13.3

3. $r(t) = \langle 2t, \ln t, t^2 \rangle \qquad 1 \le t \le 4$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$|r'(t)| = \sqrt{2^2 + \frac{1}{t^2} + 4t^2}$

$$\int_{t=1}^{t=4} \sqrt{4t^2 + 4 + \frac{1}{t^2}} = 16.39$$

9. $s(t) = \int_a^t \|r'(u)\| \, du$ for given a

$r(t) = \langle t^2, 2t^2, t^3 \rangle$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$|r'(t)| = \sqrt{4t^2 + 16t^2 + 9t^6}$

$$\int_0^t (4t^2 + 16t^2 + 9t^6)^{1/2}$$

$$\int_0^t t^2 + \int_0^t (20 + 9t^4)^{1/2}$$

$u = 20 + 9t^4$

$du = 36t^3 \, dx$

$dx = \frac{du}{36t^3}$

$$\int_0^t u^{1/2} = \frac{2u^{3/2}}{3}$$

$$\frac{t^3}{3} + \frac{2(20+9t^4)^{3/2}}{3} \cdot \frac{1}{36t^3} + C \Big|_0^t$$

$$\left(\frac{t^3}{3} + \frac{2(20+9t^4)^{3/2}}{108t^3} \right) - \left(\frac{0^3}{3} + \frac{2(20+9(0^4))^{3/2}}{108(0)^3} \right)$$

$$\frac{t^3}{3} + \frac{2(20+9t^4)^{3/2}}{108t^3}$$

11. $r(t) = \langle 2t+3,\ 4t-3,\ 5-t \rangle$ $t=4$

$r'(t) = \langle 2,\ 4,\ -1 \rangle$

13. $r(t) = \langle t,\ \ln t,\ (\ln t)^2 \rangle$ $t=1$

$r'(t) = \langle 1,\ \frac{1}{t},\ 2(\ln t)\cdot\frac{1}{t} \rangle$

$r'(1) = \langle 1,\ 1,\ 0 \rangle$

15. $r(t) = \langle \sin 3t,\ \cos 4t,\ \cos 5t \rangle$ $t = \frac{\pi}{2}$

$r'(t) = \langle \frac{1}{3}\cos 3t,\ \frac{-1}{4}\sin 4t,\ \frac{-1}{5}\sin 5t \rangle$

$r'(\frac{\pi}{2}) = \langle \frac{1}{3}\cos\frac{3\pi}{2},\ \frac{-1}{4}\sin\frac{4\pi}{2},\ \frac{-1}{5}\sin\frac{5\pi}{2} \rangle$

$r'(\frac{\pi}{2}) = \langle 0,\ 0,\ 0 \rangle$

13.4

1. $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$

$T(t) = \dfrac{\langle 8t, 9 \rangle}{\sqrt{64t^2 + 81}}$

$T(1) = \dfrac{\langle 8, 9 \rangle}{\sqrt{64(1) + 81}} = \left\langle \dfrac{8}{\sqrt{145}}, \dfrac{9}{\sqrt{145}} \right\rangle = \left\langle \dfrac{8\sqrt{145}}{145}, \dfrac{9\sqrt{145}}{145} \right\rangle$

5. $r(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$r'(t) = \langle \frac{-1}{\pi} \sin \pi t, \frac{1}{\pi} \cos \pi t, 1 \rangle$

$T(t) = \dfrac{\langle \frac{-1}{\pi} \sin \pi t, \frac{1}{\pi} \cos \pi t, 1 \rangle}{\sqrt{(\frac{-1}{\pi} \sin \pi t)^2 + (\frac{1}{\pi} \cos \pi t)^2 + 1^2}}$

$T(1) = \dfrac{\langle \frac{-1}{\pi} \sin \pi, \frac{1}{\pi} \cos \pi, 1 \rangle}{\sqrt{(\frac{-1}{\pi} \sin \pi)^2 + (\frac{1}{\pi} \cos \pi)^2 + 1}} = \left\langle \dfrac{0}{\sqrt{\frac{1}{\pi}+1}}, \dfrac{1}{\sqrt{\frac{1}{\pi}+1}}, \dfrac{1}{\sqrt{\frac{1}{\pi}+1}} \right\rangle = \langle 0, .87, .87$

7. $r(t) = \langle 1, e^t, t \rangle$

$r'(t) = \langle 0, e^t, 1 \rangle$ $\qquad |r'(t)| = \sqrt{0^2 + (e^t)^2 + 1^2}$

$r''(t) = \langle 0, e^t, 0 \rangle$ $\qquad = (\sqrt{e^{2t} + 1})^3$

$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix} = -e^t i + 0j + 0k$

$\sqrt{e^t}$

$\dfrac{\sqrt{e^t}}{(\sqrt{e^t + 1})^3}$

11. $r(t) = \langle \frac{1}{t}, \frac{1}{t^2}, t^2 \rangle$ $\quad t = -1$

$r'(t) = \langle \frac{-1}{t^2}, \frac{-2}{t^3}, 2t \rangle$ $\quad r'(-1) = \langle -1, 2, -2 \rangle$

$r''(t) = \langle \frac{2}{t^3}, \frac{6}{t^4}, 2 \rangle$ $\quad r''(-1) = \langle -2, 6, 2 \rangle$

$$r'(-1) \times r''(-1) = \begin{vmatrix} i & j & k \\ -1 & 2 & -2 \\ -2 & 6 & 2 \end{vmatrix} = \langle 16, -5, -2 \rangle$$

$\sqrt{16^2 + (-5)^2 + (-2)^2}$

$$\frac{\sqrt{285}}{(\sqrt{(-1)^2 + (2^2) + (-2)^2})^3} = \frac{\sqrt{285}}{27}$$

17. $y = t^4$ $\quad t = 2$

$r(t) = \langle 0, t^4, 0 \rangle$

$r'(t) = \langle 0, 4t^3, 0 \rangle$ $\quad r'(2) = \langle 0, 32, 0 \rangle$

$r''(t) = \langle 0, 12t^2, 0 \rangle$ $\quad r''(2) = \langle 0, 48, 0 \rangle$

$$r'(2) \times r''(2) = \begin{vmatrix} i & j & k \\ 0 & 32 & 0 \\ 0 & 48 & 0 \end{vmatrix} = \langle 0, 0, 0 \rangle$$

$$\frac{\sqrt{0+0+0}}{\sqrt{0 + (48^2) + 0}} = 0$$

21 $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle = (1 - \tanh t)i + \operatorname{sech} t\, j$

$(-\operatorname{sech}^2 t \cdot 1 i + \operatorname{sech} t \cdot \tanh t\, j)$

$$k(x) = \frac{\sqrt{(-2\operatorname{sech}^2 t \tanh t)^2 + (\operatorname{sec}^3 h t \ldots)^2}}{(1 + (-\operatorname{sech}^2 t)^2 + (\operatorname{sech} t \tanh t)^2)} \quad (-2\operatorname{sech} t \cdot \operatorname{sech} t \tanh t\, i + \operatorname{sech}^3 t \tanh t + \operatorname{sech}^3 t)$$

$$k(x) = \frac{\sqrt{4\operatorname{sech}^4 t \tanh^2 t + 2\operatorname{sech}^2 t \tanh t \operatorname{sech}^3 t + \operatorname{sech}^6 t + \operatorname{sech}^4 t \tanh t)}}{1 + \operatorname{sech}^4 t + \operatorname{sech}^2 t \tanh^2 t}$$

$$k(x) = \frac{\operatorname{sech}^2 t \sqrt{4\tanh^2 t + 2\operatorname{sech} t \tanh t + \operatorname{sech}^2 t + \tanh t}}{1 + \operatorname{sech}^6 t + \tanh^2 t}$$

$k(x) = \operatorname{sech} t$

13.5

3. $r(t) = \langle t^3, 1-t, 4t^2 \rangle$

$r'(t) = \langle 3t^2, -1, 8t \rangle$ $\quad r'(1) = \langle 3, -1, 8 \rangle$ $\quad \sqrt{3^2+(-1)^2+8^2} = \sqrt{74}$

$r''(t) = \langle 6t, 0, 8 \rangle$ $\quad r''(1) = \langle 6, 0, 8 \rangle$

velocity $= \langle 3, -1, 8 \rangle$, acceleration $= \langle 6, 0, 8 \rangle$, speed $= \sqrt{74}$ or 8.6

5. $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$ $\quad \theta = \frac{\pi}{3}$

$r'(\theta) = \langle \cos\theta, -\sin\theta, \frac{-1}{3}\sin 3\theta \rangle$ $\quad r'(\frac{\pi}{3}) = \langle \cos\frac{\pi}{3}, -\sin\frac{\pi}{3}, \frac{-1}{3}\sin\pi \rangle$

$r''(\theta) = \langle -\sin\theta, -\cos\theta, \frac{-1}{9}\cos 3\theta \rangle$ $\quad r''(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, -\cos\frac{\pi}{3}, \frac{-1}{9}\cos\pi \rangle$

velocity $= \langle \frac{1}{2}, \frac{-\sqrt{3}}{2}, 0 \rangle$ acceleration $= \langle \frac{-\sqrt{3}}{2}, \frac{-1}{2}, \frac{1}{2} \rangle$ speed $= \sqrt{(\frac{1}{2})^2 + (\frac{-\sqrt{3}}{2})^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= \sqrt{1} = 1$

15 $a(t) = \langle t, 4 \rangle$, $\quad v(0) = \langle 3, -2 \rangle$, $\quad r(0) = \langle 0, 0 \rangle$

$v(t) = \int \langle t, 4 \rangle$

$v(t) = \langle \frac{t^2}{2}, 4t \rangle + C$

$\langle 3, -2 \rangle = \langle 0, 0 \rangle + C$

$C = \langle 3, -2 \rangle$

$r(t) = \int \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$r(t) = \langle \frac{t^3}{6} + 3t, \frac{4t^2}{2} - 2t \rangle + C$

$C = \langle 0, 0 \rangle$

$r(t) = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

17. $a(t) = tk$, $\quad v(0) = i$ $\quad r(0) = j$

$a(t) = \langle 0, 0, t \rangle$ $\quad v(0) = \langle 1, 0, 0 \rangle$ $\quad r(0) = \langle 0, 1, 0 \rangle$

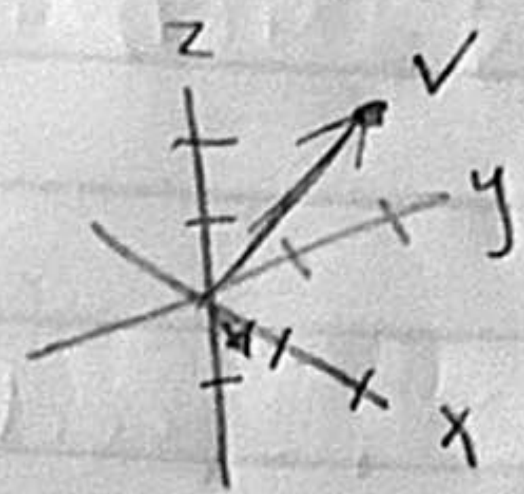
$v(t) = \int \langle 0, 0, t \rangle$

$v(t) = \langle 1, 0, \frac{t^2}{2} \rangle$

$r(t) = \int \langle 1, 0, \frac{t^2}{2} \rangle$

$r(t) = \langle t, 1, \frac{t^3}{6} \rangle$

31. $v = \langle 12, 20, 20 \rangle$ $a = \langle 2, 1, -3 \rangle$



Slowing down because a is neg to v

14.2

9. $\lim_{(x,y)\to(2,5)} (g(x,y) - 2f(x,y)) = 7 - 6 = 1$

11. $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)} = e^{9-7} = e^2$

15. $f(x,y) = \frac{x^3 + y^3}{xy^2}$ $\quad y = mx$

$f(x,y) = \frac{x^3 + (mx)^3}{x(mx^2)} = \frac{x^3(1+m^3)}{m(\cancel{x^3})} = \frac{1+m^3}{m}$

$\lim f(x,y) = \frac{1+m^3}{m}$

21. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = \frac{0}{0}$

$\lim_{(x,y)\to(0,0)} \frac{xc\cancel{x}}{3x^2 + 2(c\cancel{x})^2} =$

$\lim_{(x,y)\to(0,0)} \frac{c}{3+2c^2}$ The limit does not exist because you get different limits when you approach (0,0)

23. $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2} = \frac{0}{0}$

$\lim_{(x,y,c)\to(0,0,c)} \frac{x+xc}{x^2+(x^2c^2)} = \frac{\cancel{x}(1+c)}{\cancel{x^2}(1+c^2)} = \frac{1+c}{1+c^2}$

$\lim_{(x,c,z)\to(0,c,0)} \frac{y+yc}{y^2+(y^2c^2)} = \frac{1+c}{1+c^2}$

$\lim_{(c,y,z)\to(c,0,0)} \frac{z+zc}{z^2+z^2c^2} = \frac{1+c}{1+c^2}$

In all planes the limit depends on the slope and therefore has multiple limits

27. $\lim\limits_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \frac{(-2)^4\cos\pi}{e^{-1}} = -43.49$

$f(r\cos\theta, r\sin\theta) = \frac{(r\cos\theta)^4\cos\pi(r\sin\theta)}{e^{r\cos\theta + r\sin\theta}} = \frac{r^2\cos^5\theta\sin\theta}{e^r}$

$(-2)^2 + (1^2) = r^2$

$r = \sqrt{5}$

$\lim\limits_{r\to\sqrt{5}} \frac{r^2\cos^5\theta\sin\theta}{e^r} = \frac{5}{e^5} = -43.49$ The limit exists at -43.49

31. $\lim\limits_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{5}$

$f(r\cos\theta, r\sin\theta) = \frac{1}{\sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}} = \frac{1}{r^2 \cdot 1} = \frac{1}{5}$

$3^2 + 4^2 = r^2$ The limit exists at $\frac{1}{5}$

$r = 5$

35. $\lim\limits_{(x,y)\to(-3,-2)} (x^2y^3 + 4xy) = (-3^2)(-2^3) + 4(-3)(-2) = -48$

$f(r\cos\theta, r\sin\theta) = (r\cos\theta)^2(r\sin\theta)^2 + 4r\sin\theta\, r\cos\theta$

$= r^4\cos^2\theta\sin^2\theta + 4r^2\sin\theta\cos\theta$

$r^2 = (-3^2) + (-2^2)$

$r = \sqrt{13}$

$\lim\limits_{r\to\sqrt{13}} r^4\cos^2\theta\sin^2\theta + 4r^2\sin\theta\cos\theta = -48$

The limit exists at -48.