \cos\theta, \cos 3\theta)$, $\theta = \frac{\pi}{3}$

$v(\theta) = r'(\theta) = (\cos\theta, -\sin\theta, -3\sin 3\theta)$

$v(\frac{\pi}{3}) = (\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)$

$\|v(\frac{\pi}{3})\| = \sqrt{(\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = 1$

$a(\theta) = r''(\theta) = (-\sin\theta, -\cos\theta, -9\cos 3\theta)$

$a(\frac{\pi}{3}) = (\frac{\sqrt{3}}{2}, -\frac{1}{2}, -\frac{9}{2})$

15. $a(t) = (t, 4)$, $v(0) = (3, -2)$, $r(0) = (0, 0)$

$\int a(t)dt = \frac{1}{2}t^2 i + 4t + C$

$C = 3i - 2j$

$v(t) = (\frac{t^2}{2} + 3, 4t - 2)$

$\int v(t)\,dt = \frac{t^3}{6} + 3t\, i + 2t^2 - 2t j + C$

$C = 0$

$r(0) = (\frac{t^3}{6} + 3t, 2t^2 - 2t)$

17. $a(t) = tk$, $v(0) = i$, $r(0) = j$

$\int a(t) = tk = \frac{t^2}{2}k + C$

$v(t) = i + \frac{t^2}{2}k$

$\int v(t)\,dt = ti + \frac{t^3}{6}k + C$

$r(t) = ti + j + \frac{t^3}{6}k$

31. the partical is slowing down.

33. $r(t) = (t, \cos t, \sin t)$

14.1

1. $f(x,y)=x+yx^3$

$f(2,2)=2+2\cdot 2^3=18$

$f(-1,4)=-1+4\cdot(-1)^3=-5$

3. $h(x,y,z)=xyz^{-2}$.

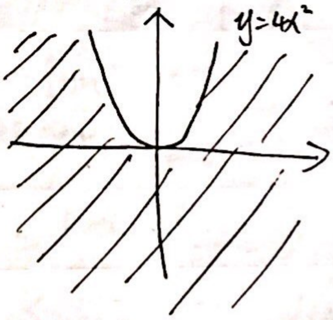
$h(3,8,2)=3\cdot 8\cdot(2)^{-2}=6$

$h(3,-2,-6)=3\cdot(-2)\cdot(-6)^{-2}=-\frac{1}{6}$

7. $f(x,y)=\ln(4x^2-y)$

$4x^2-y>0$

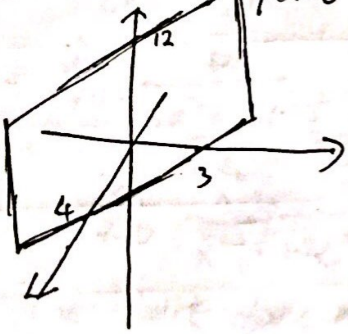
$y=4x^2$



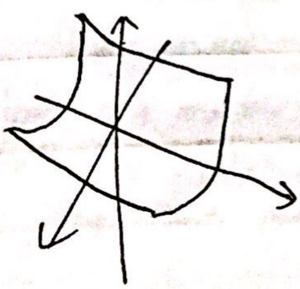
21. $f(x,y)=12-3x-4y$

$12-3x-4y=c$ in plane $z=c$

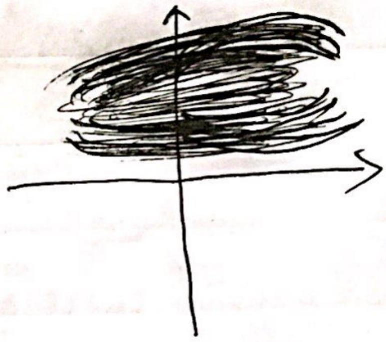
$z=(12-3a)-4y$ and $z=-3x+(12-4a)$ in plane $x=a$, and $y=a$.



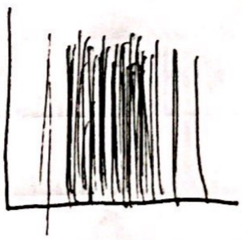
23. $f(x,y)=x^2+4y^2$



33. $f(x,y)=x^2+4y^2$



35. $f(x,y)=x^2$



14.2

9. $\lim_{(x,y)\to(2,5)} \cancel{f(x,y)^2 \cdot g(x,y)}$

$(g(x,y)-2f(x,y))$

$= 7-2\cdot 3$

$=1$

11. $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2-g(x,y)}$

$= e^{9-7} = e^2$

15. $f(x,y)=\frac{xy}{(x^2+y^2)}$

$\lim_{(x,y)\to(0,0)} f(x,y)$

Let $y=cx$

$\lim_{x\to 0} \frac{x\cdot cx}{x^2+(cx)^2} = \frac{x^2 c}{x^2(1+c^2)}$

$= \lim_{x\to 0} \frac{c}{1+c^2}$

Since ~~ft~~ it depend on c, $\lim_{(x,y)\to(0,0)}$ does not exist.

21. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$

Let $y=cx$

$\lim_{x\to 0} \frac{x\cdot cx}{3x^2+2(cx)^2} = \frac{x^2\cdot c}{3x^2+2c^2\cdot x^2}$

$= \lim_{x\to 0} \frac{x^2 c}{x^2(3+2c^2)}$

the limit does not exist since it depends on c

23. $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$

Let $y=mx \quad z=nx$

$\lim_{x\to 0} \frac{x+\cancel{mx}\,mx+nx}{x^2+m^2x^2+n^2x^2} = \lim_{x\to 0} \frac{x(m+n)}{x^2(m^2+n^2)}$

the limit does not exist

27. $\lim_{(z,w)\to(2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}}$

$= \frac{2^4\cos\pi}{e^3}$

$= \frac{-16}{e^3}$

31. $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}$

$= \frac{1}{\sqrt{3^2+4^2}}$

$= \frac{1}{5}$

35. $\lim_{(x,y)\to(-3,-2)} (x^2y^3+4xy)$

$= (-3)^2(-2)^3+4(-3)(-2)$

$= -72+24$

$= -48$