Afsana Rahman
Dr Z - Calc 3
27 September 2020
HW3 : 13.3-5, 14.1-2

$\boxed{13.3}$ 8) $r(t) = \langle 2t, \ln t, t^2 \rangle$

$v(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$\int_1^4 \sqrt{4 + \frac{1}{t^2} + 4t^2}\, dt$

$= \int_1^4 2t + \frac{1}{t} = \int_1^4 2t\,dt + \int_1^4 \frac{1}{t}$

$= t^2 \Big|_1^4 + \ln t \Big|_1^4$

$= (16-1) + (\ln 4 - \ln 1)$

$= 15 + \ln 4$

9) $r(t) = \langle t^2, 2t^2, t^3 \rangle$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$\int_0^t \sqrt{4u^2 + 16u^2 + 9u^4}\, du$

$\int_0^t \sqrt{20u^2 + 9u^4}\, du$

$\int_0^t u\sqrt{9u^2 + 20}\, du$

$v = 9u^2 + 20,\ dv = 18u\,du$

$\frac{1}{18}\int_0^t \sqrt{v}\, dv = \frac{2}{54} v^{3/2}$

$= \frac{1}{27}\left((9u^2+20)^{3/2}\right)\Big|_0^t$

$= \frac{1}{27}\left((9t^2+20)^{3/2} + 20^{3/2}\right)$

11) $r(t) = \langle 2t+3, 4t-3, 5-t \rangle$

$r'(t) = \langle 2, 4, -1 \rangle$

$v(4) = \sqrt{4+16+1} = \sqrt{21}$

13) $r(t) = \langle t, \ln t, (\ln t)^2 \rangle$

$r'(t) = \langle 1, \frac{1}{t}, 2\ln t / t$

$v(1) = \sqrt{1+1+0} = \sqrt{2}$

15) $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle$

$r'(t) = \langle 3\cos 3t, -4\sin 4t, 5\sin 5t \rangle$

$v(\frac{\pi}{2}) = \sqrt{9\cos^2\frac{3\pi}{2} + 16\sin^2 2\pi + 25\sin^2\frac{5\pi}{2}}$

$= \sqrt{0+0+25} = 5$

21) $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$

$r'(t) = \langle 1 - 1 + \tanh^2 t, -\tanh t \operatorname{sech} t \rangle$

$r''(t) = \langle 2\tanh t * (1 - \tanh^2 t),$
$-\operatorname{sech} t(1 - \tanh^2 t) + \tanh t(\tanh t \operatorname{sech} t) \rangle$

$= \langle 2\tanh t - 2\tanh^3 t,$
$\tanh^2 t \operatorname{sech} t + \operatorname{sech} t + \tanh^2 t \operatorname{sech} t \rangle$

$= \langle 2\tanh t - 2\tanh^3 t, 2\tanh^2 t \operatorname{sech} t + \operatorname{sech} t \rangle$

$r'(t) \times r''(t) = \langle \tanh^2 t(2\tanh^2 t \operatorname{sech} t + \operatorname{sech} t)$
$- \tanh t \operatorname{sech} t(2\tanh t - 2\tanh^3 t) \rangle$

$= \langle 2\tanh^4 t \operatorname{sech} t + \tanh^2 t \operatorname{sech} t,$
$2\tanh^4 t \operatorname{sech} t - 2\tanh^2 t \operatorname{sech} t \rangle$

??! i feel like this isn't right

$\boxed{13.4}$ 1) $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$

$T(t) = \frac{1}{\sqrt{64t^2+81}} \langle 8t, 9 \rangle$

$T(1) = \frac{1}{\sqrt{64+81}} \langle 8, 9 \rangle$

$= \left\langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \right\rangle$

5) $r(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$r'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$T(t) = \frac{1}{\sqrt{\pi^2 \sin^2 \pi t + \pi^2 \cos^2 \pi t + 1}} \cdot r'(t)$

$= \frac{1}{\sqrt{\pi^2(\sin^2 \pi t + \cos^2 \pi t) + 1}} \cdot r'(t)$

$= \frac{1}{\sqrt{\pi^2+1}} \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$

$T(1) = \frac{1}{\sqrt{\pi^2+1}} \langle -\pi \sin \pi, \pi \cos \pi, 1 \rangle$

$= \left\langle 0, \frac{\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \right\rangle$

7) $\kappa = \frac{\| r'(t) \times r''(t) \|}{\| r'(t) \|^3}$

$r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$K = \frac{\| \langle 0 - e^t, 0 - 0, 0 - 0 \rangle \|}{(\sqrt{e^{2t}+1})^3}$

$= \frac{\sqrt{e^{2t}}}{\sqrt{e^{2t}+1}^3} = \frac{e^t}{(e^{2t}+1)^{3/2}}$

11) $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle$

$r'(t) = \langle -\frac{1}{t^2}, -\frac{2}{t^3}, 2t \rangle$

$r''(t) = \langle \frac{2}{t^3}, \frac{6}{t^4}, 2 \rangle$

$r'(t) \times r''(t) = \langle -\frac{4}{t^3} - \frac{12}{t^3}, \frac{4}{t^2} + \frac{2}{t^4}, -\frac{6}{t^6} + \frac{4}{t^6} \rangle$

$K = \frac{\| \langle -\frac{16}{t^3}, \frac{6}{t^2}, -\frac{2}{t^6} \rangle \|}{\sqrt{\frac{1}{t^4} + \frac{4}{t^6} + 4t^2}^3}$

$= \frac{\sqrt{\frac{256}{t^6} + \frac{36}{t^4} + \frac{4}{t^{12}}}}{(\frac{1}{t^4} + \frac{4}{t^6} + 4t^2)^{3/2}}$

$= \frac{\sqrt{256+36+4}}{\sqrt{1+4+4}^3} = \frac{\sqrt{296}}{3^3} = \frac{2\sqrt{74}}{27}$

7) $K = \frac{|f''(t)|}{(1 + (f'(t))^2)^{3/2}}$ $y'(t) = 4t^3$ $y''(t) = 12t^2$

$= \frac{\sqrt{144t^4}}{(1+16t^6)^{3/2}} \Rightarrow \frac{\sqrt{144 \cdot 16}}{(1+1024)^{3/2}} = \frac{\sqrt{2304}}{1025^{3/2}}$

$\boxed{13.5}$ 3) $r(t) = \langle t^3, 1-t, 4t^2 \rangle$

$v(t) = \langle 3t^2, -1, 8t \rangle$

$v(1) = \langle 3, -1, 8 \rangle$

$a(t) = \langle 6t, 0, 8 \rangle$

$a(1) = \langle 6, 0, 8 \rangle$

5) $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$

$v(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$v(\frac{\pi}{3}) = \langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \rangle$

$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$a(\frac{\pi}{3}) = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 9 \rangle$

15) $v(t) = \int a(t) = \int \langle t, 4 \rangle = \langle \frac{1}{2}t^2, 4t \rangle + C$

$= \langle \frac{1}{2}t^2, 4t \rangle + \langle 3, -2 \rangle = \langle \frac{1}{2}t^2 + 3, 4t - 2 \rangle$

$r(t) = \int v(t) = \int \langle \frac{1}{2}t^2 + 3, 4t - 2 \rangle = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

$= \langle \frac{1}{6}t^3 + 3t, 2t(t-1) \rangle$

17) $v(t) = \int a(t) = \int \langle 0, 0, 1 \rangle = \langle 0, 0, t \rangle + C$

$= \langle 0, 0, t \rangle + \langle 1, 0, 0 \rangle = \langle 1, 0, t \rangle$

$r(t) = \int v(t) = \int \langle 1, 0, t \rangle = \langle t, 0, \frac{1}{2}t^2 \rangle + C$

$= \langle t, 0, \frac{1}{2}t^2 \rangle + \langle 0, 1, 0 \rangle = \langle t, 1, \frac{1}{2}t^2 \rangle$

31) slowing down

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$\boxed{14.1}$

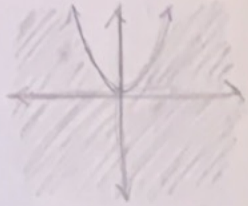
1) $f(2,2) = 2+2(2^3) = 18$

$f(-1,4) = -1+4(-1^3) = -5$

3) $h(3,8,2) = \frac{24}{4} = 6$

$h(3,-2,-6) = \frac{-6}{36} = -\frac{1}{6}$

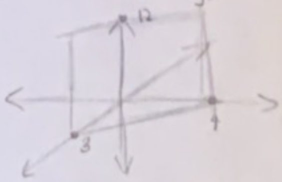
7)



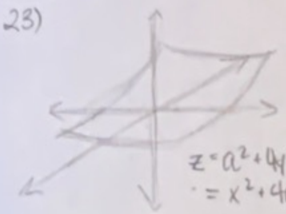
21) $3x+4y = 12-c,\ z=c$

$z = (12-3a)-4y$

$= 3x+(12-4y)$



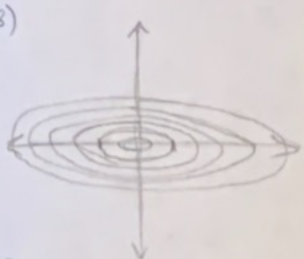
23)



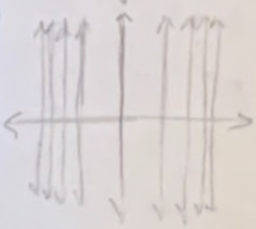
$z = a^2+4y^2$

$= x^2+4a^2$

33)



35)



$\boxed{14.2}$

9) $\lim\limits_{(x,y)\to(2,5)} g(x,y)-2f(x,y)$

$= \lim\limits_{\to} (7-2(3)) = 1$

11) $\lim\limits_{(x,y)\to(2,5)} e^{f(x,y)^2-g(x,y)}$

$= \lim\limits_{\to} e^{9-7} = e^2$

15) $f(x,mx) = \frac{x^3+m^3x^3}{xm^2x^2} = \frac{x^3(1+m^3)}{x^3m^2} = \frac{1+m^3}{m^2}$

DEPENDS ON SLOPE M - LIMIT DOES NOT EXIST

21) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = \frac{0\cdot 0}{3(0)+2(0)} = \frac{0}{0}$

$\lim\limits_{x\to 0} \frac{x\cdot cx}{3x^2+2(cx)^2} = \frac{cx^2}{3x^2+2c^2x^2} = \frac{cx^2}{x^2(3+2c)} = \frac{c}{3+2c}$

DEPENDS ON SLOPE C - LIMIT DOES NOT EXIST

23) $\lim\limits_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2} = \frac{0+0+0}{0+0+0} = \frac{0}{0}$

SET $y=z=0$ (approach from x-axis)

$\lim\limits_{x\to 0} \frac{x}{x^2} = \lim\limits_{x\to 0} \frac{1}{x}$ = DOES NOT EXIST

27) $\lim\limits_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \frac{(-2^4)\cos\pi}{e^{-2+1}} = -16e$

31) $\lim\limits_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{9+16}} = \frac{1}{5}$

35) $\lim\limits_{(x,y)\to(-3,-2)} x^2y^3+4xy = 3^2(-2)^3+4(3)(2) = 9(-8)+24 = 24-72 = -48$