

Fayed Raza 9/27/20

13.3: 3, 9, 11, 13, 15

3) $r(t) = \langle 2t, \ln t, t^2 \rangle \quad | < t \leq 4$

$$L = \int_1^4 \sqrt{(2)^2 + \left(\frac{1}{t}\right)^2 + (2t)^2} dt$$

$$L = \int_1^4 \sqrt{4 + \frac{1}{t^2} + 4t^2} dt = \int_1^4 \sqrt{\frac{4t^4 + 1 + 4t^6}{t^2}} dt$$

$$= \int_1^4 \frac{\sqrt{4t^6 + 4t^2 + 1}}{t} dt = \int_1^4 \left(2t + \frac{1}{t}\right) dt = 15 + \ln(4)$$

$$= \int_1^4 2t dt + \int_1^4 \frac{1}{t} dt = \left. \frac{2t^2}{2} \right|_1^4 + \ln(4) - \ln(1) = \ln(4)$$

9) $r(t) = \langle t^2, 2t^2 + 3, 9t \rangle \quad 0 \leq t \leq 2$

$$= \int_0^2 \sqrt{4t^4 + 16t^4 + 9t^4} dt \quad u = 20t^2 + 9 \quad du = 40t dt$$

$$= \int_0^2 \sqrt{20t^2 + 9} dt \quad \frac{du}{40} = t dt$$

$$= \int_0^2 \sqrt{t^2(20 + 9t^2)} dt = \frac{2}{3} u^{3/2} \Big|_{9}^{20+36}$$

$$= \int_0^2 \sqrt{20 + 9t^2} t dt = \frac{2}{3} (20 + 9t^2)^{3/2} \Big|_0^2$$

$$= \frac{2}{3} \int_0^2 \sqrt{u} du = \frac{2}{3} \left(\frac{2}{3} (20 + 9t^2)^{3/2} - \frac{2}{3} (20)^{3/2} \right)$$

$$11. \quad r(t) = \langle 2t+3, 4t-3, 5-t \rangle$$

$$r'(t) = \langle 2, 4, -1 \rangle$$

$$\|r'(t)\| = \sqrt{2^2 + 4^2 + 1^2}$$

$$= \sqrt{4+16+1}$$

$$= \sqrt{21}$$

$$B \quad r(t) = \langle t, \ln t, \ln(t)^2 \rangle$$

$$r'(t) = \left\langle 1, \frac{1}{t}, \frac{2(\ln t)^1}{t} \right\rangle$$

$$\sqrt{1^2 + 1^2 + 0} = \sqrt{2}$$

$$15. \langle \sin 3t, \cos 4t, \cos 5t \rangle \quad t = \frac{\pi}{2}$$

$$r'(t) = \sqrt{3(\cos(3t))^2 + (-4\sin(4t))^2 + (-5\sin(5t))^2}$$

$$r'(0) = \sqrt{0 + 0 + 25} = 5$$

$$13. 4: 1, 5, 7, 11, 17, 21$$

$$1. r(t) = \langle 4t^2, 9t \rangle$$

$$r'(t) = \langle 8t, 9 \rangle$$

$$r'(t) = \langle 8t, 9 \rangle$$

$$r''(t) = 8$$

$$\sqrt{(8t+9)^2} = \sqrt{64t^2 + 144t + 81}$$

$$T(t) = \frac{\langle 8t, 9 \rangle}{\sqrt{64t^2 + 144t + 81}}$$

$$T(1) = \left\langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \right\rangle$$

$$5 \quad r'(t) = \langle -\pi \sin(\pi t), \pi \cos(\pi t), 1 \rangle$$

$$T(t) = \frac{1}{\pi^2 + 1} \langle -\pi \sin(\pi t), \pi \cos(\pi t), 1 \rangle$$

$$(-\pi \sin(\pi t))^2 = \pi^2 \sin^2(\pi t) \quad T(1) < 0, \frac{-\pi}{\sqrt{\pi^2 + 1}}, \frac{1}{\sqrt{\pi^2 + 1}}$$

$$(\pi \cos(\pi t))^2 = \pi^2 \cos^2(\pi t)$$

$$1^2 = 1$$

$$\sqrt{\pi^2 \sin^2(\pi t) + \pi^2 \cos^2(\pi t) + 1}$$

$$\sqrt{\pi^2 + 1}$$

$T(1)$

$$-\pi \sin \pi$$

$$\downarrow$$

$$0$$

$$\pi \cos \pi$$

$$\downarrow$$

$$\pi$$

$$1$$

$$\downarrow$$

$$1$$

$$K(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

$$\|r'(t)\|^3$$

$$r(t) = \langle 1, e^t, t \rangle$$

$$r''(t) = \begin{matrix} & j & k \\ \begin{matrix} i \\ 0 \\ 0 \end{matrix} & \begin{matrix} e^t & 1 \\ e^t & 0 \\ -e^t & 0 \end{matrix} \end{matrix}$$

$$r'(t) = \langle 0, e^t, 1 \rangle$$

$$r''(t) = \langle 0, e^t, 0 \rangle$$

$$K(t) = \frac{e^{2t}}{(e^{2t} + 1)^{\frac{3}{2}}}$$

$$\sqrt{0^2 + (e^t)^2 + 1^2}$$

$$\left(\sqrt{e^{2t} + 1} \right)^3$$

$$11. \quad r(t) = \left\langle \frac{1}{t}, \frac{1}{t^2}, t^2 \right\rangle \quad t=1$$

$$r'(t) = \left\langle -\frac{1}{t^2}, -\frac{2}{t^3}, 2t \right\rangle$$

$$r''(t) = \left\langle \frac{2}{t^3}, -\frac{6}{t^4}, 2 \right\rangle$$

$$\frac{6}{t^6} + \frac{4}{t^6}$$

$$\sqrt{1+4+4}$$

$$3^3 = 27$$

$$\sqrt{7128 \cdot 36 + 100}$$

$$27$$

$$\frac{6}{t^6} + \frac{4}{t^6}$$

$$\frac{4}{t^3} + \frac{12}{t^3}$$

$$\frac{\sqrt{200}}{27}$$

$$-\frac{1}{t^2}$$

$$\frac{2}{t^3}$$

$$2t$$

$$\frac{6}{t^6} + \frac{4}{t^6}$$

$$\frac{2}{t^2} - \frac{4}{t^2}$$

$$\frac{2}{t^2}$$

$$-\frac{4}{t^2}$$

$$\frac{2}{t^3}$$

$$\frac{-6}{t^4}$$

$$2$$

$$\frac{-2}{t^2} - \frac{4}{t^2}$$

$$\frac{4}{t^3} + \frac{12}{t^3}$$

$$\left(\frac{8}{t^3}\right)^2 - \left(\frac{6}{t^2}\right)^2 + \left(\frac{10}{t^6}\right)^2$$

$$\frac{64}{t^6} + \frac{36}{t^4} + \frac{10}{t^6}$$

$$-\frac{4}{t^3} + \frac{12}{t^3}$$

$$\frac{74}{t^6} + \frac{36}{t^4} + \frac{100}{t^6}$$

17.

$$y = t^4$$

$$y' = 4t^3$$

$$y'' = 12t^2$$

$$\| \| 2t^2 \| \|$$

$$(1 + (t^3)^2)^{\frac{3}{2}} \quad t=2$$

$$\frac{\sqrt{12} t^2}{(1 + 16t^6)^{\frac{3}{2}}} = \frac{\sqrt{48}}{(1025)^{\frac{3}{2}}}$$

21

$$r'(t) = \langle 1 - \operatorname{sech}^2, -\operatorname{sech}^2 \tanh \rangle$$

$$r''(t) = \langle -2\operatorname{sech}^2 \tanh, -\operatorname{sech}^3 - \operatorname{sech} \tanh^3 \rangle$$

$$\sqrt{(1 - 2\operatorname{sech}^2 \tanh)^2 + (-\operatorname{sech}^3 - \operatorname{sech} \tanh^3)^2}$$

$$\sqrt{(1 - \operatorname{sech}^2)^2 + (\operatorname{sech} \tanh^2)^2}$$

$$\sqrt{\tanh^4 + \operatorname{sech}^3}$$

$$\sqrt{\tanh^4 + \operatorname{sech}^3}^{\frac{3}{2}}$$

$$\sqrt{\tanh^4 + \operatorname{sech}^3}$$

sech^2

$$13.5. \quad 3, 5, 15, 17, 3)$$

3,

velocity

$$r'(t) = \langle 3t^2, -1, 8t \rangle \quad r''(t) = \langle 6t, 0, 8 \rangle$$

$$v(t) = \langle 3, -1, 8 \rangle \quad a(t) = \langle 6, 0, 8 \rangle$$

$$\text{Speed } \sqrt{9+1+64} = \sqrt{74}$$

$$5 \quad r'(t) = \langle \cos t, -\sin t, 3\sin t \rangle$$

$$r''(t) = \langle -\sin t, -\cos t, 9\cos t \rangle$$

$$v\left(\frac{\pi}{3}\right) = \langle \cos\frac{\pi}{3}, -\sin\frac{\pi}{3}, 3\sin\left(\frac{\pi}{3}\right) \rangle$$

$$a\left(\frac{\pi}{3}\right) = \langle -\sin\frac{\pi}{3}, -\cos\frac{\pi}{3}, 9\cos\left(\frac{\pi}{3}\right) \rangle$$

$$\text{Velocity: } \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \right\rangle$$

$$\text{acceleration: } \left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9 \right\rangle$$

$$\text{Speed: } 1$$

$$\frac{1}{4} + \frac{3}{4} + 0$$

15.

$$a(t) = \langle t, t \rangle$$

$$v(t) = \int a(t)$$

$$v(t) = \left\langle \frac{t^2}{2} + C, 4t + D \right\rangle$$

$$v(0) = \langle 3, -2 \rangle$$

$$3 = \frac{t^2}{2} + C$$

$$-2 = 4(0) + C$$

$$C = -2$$

$$3 = C$$

$$-2 = 4t$$

$$t = -\frac{1}{2}$$

$$v(t) = \left\langle \frac{t^2}{2} + 3, 4t - 2 \right\rangle$$

$$v(0) = \langle 0, 0 \rangle$$

$$0 = \frac{t^3}{9} + 3t + C$$

$$0 = \frac{4t^2}{2} - 2t + C$$

$$0 = C$$

$$0 = C +$$

$$r(t) = \left\langle \frac{t^3}{3} + 3, 2t^2 - 2t \right\rangle$$

$$v(t) = \left\langle \frac{t^2}{2} + 3, 4t - 2 \right\rangle$$

$$17. \quad a(t) = t^2 k \quad i, j, k$$

$$v(t) = \frac{t^2}{2} k + C$$

$$v(i) = C \quad C_1, C_2, C_3$$

$$v(t) = \frac{t^2}{2} k + j$$

$$j = \frac{2}{2} C$$

$$2. \quad r(t) = \frac{t^3}{3} k + \frac{t^2}{2} j + C$$

$$C = \frac{2^3}{3} = 0 + 0 + C$$

$$0 = C$$

$$r(t) = \frac{t^3}{3} k + \frac{t^2}{2} j + j$$

$$v(t) = \frac{t^2}{2} k + j$$

31.

$$\langle 12, 20, 20 \rangle$$

$$\langle 2, 1, -3 \rangle$$

$$12(2) + 20 = 60$$

$$24 + 20 = 60$$

$$44 = 60$$

$$-16 < 0$$

sgn

decreasing

4.) : 1, 3, 7, 21, 23, 33, 35,

1. $f(x, y) = x + yx^3$

$$f(2, 2) = 2 + (2)(8) = 18$$

$$f(-1, 4) = -1 + 4(-1) = -1 - 4 = -5$$

3. $h(x, y, z) = xyz^{-2}$

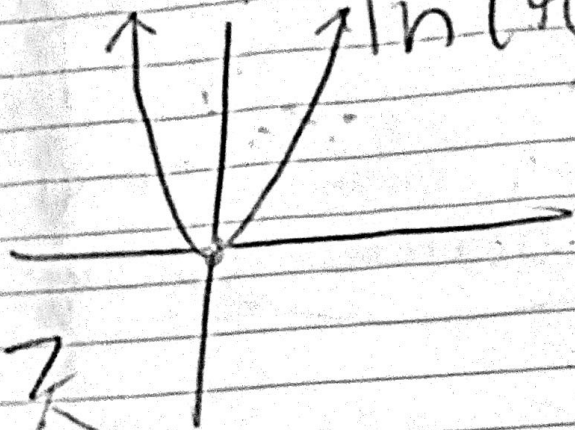
$$h(3, 8, 2) = \frac{3(8)}{4} = \frac{24}{4} = 6$$

$$h(3, -2, -6) = \frac{3(-2)}{36} = -\frac{6}{36} = -\frac{1}{6}$$

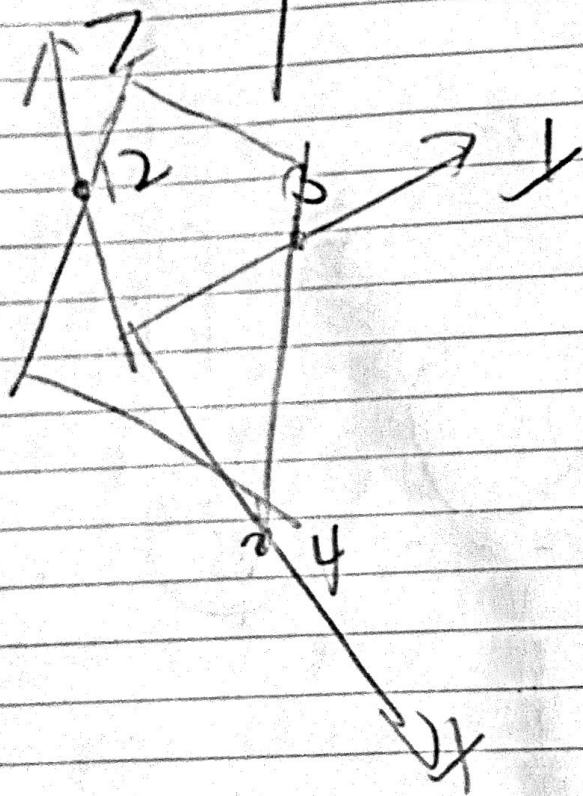
$$7. \ln(0) = 0$$

$$\ln(4-1) = \ln(3)$$

$$\ln(4(1)-1) = \ln(3)$$



21.



$$P(x, y) = 12 - 3x - 4y$$

$$P(0, 0) = 12$$

$$P(0, y) = 12 - 4y$$

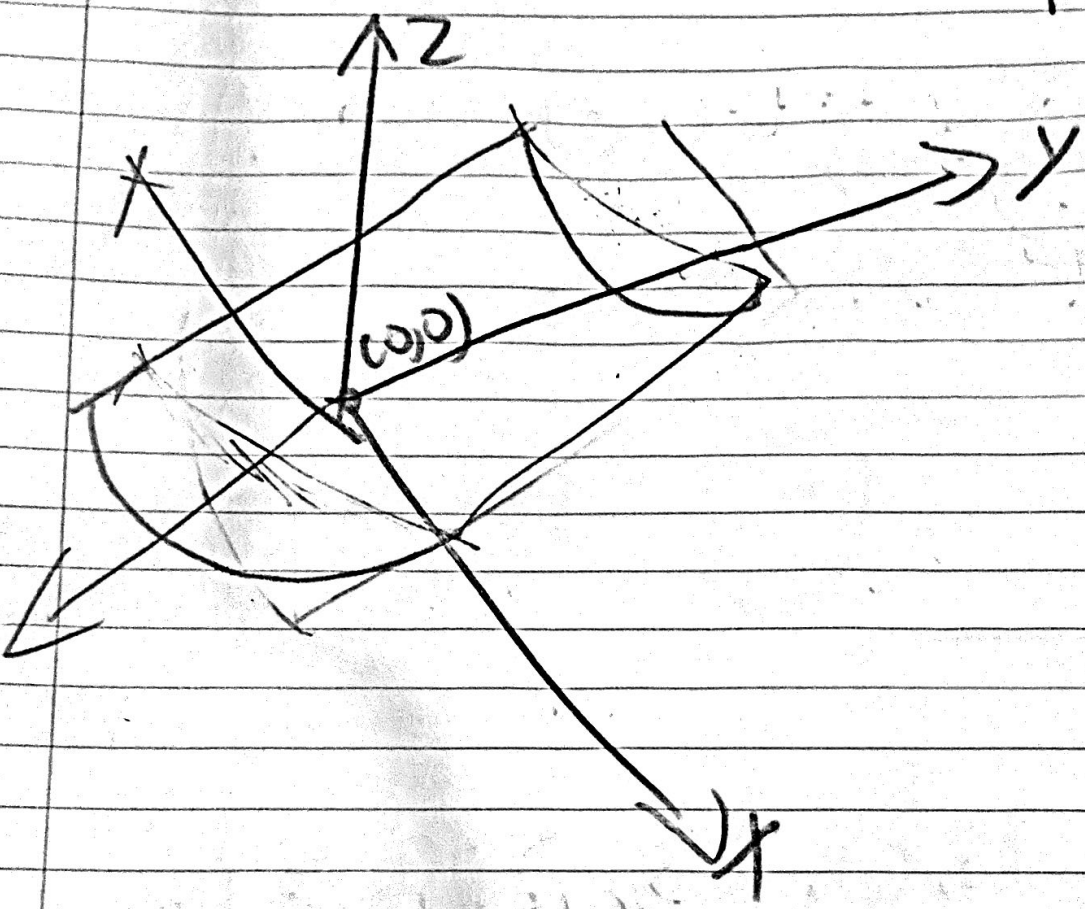
$$\frac{12 = -4y}{-4} = \frac{-4y}{-4}$$

$$y = 3$$

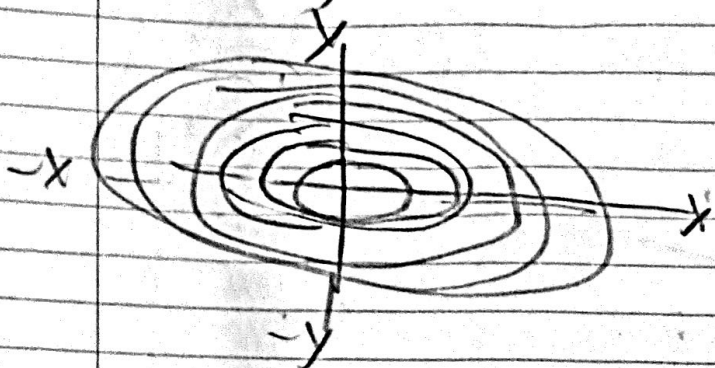
23

$$f(x, y) = x^2 + 4y^2$$

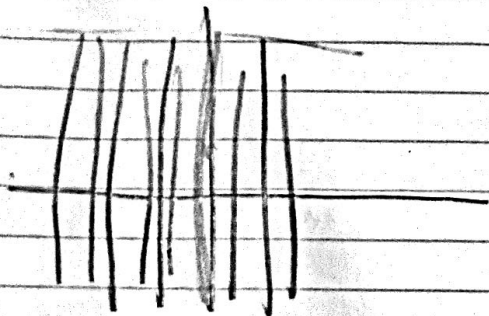
$$f(0, 0) = 0, 0 = 0$$



33 $f(x, y) = x^2 + 4y^2$



35



14. $1, 2, 9, 11, 15, 21, 23, 27, 31, 35, \dots$

Q. $\lim_{(x,y) \rightarrow (2,5)} (g(x,y) - 2f(x,y))$

$$\lim_{(x,y) \rightarrow (2,5)} g(x,y) - 2 \cdot \lim_{(x,y) \rightarrow (2,5)} f(x,y)$$

$$7 - 2(3) = 1$$

$$11. \lim_{(x,y) \rightarrow (2,5)} f(x,y) = 3$$

$$\lim_{(x,y) \rightarrow (2,5)} g(x,y) = 7$$

$$\lim_{(x,y) \rightarrow (2,5)} e^{f(x,y)^2 - g(x,y)}$$

$$= e^{9-7} = e^2$$

$$15. \lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{xy^2} = \frac{0}{0} \quad y = mx$$

$$\lim_{(x,y) \rightarrow (0,0) \text{ } y=mx} \frac{x^3 + y^3}{xy^2} = \lim_{\substack{(x,y) \rightarrow (0,0) \\ y=mx}} \frac{x^3 + m^3 x^3}{x \cdot m^2 y^2}$$

$$= \lim_{(x,y) \rightarrow (0,0) \text{ } y=mx} \left(\frac{x^2}{m^2 y^2} + my \right)$$

it depends
on value of
m so limit
dne

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) \text{ dne}$$

$$26. \lim_{(x,y) \rightarrow (0,0)} = \frac{0}{0}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{3x^2 + 2y^2} = \lim_{x \rightarrow 0} \frac{mx}{3x^2 + 2m^2x^2} = \lim_{x \rightarrow 0} \frac{m}{3x + 2m^2} = \frac{m}{2m^2 + 3}$$

Since it depends on the value of m it does not exist.

27.

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2y + z}{x^2 + y^2 + z^2} = \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{1}{x} = \infty$$

Let's say $y = z = 0$

Goes to ∞ so limit does not exist

$$28. \lim_{(z,w) \rightarrow (-2,1)} \frac{z^4 \cos(\pi w)}{e^{z+w}} = \frac{-2^4 \cos(\pi)}{e^{-2+1}} = \frac{-16}{e^{-1}} = -16e$$

$$31. \lim_{(x,y) \rightarrow (3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{9+16}} = \frac{1}{5}$$

$$35. \lim_{(x,y) \rightarrow (1,-3)} (x^2y^3 + 4xy) = (1)(-27) + 4(1)(-3) \\ = -27 - 12 = -39$$