Jennifer Gonzalez

Math 251

Dr. Z

Sec 23

13.3 #3 $r(t)$ $\langle 2t, \ln t, t^2 \rangle$ $1 \leq t \leq 4$

$$s = \int_a^b \|r'(t)\|\, dt = \int_1^4 \sqrt{2^2 + \left(\tfrac{1}{t}\right)^2 + (2t)^2}\, dt$$

$$= \int_1^4 \sqrt{4 + \tfrac{1}{t^2} + 4t^2}$$

$$= \int_1^4 2t + \tfrac{1}{t}\, dt$$

$$= t^2 \Big|_1^4 + \ln t \Big|_1^4$$

$$= (16 - 1) + (\ln 4 - \overset{0}{\ln(1)})$$

$$15 + \ln 4$$

#9 $r(t) = \langle t^2, 2t^2, t^3 \rangle$ $a = 0$

$$s = \int_0^t \sqrt{(2t)^2 + (4t)^2 + (3t^2)^2}\, dt$$

$$= \int_0^t \sqrt{4t^2 + 16t^2 + 9t^2}\, dt$$

$$= \int_0^t \sqrt{29t^2}\, dt$$

$$= \sqrt{29} \int_0^t t\, dt$$

$$S = -\sqrt{29}\, t$$

#11 $r(t) = \langle 2t+3, 4t-3, 5-t \rangle$
$t = 4$

$r(t) = (2t+3)i + (4t-3)j + (5-t)K$

$r'(t) = 2i + 4j - K$

$v(2) = 2i + 4j - K$

$\|r'(t)\| = \sqrt{4 + 16 - 0}$

$s = \sqrt{20}$

#13 $r(t) = \langle t, \ln t, (\ln t)^2 \rangle$
$t = 1$

$r'(t) = \langle 1, \tfrac{1}{t}, 2\ln t \rangle$

$v(t) = \langle 1, 1, 0 \rangle$

$s = \sqrt{1 + 1 + 0} = \sqrt{2}$

#15 $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle$
$t = \pi/2$

$r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$v(\pi/2) = \langle 0, 0, -5 \rangle$

$s = \sqrt{0^2 + 0^2 + (-5)^2} = \sqrt{25} = 5$

13.4

#1 $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$

$T(t) = \dfrac{8t+9}{\sqrt{(8t)^2 + 9^2}}$

$= \dfrac{8t+9}{\sqrt{64t^2 + 81}}$

$T(t) = \dfrac{17}{145}$

#5 $r(t) = \langle \cos\pi t, \sin\pi t, t \rangle$

$r'(t) = \langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle$

$T(t) = \dfrac{-\pi\sin\pi t + \pi\cos\pi t + 1}{\sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1}}$

$T(1) \quad \dfrac{-\pi + 1}{\sqrt{\pi^2 + 1}}$

#7 $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$\|r'(t)\| = \sqrt{0^2 + e^{2t} + 1} = \sqrt{e^{2t} + 1}$

$r'(t) \times r''(t) = \begin{bmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{bmatrix}$

$= i\begin{bmatrix} e^t & 1 \\ e^t & 0 \end{bmatrix} + j\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - k\begin{bmatrix} 0 & e^t \\ 0 & e^t \end{bmatrix}$

$= \langle -e^t, 0, 0 \rangle$

$K(t) = \dfrac{\sqrt{(-e^t)^2 + 0^2 + 0^2}}{\left((e^{2t}+1)^{1/2}\right)^3}$

$\kappa(t) = \dfrac{e^t}{(e^{2t}+1)^{3/2}}$

#11 $r(t) = \langle 1/t, 1/t^2, t^2 \rangle \quad t = -1$

$r'(t) = \langle -\frac{1}{t^2}, -\frac{2}{t^3}, 2t \rangle$

$r''(t) = \langle \frac{2}{t^3}, \frac{6}{t^4}, 2 \rangle$

$\|r'(t)\| = \sqrt{(-1/t^2)^2 + (-2/t^3)^2 + (2t)^2}$

$= \sqrt{1/t^4 + 4/t^6 + 4t^2}$

$r'(t) \times r''(t) = \begin{bmatrix} i & j & k \\ -1/t^2 & -2/t^3 & 2t \\ 2/t^3 & 6/t^4 & 2 \end{bmatrix}$

$= i\begin{bmatrix} \frac{-2}{t^3} & 2t \\ 6/t^4 & 2 \end{bmatrix} + j\begin{bmatrix} -1/t^2 & 2t \\ 2/t^3 & 2 \end{bmatrix} - k\begin{bmatrix} -1/t^2 & -2/t^3 \\ 2/t^3 & 6/t^4 \end{bmatrix}$

$= \langle -\frac{16}{t^3}, \frac{6}{t^2}, -\frac{2}{t^6} \rangle$

$= \dfrac{\sqrt{(-\frac{16}{t^3})^2 + (\frac{6}{t^2})^2 + (-\frac{2}{t^6})^2}}{\left(\sqrt{1/t^4 + 4/t^6 + 4t^2}\right)^3}$

$K(t) = \dfrac{\sqrt{256 + 36 + 4}}{(1 + 4 + + 4)^{3/2}} = \dfrac{\sqrt{296}}{27}$

#17 $y = t^4$, $t = 2$

#21

$y' = 4t^3$

$y'' = 12t^2$

$$\frac{|12t^2|}{(1+(4t^3)^2)^{3/2}} = \frac{12t^2}{(1+16(t)^6)^{3/2}}$$

$$K(2) = \frac{48}{(1025)^{3/2}}$$

13.5

#3 $r(t) = \langle t^3, 1-t, 4t^2 \rangle$, $t = 1$

$v(t) = r'(t) = \langle 3t^2, -1, 8t \rangle$

$v(1) = \langle 3, -1, 8 \rangle$

$a(t) = r''(t) = \langle 9t, 0, 8 \rangle$

$a(1) = \langle 9, 0, 8 \rangle$

#5 $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$ $\theta = \frac{\pi}{3}$

$v(\theta) = r'(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$v(\pi/3) = \langle 1/2, -\frac{\sqrt{3}}{2}, 0 \rangle$

$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$a(\pi/3) = \langle -\frac{\sqrt{3}}{2}, -1/2, 9 \rangle$

#15 $a(t) = \langle t, 4 \rangle$ $v(0) = \langle 3, -2 \rangle$ $r(0) = \langle 0, 0 \rangle$

$v(t) = \langle t^2/2 + C, 4t + C \rangle$

$v(0) = \langle \underbrace{0 + C}_{=3}, \underbrace{0 + C}_{=-2} \rangle$

$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$r(t) = \langle \frac{t^3}{6} + 3t + C, 2t^2 - 2t + C \rangle$

$r(0) = \langle \underbrace{0 + 0 + C}_{=0}, \underbrace{0 + 0 + C}_{=0} \rangle$

$r(t) = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

#17 $a(t) = tK$ $v(0) = i$ $r(0) = j$

$a(t) = \langle 0, 0, t \rangle$

$v(0) = \langle 1, 0, 0 \rangle$

$r(0) = \langle 0, 1, 0 \rangle$

$v(t) = \langle c, c, \frac{t^2}{2} + C \rangle$

$v(0) = \langle \underbrace{c}_{=1}, \underbrace{c}_{=0}, \underbrace{c}_{=0} \rangle$

$v(t) = \langle 1, 0, t^2/2 \rangle$

$r(t) = \langle t + c, c, \frac{t^3}{6} + C \rangle$

$r(0) = \langle \underbrace{c}_{=0}, \underbrace{c}_{=1}, \underbrace{c}_{=0} \rangle$

$r(t) = \langle t, 1, \frac{t^3}{6} \rangle$

14.1

#1 - $f(x,y) = x + yx^3$ @ $(2,2), (-1,4)$

$f(2,2) = 18$

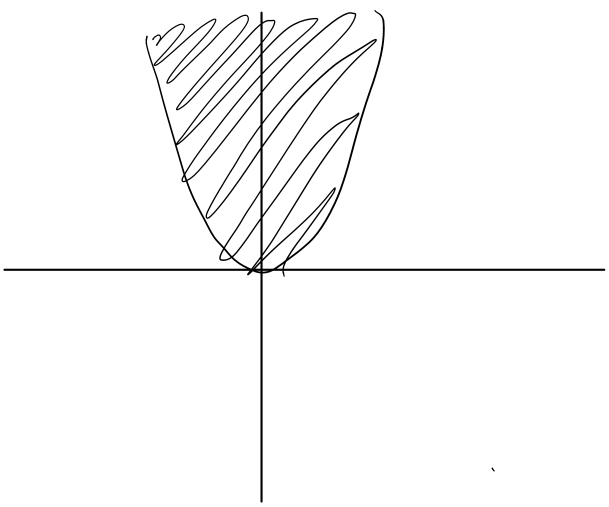
$f(-1,4) = -1 + 4(-1) = -5$

#3 $h(x,y,z) = xyz^{-2}$ $(3,8,2)$ $(3,-2,-6)$

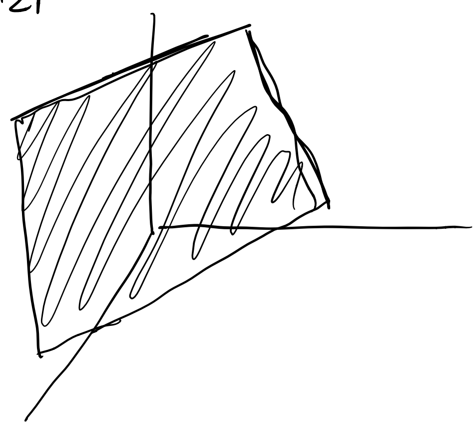
$h(3,8,2) = 3 \cdot 8 \cdot 2^{-2}$
$= 6$

$h(3,-2,-6) = 3(-2)(-6)^{-2}$
$= -1/6$

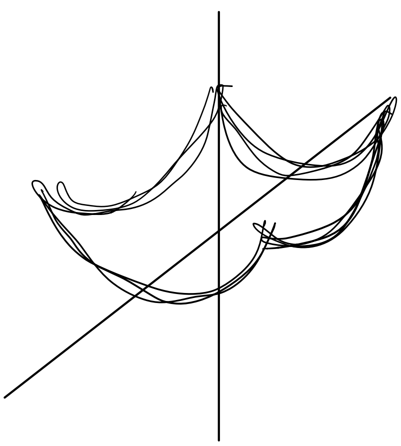
#7 $f(x,y) = \ln(4x^2 - y)$



#21



#23 $f(x,y) = x^2 + 4y^2$



#33 $f(x,y) = x^2 + 4y$