Fayed Raza 9/27/20

13.3: 3, 9, 11, 13, 15

3.) $r(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \le t \le 4$

$$L = \int_1^4 \sqrt{(2)^2 + \left(\frac{1}{t}\right)^2 + (2t)^2}\, dt$$

$$L = \int_1^4 \sqrt{4 + \frac{1}{t^2} + 4t^2}\, dt = \int_1^4 \sqrt{\frac{4t^2 + 1 + 4t^4}{t^2}}\, dt$$

$$= \int_1^4 \frac{(2t^2+1)^2}{t^2} = \int_1^4 \frac{2t^2+1}{t}\, dt = \int_1^4 \left(2t + \frac{1}{t}\right) dt$$ $15 + \ln 4$

$$= \int_1^4 2t\, dt + \int_1^4 \frac{1}{t}\, dt \quad \frac{2t^2}{2}\Big|_1^4 = \frac{2(16)}{2} - \frac{2}{2} = 15$$

$\ln(4) - \ln(1) = \ln(4)$

9.) $r(t) = \langle t^2, 2t, t^3 \rangle \quad 0 \le t \le 9$

$$= \int_0^t \sqrt{4t^2 + 16t^2 + 9t^4}\, dt \quad u = 20 + 9t^2$$

$du = 18t\, dt$

$$= \int_0^t \sqrt{20t^2 + 9t^4}\, dt \quad \frac{du}{18} = t\, dt$$

$$= \int_0^t \sqrt{t^2(20 + 9t^2)}\, dt \quad \frac{2}{3} u^{\frac{3}{2}} \Big|_{20}^{20+9t^2}$$

$$= \int_0^t \sqrt{(20 + 9t^2)}\, t\, dt$$

$$= \frac{1}{18} \int_{20}^{20+9t^2} \sqrt{u}\, du$$

$\frac{2}{3}(20 + 9t^2)^{\frac{3}{2}} - \frac{2}{3} 20^{\frac{3}{2}}$

$\frac{1}{27}\left((20 + 9t^2)^{\frac{3}{2}} - 20^{\frac{3}{2}}\right)$

11. $r(t) = \langle 2t+3, 4t-3, 5-t \rangle$

$r'(t) = \langle 2, 4, 5 \rangle$

$\|r'(t)\| = \sqrt{2^2+4^2+15^2}$

$= \sqrt{4+16+15}$

$= \sqrt{25}$

B $r(t) = \langle t, \ln t, \ln(t)^2 \rangle$

$r'(t) = \langle 1, \frac{1}{t}, \frac{2(\ln t)^1}{t} \rangle$

$\sqrt{1^2+1^2+0} = \sqrt{2}$

15. $\langle \sin 3t, \cos 4t, \cos 5t \rangle$ $t = \frac{\pi}{2}$

$r'(t) = \sqrt{3\cos(3t)^2 + (-4\sin(4t))^2 + (-5\sin 5t)^2}$

$r'(0) = \sqrt{0 + 0 + 25} = 5$

13.4: 1, 5, 7, 11, 17, 21

1. $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle$

$r'(t) \langle 8t, 9 \rangle$ $r''(t) = 8$

$\sqrt{(8t + 9)^2} = \sqrt{64t^2 + 1}$

$T(t) = \frac{8 + 9}{\sqrt{64t^2 + 81}} \langle 8t, 9 \rangle$

$T(1) = \left\langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \right\rangle$

5. $r'(t) = \langle -\pi \sin(\pi t), \pi \cos(\pi t), 1 \rangle$

$$T(t) = \frac{1}{\sqrt{\pi^2+1}} \langle -\pi \sin(\pi t), \pi \cos(\pi t), 1 \rangle$$

$$T(1) = \langle 0, \frac{-\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \rangle$$

$(-\pi \sin(\pi t))^2 = \pi^2 \sin(\pi t)^2$

$(\pi \cos(\pi t)^2) = \pi^2 \cos(\pi t)^2$

$1^2 = 1$

$$\sqrt{\pi^2 \sin(\pi t)^2 + \pi^2 \cos(\pi t)^2 + 1}$$

$$\sqrt{\pi^2 + 1}$$

$T(1)$

$-\pi \sin \pi \quad \pi \cos \pi \quad 1$

$\downarrow \quad \downarrow \quad \downarrow$

$0 \quad \pi \quad 1$

7.

$$k(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

$r(t) = \langle 1, e^t, t \rangle$

$$\begin{array}{ccc} i & j & k \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{array}$$

$-e^t i + 0j + 0k$

$r'(t) = \langle 0, e^t, 1 \rangle \quad r''(t) = \langle 0, e^t, 0 \rangle$

$$k(t) = \frac{e^{2t}}{(e^{2t}+1)^{\frac{3}{2}}}$$

$\sqrt{0^2 + (e^t)^2 + 1^2}$

$\left(\sqrt{e^{2t}+1}\right)^3$

11. $r(t) = \langle 1/t,\ 1/t^2,\ t^2 \rangle$ $t = 1$

$r'(t) = \langle -\frac{1}{t^2}, \frac{-2}{t^3}, 2t \rangle$

$\frac{6}{t^6} + \frac{4}{t^6}$

$r''(t) = \langle \frac{2}{t^3}, \frac{-6}{t^4}, 2 \rangle$ $4t$ 3

$\sqrt{1+4+4}$

$3^3 = 27$

$\frac{\sqrt{128 - 36 + 100}}{27}$

21

$\frac{6}{t^5} + \frac{4}{t^6}$

$-\frac{4}{t^3} + \frac{12}{t^3}$ $\frac{\sqrt{200}}{27}$

$-\frac{1}{t^2}$ $\frac{-2}{t^3}$ K $2t$ $\frac{6}{t^6} + \frac{4}{t^6}$

$-\frac{2}{t^2} - \frac{4}{t^2}$ $\frac{-2}{t^2}$ $-\frac{4}{t^2}$ $\frac{6}{t^6} + \frac{4}{t^6}$ $\frac{2}{t^3}$ $\frac{-6}{t^4}$ 2 $\frac{-2}{t^2} - \frac{4}{t^2}$

$\frac{6}{t^2}$ $\frac{4}{t^2}$ $-\frac{4}{t^3} + \frac{12}{t^3}$

$\left(\frac{8}{t^3}\right)^2 - \left(\frac{6}{t^2}\right)^2 + \left(\frac{10}{t^6}\right)$ $-\frac{4}{t^3} + \frac{12}{t^3}$

$-\frac{6}{t^2}$

$\frac{64}{t^6} + \frac{36}{t^4} + \frac{10}{t^6}$

$64 - 36 + 10$ $\frac{74}{t^6} + \frac{36}{t^4} + \frac{100}{t^6}$

17. $y = t^4$

$y' = 4t^3$

$y'' = 12t^2$

$$\frac{\|12t^2\|}{(1+(4t^3)^2)^{\frac{3}{2}}}$$

$t = 2$

$$\frac{\sqrt{12t^2}}{(1+16t^6)^{\frac{3}{2}}} = \frac{\sqrt{48}}{(1025)^{1.5}}$$

21. $r'(t) = \langle 1 - \operatorname{sech}^2 t, -\operatorname{sech} x \tanh x \rangle$

$r''(t) = \langle 1 - 2\operatorname{sech}^2 t \tan x, -\operatorname{sech}^3 x - \operatorname{sech} x \tanh^2 x$

$$\cancel{\frac{\sqrt{(1-2\operatorname{sech}^2 x)^2 + (-\operatorname{sech}^3 x - \operatorname{sech} x \tanh^2 x)^2}}{(\sqrt{(1-\operatorname{sech}^2 t)^2 + (\operatorname{sech} x \tanh h)^2})^3}}$$

$$\frac{\sqrt{\tan^4 h + \operatorname{sech}^3 x}}{(\sqrt{\tan^4 h + \operatorname{sech}^3 x})^{\frac{3}{2}}}$$

$$\frac{1}{\sqrt{\tan^4 hx + \operatorname{sech} x}}$$

sech x

13-5: 3, 5, 15, 17, 31

3. velocity

$r'(t) = \langle 3t^2, -1, 8t \rangle$ $r''(t) = \langle 6t, 0, 8 \rangle$

$V(1) = \langle 3, -1, 8 \rangle$ $a(1) = \langle 6, 0, 8 \rangle$

Speed $\sqrt{9+1+64} = \sqrt{74}$

5. $r'(t) = \langle \cos\theta, -\sin\theta, -3\sin(3\theta) \rangle$

$r''(t) = \langle -\sin\theta, -\cos\theta, -9\cos(3\theta) \rangle$

$V(\frac{\pi}{3}) = \langle \cos\frac{\pi}{3}, -\sin\frac{\pi}{3}, -3\sin(\pi) \rangle$

$a(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, -\cos\frac{\pi}{3}, -9\cos(\pi) \rangle$

Velocity: $\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \rangle$ $\frac{1}{4} + \frac{3}{4} + 0$

acceleration: $\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9 \rangle$

Speed: 1

15.

$a(t) = \langle t, 4 \rangle$

$V(t) = \int a(t)$

$V(t) = \langle \frac{t^2}{2} + c, 4t + c \rangle$ $\quad V(0) = \langle 3, -2 \rangle$

$-2 = 4(0) + c$

$c = -2$

$3 = \frac{t^2}{2} + c$ $\quad -2 = 4t$

$3 = c$ $\quad t = -\frac{1}{2}$

$V(t) = \langle \frac{t^2}{3} + 3, 4t - 2 \rangle$

$r(0) = \langle 0, 0 \rangle$

$0 = \frac{t^3}{9} + 3t + c$ $\quad 0 = \frac{4t^2}{2} - 2t + c$

$0 = c$ $\quad 0 = c$

$r(t) = \langle \frac{t^3}{3} + 3, 2t^2 - 2t \rangle$

$V(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

17. $a(t) = t k$ $\quad i\ j\ k$

$v(t) = \langle \frac{t^2}{2} k + C \rangle$

$v(1) = C \langle C, C, t \rangle$

$v(t) = \frac{t^2}{2} k + i$

$r(t) = \frac{t^3}{3} k + ti + C$

$j = 0 + 0 + C$

$j = C$

$r(t) = \frac{t^3}{3} k + ti + j$

$v(t) = \frac{t^2}{2} k + i$

31. $\langle 12, 20, 20\rangle$ $12(2)+20-60$

$\langle 2, 1, -3\rangle$ $24+20-60$

$44-60$

$-16<0$

decreasing

(4.) : 1, 3, 7, 21, 23, 33, 35,

1. $f(x,y) = x + y x^3$

$f(2,2) = 2 + (2)(8) = 18$

$f(-1,4) = -1 + 4(-1) = -1 - 4 = -5$

3. $h(x,y,z) = xyz^{-2}$

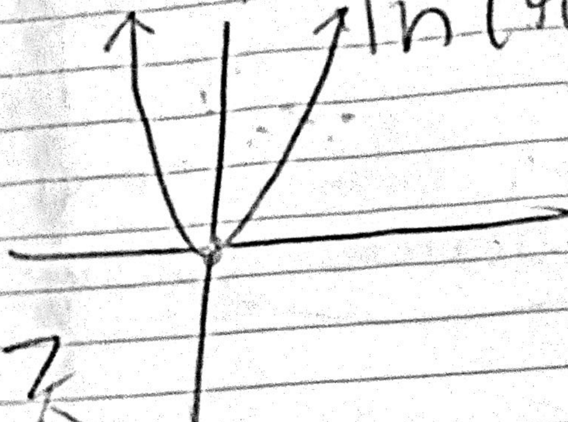
$h(3,8,2) = \frac{3(8)}{4} = \frac{24}{4} = 6$

$h(3,-2,-6) = \frac{3(-2)}{36} = \frac{-6}{36} = -\frac{1}{6}$

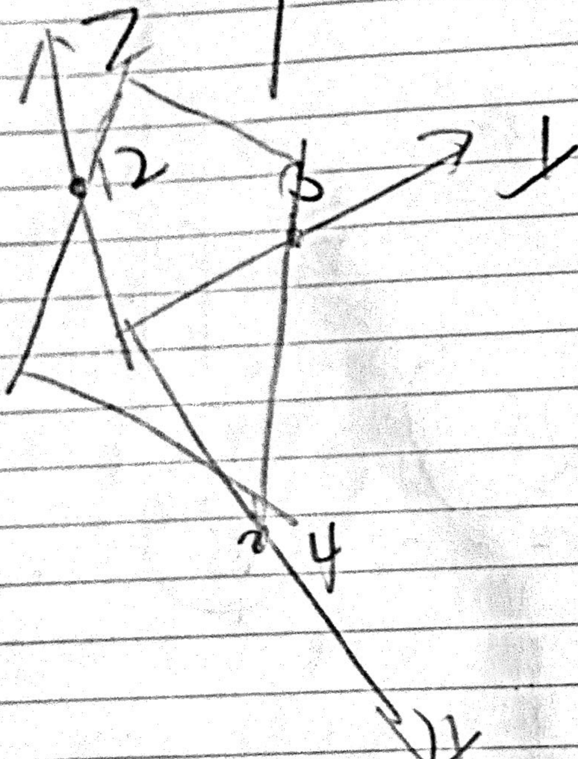
7. $\ln(0)=0$

$\ln(4-1)=\ln(3)$

$\ln(4(1)-1)=\ln(3)$



21.



$f(x,y)=12-3x-4y$

$f(0,0)=12$

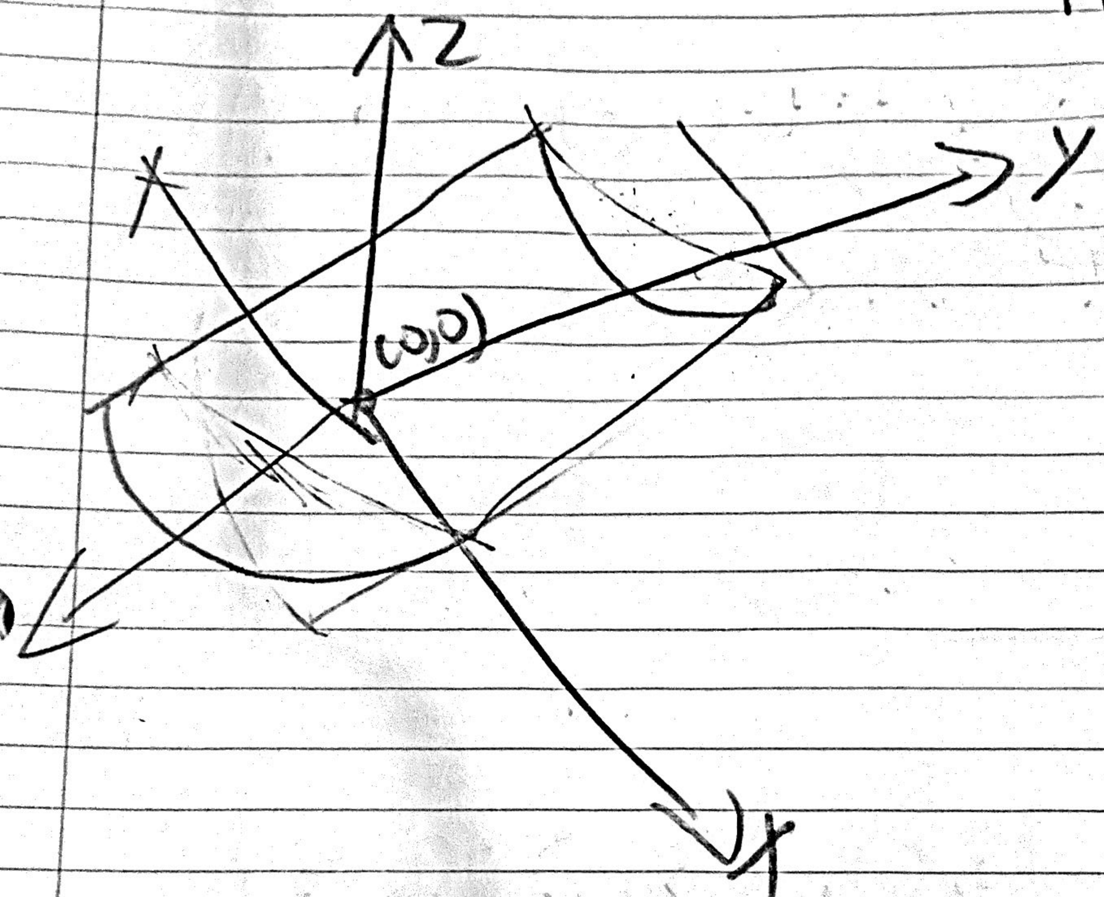
$f(0,y)=12-4y$

$\frac{-12}{-4}=\frac{-4y}{-4}$

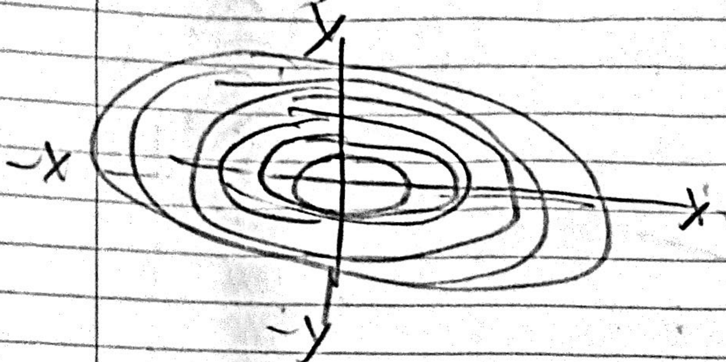
$y=3$

23 $f(x,y) = x^2 + 4y^2$

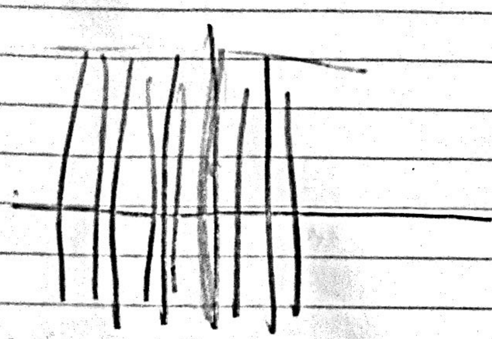
$f(0,0) = 0 + 0$
$= 0$



33) $f(x,y) = x^2 + 4y^2$



35



14. 1,2, 9, 11, 15, 21, 23, 27, 31, 35

9. $\lim\limits_{(x,y)\to(2,5)} (g(x,y) - 2f(x,y))$

$\lim\limits_{(x,y)\to(2,5)} g(x,y) - 2\cdot\lim\limits_{(x,y)\to(2,5)} f(x,y)$

$7 - 2(3) = 1$

11. $\lim_{(x,y)\to(2,5)} f(x,y) = 3$

$\lim_{(x,y)(2,5)} g(x,y) = 7$

$\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)}$

$= e^{9-7} = e^2$

15. $\lim_{(x,y)\to(0,0)} = \frac{x^3+x^3}{xy^2} = \frac{0}{0}$ $\quad y = mx$

$\lim_{(x,y)\to(0,0)\, y=mx} = \frac{x^3+y^3}{xy^2} = \lim_{\substack{(x,y)\to(0,0)\\ y=mx}} \frac{x^3+m^3x^3}{x\,m^2y^2}$

$= \lim_{(x,y)\to(0,0)\, y=mx} \left(\frac{x^2}{m^2x^2} + my\right)$ it depends on value of m so limit dne

$\lim_{(x,y)\to(0,0)} f(x,y)$ dne

21. $\lim_{(x,y)\to(0,0)} = \frac{0}{0}$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=mx}} \frac{xy}{3x^2+2y^2} = \lim_{x\to 0} m \frac{x^2}{3x^2+2m^2x^2} = \frac{m}{2m^2+3}$$

Since it depends on the value of m it does not exist.

22. $$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2} = \lim_{(x,y,z)\to(0,0,0)} \frac{1}{x} = \infty$$

Lets say $y=z=0$

Goes to ∞ so limit doo not exist

23. $$\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \frac{-2^4\cos(\pi)}{e^{-2+w}} = \frac{-16}{e^{-1}}$$

$= -16e$

31. $\lim\limits_{(x,y)\to(3,4)} \dfrac{1}{\sqrt{x^2+y^2}} = \dfrac{1}{\sqrt{9+16}} = \dfrac{1}{5}$

35. $\lim\limits_{(x,y)\to(1,-3)} (x^2y^3+4xy) = (9)(-8) + 4(-3)(-2)$

$= -72 + 24 = -48$