13.3

~~3.~~ $\cancel{\int_1^4 r(t)\,dt}$

$\cancel{=\left[\langle t^2, t\ln t - t, \frac{t^3}{3}\rangle\right]_1^4}$

$\cancel{=}$

3. $r'(t)=\langle 2, \frac{1}{t}, 2t\rangle$.

$|r'|=\sqrt{4+t^{-2}+4t^2}$

$L=\int_1^4 |r'|\,dt$

$=\int_1^4 \sqrt{\left(\frac{2t^2+1}{t}\right)^2}\,dt$

$=\int_1^4 (2t+t^{-1})\,dt$

$=\left[t^2+\ln t\right]_1^4$

$=16+\ln 4-1$

$=15+\ln 4$.

9. $r'(t)=\langle 2t, 4t, 3t^2\rangle$.

$S(t)=\int_0^t |r'(t)|\,dt$

$=\int_0^t \sqrt{9t^4+20t^2}\,dt$

$=\int_0^t t\cdot(9t^2+20)^{\frac{1}{2}}\,dt$.

Let ~~$u=t^2$, $du=$~~

$u=9t^2+20 \quad du=18t\,dt$.

$S(t)=\int_0^t \frac{1}{18}u^{\frac{1}{2}}\,du$.

$=\left[\cancel{\tfrac{1}{27}u^{\frac{3}{2}}}\right]^t$

$=\frac{1}{18}\cancel{\left[\frac{1}{27}u^{\frac{3}{2}}\right]_0^t}\ \left[\frac{1}{27}(9t^2+20)^{\frac{3}{2}}\right]_0^t$

$=\frac{1}{27}\left[(9t^2+20)^{\frac{3}{2}}-20^{\frac{3}{2}}\right]$

11. $V(t)=\langle 2,4,-1\rangle$.

$|V|=\sqrt{2^2+4^2+(-1)^2}=\sqrt{21}$

13. $V(t)=\langle 1, \frac{1}{t}, \frac{2\ln t}{t}\rangle$

$|V|=\sqrt{1+t^{-2}+\frac{4(\ln t)^2}{t^2}}$

$|V(1)|=\sqrt{1+1}=\sqrt{2}$

15. ~~$V(t)=\sqrt{\quad}$~~

$V(t)=\langle 3\cos 3t, -4\sin 4t, \cancel{-5\cos 5t}\rangle$ $-5\sin 5t$

$|V(t)|=\sqrt{9\cos^2(3t)+16\sin^2(4t)+\cancel{25\cos^2(5t)}}$ $25\sin^2(5t)$

$|V(\frac{\pi}{2})|=\sqrt{25\times 1}$

$=5$.

13.4.

1. $r'(t)=\langle 8t, 9\rangle$

$T(t)=\frac{r'(t)}{|r'(t)|}=\frac{\langle 8t,9\rangle}{\sqrt{64t^2+81}}$

$T(1)=\frac{\langle 8,9\rangle}{\sqrt{145}}=\langle \frac{8\sqrt{145}}{145}, \frac{9\sqrt{145}}{145}\rangle$.

5. $r'(t)=\langle -\pi\sin\pi t, \pi\cos\pi t, 1\rangle$.

$T(t)=\frac{r'(t)}{\sqrt{\pi^2(\sin^2\pi t+\cos^2\pi t)+1}}$

$=\frac{\langle -\pi\sin\pi t, \pi\cos\pi t, 1\rangle}{\sqrt{\pi^2+1}}$

$T(1)=\langle 0, \frac{-\pi\sqrt{\pi^2+1}}{\pi^2+1}, \frac{\sqrt{\pi^2+1}}{\pi^2+1}\rangle$.

7. $r'(t) = \langle 0, e^t, 1 \rangle$.

$r''(t) = \langle 0, e^t, 0 \rangle$.

$$K(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$= \frac{e^t}{(\sqrt{e^{2t}+1})^3}$$

$$= \frac{e^t}{(e^{2t}+1)^{\frac{3}{2}}}$$

11. $r'(t) = \langle -t^{-2}, -2t^{-3}, 2t \rangle$.

$r'(-1) = \langle -1, 2, -2 \rangle$.

$r''(t) = \langle 2t^{-3}, 6t^{-4}, 2 \rangle$.

$r''(-1) = \langle -2, 6, 2 \rangle$.

$$K(1) = \frac{|r'(-1) \times r''(-1)|}{|r'(-1)|^3}$$

$$= \frac{\sqrt{16^2+6^2+(-2)^2}}{(\sqrt{(-1)^2+2^2+(-2)^2})^3}$$

$$= \frac{2\sqrt{74}}{27}$$

17. $r(t) = \langle t, t^4, 0 \rangle$

$r'(t) = \langle 1, 4t^3, 0 \rangle$.

$r'(2) = \langle 1, 32, 0 \rangle$.

$r''(t) = \langle 0, 12t^2, 0 \rangle$.

$r''(2) = \langle 0, 48, 0 \rangle$.

$$K(2) = \frac{12 \times 2^2}{(32^2+1)^{\frac{3}{2}}}$$

$$= \frac{48}{32768}$$

$$= 1.5 \times 10^{-3}$$

13.4

21.

$$r'(t) = \langle 1-\operatorname{sech}^2(t), -\tanh(t)\cdot\operatorname{sech}(t)\rangle$$

$$= \langle \tanh^2(t), -\tanh(t)\cdot\operatorname{sech}(t)\rangle$$

$$r''(t) = \langle 2\tanh(t)\cdot\operatorname{sech}^2(t), -\operatorname{sech}^3(t)+\tanh^2(t)\cdot\operatorname{sech}(t)\rangle$$

$$k(t) = \frac{-\tanh^2(t)\cdot\operatorname{sech}^3(t)+\tanh^4(t)\cdot\operatorname{sech}(t)+2\tanh^2(t)\cdot\operatorname{sech}^3(t)}{|\tanh^4(t)+\tanh^2(t)\cdot\operatorname{sech}^2(t)|^{\frac{3}{2}}}$$

$$= \frac{\tanh^4(t)\cdot\operatorname{sech}(t)+\tanh^2(t)\cdot\operatorname{sech}^3(t)}{|\tanh^2(t)\cdot[\tanh^2(t)+\operatorname{sech}^2(t)]|^{\frac{3}{2}}}$$

$$= \frac{\tanh^4(t)\cdot\operatorname{sech}(t)+\tanh^2(t)\cdot\operatorname{sech}^3(t)}{\tanh^3(t)}$$

$$= \frac{\tanh^2(t)\cdot\operatorname{sech}(t)+\cancel{\tanh^2(t)}\operatorname{sech}^3(t)}{\tanh(t)}$$

$$= \frac{\frac{\sinh^2(t)}{\cosh^2(t)}\cdot\frac{1}{\cosh(t)}+\frac{1}{\cosh^3(t)}}{\frac{\sinh(t)}{\cosh(t)}}$$

$$= \frac{\frac{\sinh^2(t)+1}{\cosh^2(t)}}{\sinh(t)}$$

$$= \frac{1}{\sinh(t)}$$

$$= \operatorname{csch}(t)$$

13.5.

1. when $h=-0.2$

$$\frac{r(0.8)-r(1)}{-0.2}=\langle 0.085, 1.91, 2.635\rangle$$

when $h=-0.1$.

$$\frac{r(0.9)-r(1)}{-0.1}=\langle -0.19, 2.07, 2.97\rangle$$

when $h=0.1$.

$$\frac{r(1.1)-r(1)}{0.1}=\langle -0.41, 2.37, 4.08\rangle.$$

when $h=0.2$

$$\frac{r(1.2)-r(1)}{0.2}=\langle -0.525, 2.505, 5.075\rangle$$

3. $v(t)=\langle 3t^2, -1, 8t\rangle$

$v(1)=\langle 3, -1, 8\rangle$

$a(t)=\langle 6t, 0, 8\rangle$

$a(1)=\langle 6, 0, 8\rangle$

$|v(1)|=\sqrt{3^2+(-1)^2+8^2}=\sqrt{74}$.

5. $v(\theta)=\langle \cos\theta, -\sin\theta, -3\sin 3\theta\rangle$

$v(\frac{\pi}{3})=\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\rangle$

$a(\theta)=\langle -\sin\theta, -\cos\theta, -9\cos 3\theta\rangle$.

$a(\frac{\pi}{3})=\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9\rangle$

$|v(\frac{\pi}{3})|=\sqrt{\cos^2\theta+\sin^2\theta+0}=1$.

15.

$v(t)=\langle \frac{t^2}{2}, 4t\rangle+C$.

$v(0)=C=\langle 3,-2\rangle$

$\therefore v(t)=\langle \frac{t^2}{2}+3, 4t-2\rangle$.

$r(t)=\langle \frac{t^3}{6}+3t, 2t^2-2t\rangle+C$

$r(0)=C=\langle 0,0\rangle$.

$\therefore r(t)=\langle \frac{t^3}{6}+3t, 2t^2-2t\rangle$

17.

$v(t)=\langle 0, 0, \frac{t^2}{2}\rangle+C$.

$v(0)=C=\langle 1,0,0\rangle$

$\therefore v(t)=\langle 1, 0, \frac{t^2}{2}\rangle$.

$r(t)=\langle t, 0, \frac{t^3}{6}\rangle+C$.

$r(0)=C=\langle 0,1,0\rangle$.

$r(t)=\langle t, 1, \frac{t^3}{6}\rangle$.

31. $v\cdot a=12\times 2+20-3\times 20$

$=-16<0$.

$\therefore$ Slowing down

33. $r'(t)=\langle 1, -\sin t, \cos t\rangle$.

$r''(t)=\langle 0, -\cos t, -\sin t\rangle$.

$$a_T=\frac{0+\sin t\cos t-\sin t\cos t}{\sqrt{\sin^2 t+\cos^2 t+1}}$$

$=0$.

$$a_N=\frac{\sqrt{1^2+\sin^2 t+(-\cos t)^2}}{\sqrt{1^2+\sin^2 t+\cos^2 t}}$$

$=1$.

14.1.

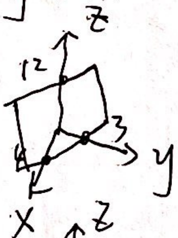
1. $f(2,2)=2+2\times2^3=18$

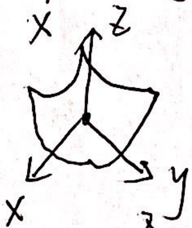
$f(-1,4)=-1-4=-5$

3. $h(3,8,2)=3\times8\times2^{-2}=6$

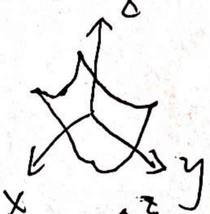
$h(3,-2,-6)=3\times(-2)\times(-6)^{-2}=-\frac{1}{6}$

7. $y<4x^2$

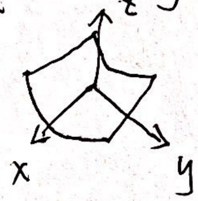
21.


23.


It pass ~~(0,0)~~ (0,0,0).

33


It pass (0.0,0).

35.


The x-axis is on the graph

37. when $m=6$. $f(0,0)=6$.

$f(-3,0)=0$.

$f(0,-1)=0$

$f(x,y)=2x+6y+6$

when $m=3$ $f(0,0)=3$

$f(-3,0)=0$

$f(0,-1)=0$.

$f(x,y)=x+3y+3$

14.2

9. $7-2\times3=1$.

11. $e^{3^2-7}=e^2$

15. $y=mx$.

$f(x,y)=\frac{(m^3+1)x^3}{m^2x^3}=\frac{m^3+1}{m^2}$

$\therefore$ the limit doesn't exist.

21. Set $y=mx$.

$\frac{xy}{3x^2+2y^2}=\frac{mx^2}{3x^2+2m^2x^2}=\frac{m}{2m^2+3}$

$\therefore$ the limit doesn't exist

23. Set $y=mx$, $z=nx$.

$\frac{x+y+z}{x^2+y^2+z^2}=\frac{(m+n+1)x}{(m^2+n^2+1)x^2}$

$=\frac{m+n+1}{(m^2+n^2+1)x}$.

As $x\to0$, it approach ∞.

$\therefore$ Not exist.

27. $\frac{(-2)^4\cos\pi}{e^{-1}}=-16e$

31. $\frac{1}{\sqrt{3^2+4^2}}=\frac{1}{5}$

35. $(-3)^2\cdot(-2)^3+4\times(-3)(-2)=-48$