CALC 3 - HW due 09/27 (Sections 13.3 -

13.3 - # 3, 9, 11, 13, 15

3. $r(t) = \langle 2t, \ln t, t^2 \rangle$
over $1 \le t \le 4$

$r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$

$L = \int_a^b |r'(t)| dt$

$= \int_1^4 \sqrt{2^2 + (\frac{1}{t})^2 + (2t)^2}\, dt$

$= \int_1^4 \sqrt{4t^2 + \frac{1}{t^2} + 4}\, dt$

$= \int_1^4 (4t^2 + \frac{1}{t^2} + 4)^{1/2} dt$

$= \int_1^4 \left(\frac{4t^4 + 4t^2 + 1}{t^2}\right)^{1/2} dt$

$= \int_1^4 \left(\frac{(2t^2+1)^2}{t^2}\right)^{1/2} dt$

$= \int_1^4 \frac{2t^2+1}{t} dt$

$= \int_1^4 2t + \frac{1}{t}\, dt$

$= t^2 + \ln(t) \Big|_1^4$

$= [(4^2 + \ln(4) - (1^2 + \ln(1))]$

$= 15 + \ln(4)$

9. $r(t) = \langle t^2, 2t^2, t^3 \rangle, a = 0$ over $s(t) = \int_a^t |r'(u)| du$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$|r'(t)| = \sqrt{(2t)^2 + (4t)^2 + (3t^2)^2}$

$= \sqrt{4t^2 + 16t^2 + 9t^4}$

$= \sqrt{9t^4 + 20t^2}$

$= \sqrt{t^2(9t^2+20)}$

$= t\sqrt{9t^2+20}$

$s(t) = \int_0^t (u\sqrt{9u^2+20})\, du = \frac{1}{18}\int_0^t v^{1/2} dv$

$v = 9u^2 + 20$
$dv = 18u$
$\frac{dv}{18} = u$

$= \frac{1}{18}\left[\frac{2}{3} v^{3/2}\right]\Big|_0^t$

$= \frac{1}{27}(9u^2+20)^{3/2}\Big|_0^t$

$s(t) = \frac{1}{27}(9t^2+20) - \frac{1}{27}(20)^{3/2}$

11. $r(t) = \langle 2t+3, 4t-3, 5-t \rangle$

$r'(t) = v(t) = \langle 2, 4, -1 \rangle$ @ $t = 4$

$|v(t)| = \sqrt{2^2 + 4^2 + (-1)^2} = \sqrt{21}$

13. $r(t) = \langle t, \ln(t), (\ln t)^2 \rangle$

$r'(t) = v(t) = \langle 1, \frac{1}{t}, \frac{2}{t} \rangle$ @ $t = 1$

$|v(t)| = \sqrt{1^2 + (\frac{1}{1})^2 + (\frac{2}{1})^2} = \sqrt{1+1+4} = \sqrt{6}$

15.
$r(t) = \langle \sin(3t), \cos(4t), \cos(5t) \rangle$

$r'(t) = v(t) = \langle 3\cos(3t), -4\sin(4t), -5\sin(5t) \rangle$ $t = \frac{\pi}{2}$

$|v(t)| = \sqrt{(3\cos(\frac{3\pi}{2}))^2 + (-4\sin(2\pi))^2 + (-5\sin(\frac{5\pi}{2}))^2}$

$|v(t)| \approx 9.598$

13.4 - # 1, 5, 7, 11, 17, 21

1. $r(t) = \langle 4t^2, 9t \rangle$

$T(t) = \dfrac{r'(t)}{|r'(t)|}$

$r'(t) = \langle 8t, 9, 0 \rangle$

$|r'(t)| = \sqrt{(8t)^2 + 9^2 + 0^2}$

$= \sqrt{16t^2 + 18}$

$T(t) = \left\langle \dfrac{8t}{\sqrt{16t^2+18}}, \dfrac{9}{\sqrt{16t^2+8}}, 0 \right\rangle$

$T(1) = \left\langle \dfrac{8}{\sqrt{34}}, \dfrac{9}{\sqrt{34}} \right\rangle$

5. $r(t) = \langle \cos(\pi t), \sin(\pi t), t \rangle$

$r'(t) = \langle -\pi \sin(\pi t), \pi \cos(\pi t), 1 \rangle$

$|r'(t)| = \sqrt{\pi^2 \sin^2(\pi t) + \pi^2 \cos^2(\pi t) + 1^2}$

$= \sqrt{\pi^2 + 1}$

$T(t) = \left\langle \dfrac{-\pi \sin(\pi t)}{\sqrt{\pi^2+1}} + \dfrac{\pi \cos(\pi t)}{\sqrt{\pi^2+1}}, \dfrac{1}{\sqrt{\pi+1}} \right\rangle$

$T(1) = \left\langle 0, \dfrac{\pi}{\sqrt{\pi^2+1}}, \dfrac{1}{\sqrt{\pi+1}} \right\rangle$

7. $r(t) = \langle 1, e^t, t \rangle$

$K = \dfrac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

$K = \dfrac{e^t}{(\sqrt{e^{t^2}+1})^3}$

$r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{vmatrix} = (-e^t)\hat{i} - 0\hat{j} + 0\hat{k}$

11. $r(t) = \left\langle \dfrac{1}{t}, \dfrac{1}{t^2}, t^2 \right\rangle$ @ $t = -1$

$r'(t) = \left\langle -\dfrac{1}{t^2}, \dfrac{1}{2t}, 2t \right\rangle$

$r''(t) = \left\langle \dfrac{2}{t^3}, \dfrac{-1}{2t^2}, 2 \right\rangle$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{1}{t^2} & \frac{1}{2t} & 2t \\ \frac{2}{t^3} & -\frac{1}{2t^2} & 2 \end{vmatrix} = \left(t + \dfrac{1}{t}\right)\hat{i} - \left(-\dfrac{2}{t^2} - \dfrac{4t}{t^3}\right)\hat{j} + \left(\dfrac{1}{2t^4} - \dfrac{2}{2t^4}\right)\hat{k}$

$= \left(\dfrac{t^2+1}{t}\right)\hat{i} + \left(\dfrac{6t}{t^3}\right)\hat{j} + \left(\dfrac{-1}{2t^4}\right)\hat{k}$

$K = \dfrac{\left\langle \frac{t^2+1}{t}, \frac{6}{t^2}, \frac{-1}{2t^4} \right\rangle}{\left(\sqrt{\left(\frac{-1}{t^2}\right)^2 + \left(\frac{1}{2t}\right)^2 + (2t)^2}\right)^3}$

$K = \dfrac{\left\langle 2, 6, -\frac{1}{2} \right\rangle}{\sqrt{\frac{21}{4}}} = \left\langle \dfrac{2}{\sqrt{\frac{21}{4}}}, \dfrac{6}{\sqrt{\frac{21}{4}}}, -\dfrac{\sqrt{\frac{21}{4}}}{2} \right\rangle$

17. $y = t^4$ @ $t = 2$

$y' = 4t^3$

$y'' = 12t^2$

$K = \dfrac{12t^2}{(1 + (4t^3)^2)^{3/2}}$

$K = \dfrac{12(2)^2}{(1 + (4(2))^2)^{3/2}} = \dfrac{48}{(65)^{3/2}}$

$K = 0.092$

21. $r(t) = \langle t - \tan(ht), \sec(ht) \rangle$

$r'(t) = \langle 1 - \sec^2(t), \sec(ht)\tan(ht) \rangle$

21. $r(t) = \langle t - \tanh(t), \operatorname{sech}(t) \rangle$

prove $K(t) = \operatorname{sech}(t)$

$$K = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$r'(t) = \langle 1 - \operatorname{sech}^2(t), -\operatorname{sech}(t)\tanh(t) \rangle$

$r'(t) = \langle \tanh^2(t), -\operatorname{sech}(t)\tanh(t) \rangle$

$r'(t) = \tanh t \langle \tanh t, -\operatorname{sech} t \rangle$

$r''(t) = \langle 2\tanh t \operatorname{sech}^2 t, \operatorname{sech} t \tanh^2 t - \operatorname{sech}^3 t \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \tanh^2 t & -\operatorname{sech} t \tanh t & 0 \\ 2\tanh \operatorname{sech}^3 t & \operatorname{sech} t \tanh^2 t - \operatorname{sech}^3 t & 0 \end{vmatrix}$$

$= \operatorname{sech} t \tanh^2 t \, \hat{k}$

$$K = \frac{\operatorname{sech} t \tanh^2 t}{(\tanh t)^3} = \operatorname{csch} t$$

13.5 – # 3, 5, 15, 17, 31, 33

3. $r(t) = \langle t^3, 1-t, 4t^2 \rangle$ @ $t=1$

$v(t) = \langle 3t^2, -1, 8t \rangle$

$|v(1)| = \sqrt{9 + 1 + 64}$

$|v(1)| = \sqrt{74}$

$a(t) = \langle 6t, 0, 8 \rangle$

$|a(1)| = \sqrt{36 + 64}$

$|a(1)| = 10$

5. $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle$ $\theta = \frac{\pi}{3}$

$v(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$|v(\theta)| = \sqrt{\cos^2\theta + \sin^2\theta + 9\sin^2(3\theta)}$

$= \sqrt{1 + 9\sin^2(\pi)}$

$|v(\frac{\pi}{3})| = 1.00$

$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$|a(\theta)| = \sqrt{1 + 81\cos^2(3\theta)}$

$|a(\frac{\pi}{3})| = \sqrt{1 + 81\cos^2(\pi)}$

$|a(\frac{\pi}{3})| = \sqrt{82}$

15. $a(t) = \langle t, 4 \rangle$ $v(t) = \langle \frac{1}{2}t^2, 4t \rangle$

$v(0) = \langle \frac{1}{3}, -2 \rangle$

$v(t) = \langle \frac{1}{2}t^2 + \frac{1}{3}, 4t - 2 \rangle$

17. $a(t) = t\hat{k}$ $v(t) = \langle 1, 0 \frac{1}{2}t^2 \rangle$

$v(0) = \hat{i}$

31. Find $|v|^2 \rightarrow 2\langle 2, 1, -3 \rangle \cdot \langle 12, 20, 20 \rangle$

$= -32 < 0 \rightarrow$ speed is decreasing

33. $r(t) = \langle t, \cos t, \sin t \rangle$

$$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|}$$

$$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|}$$

$r'(t) = \langle 1, -\sin t, \cos t \rangle$

$r''(t) = \langle 0, -\cos(t), -\sin t \rangle$

$$a_T = \frac{(1)(0) + (-\sin t)(-\cos t) + (\cos t)(-\sin t)}{\sqrt{1 + \sin^2 t + \cos^2 t}}$$

$$a_T = \frac{\sin t \cos t - \sin t \cos t}{\sqrt{2}} = 0$$

$$a_N = \frac{\sqrt{1 + \sin^2 t + \cos^2 t}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{vmatrix} = (\sin^2(t) + \cos^2(t))\hat{i} - (-\sin t - 0)\hat{j} + (-\cos t)\hat{k}$$

$= \hat{i} - \sin t \hat{j} - \cos t \hat{k}$

14.1- #1, 3, 7, 21, 23, 33, 35, 37

1. $f(x,y) = x + yx^3$

$f(2,2) = (2) + (2)(2)^3 = 18$

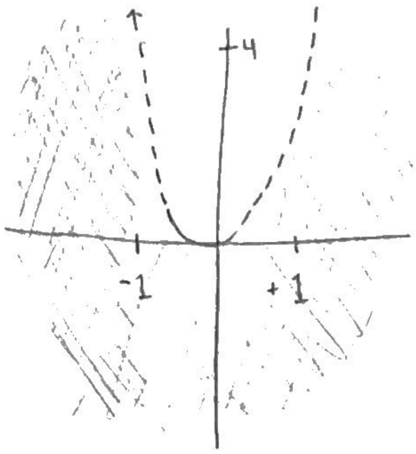
$f(-1,4) = (-1) + (4)(-1)^3 = -5$

3. $h(x,y,z) = xyz^{-2}$

$h(3,8,2) = (3)(8)(2)^{-2} = 6$

$h(3,-2,-6) = (3)(-2)(-6)^{-2} = -\frac{1}{6}$

7. $f(x,y) = \ln(4x^2 - y)$

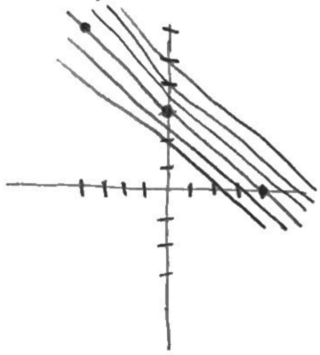


$4x^2 - y > 0$

$x^2 - \frac{y}{4} > 0$

$x^2 - \frac{1}{4}y > 0$

21. $f(x,y) = 12 - 3x - 4y$



Let $z = 0$

$0 = 12 - 3x - 4y$

$4y = 12 - 3x$

$y = -\frac{3}{4}x + 3$

Let $z = -1$

$-1 = 12 - 3x - 4y$

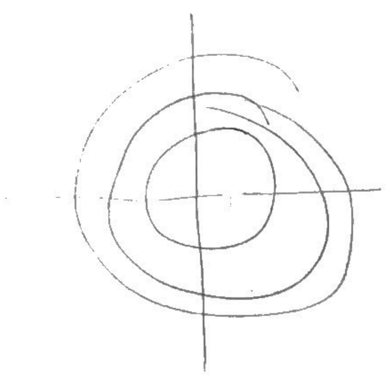
$y = -\frac{3}{4}x + \frac{13}{4}$

23. $f(x,y) = x^2 + 4y^2$

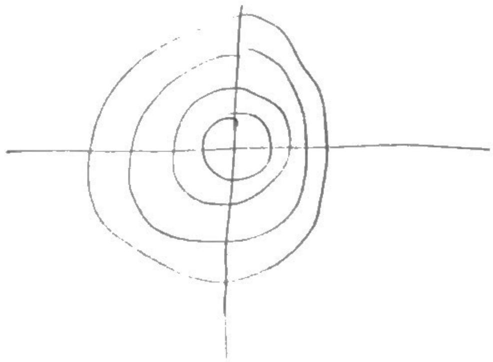
Let $z = 0$

$= x^2 + 4y^2$

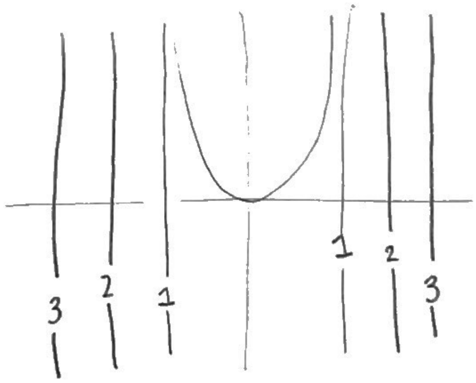
$= 4y^2$



33. $f(x,y) = x^2 + 4y^2$



35. $f(x,y) = x^2$



21. r(t) = ⟨t - tanh(t), sech(t)⟩

14.2 - #9, 11, 15, 21, 23, 27, 31, 35

9. $\lim_{(x,y)\to(2,5)} \left(g(x,y) - 2f(x,y)\right) = 7 - 2(3) = 1$

11. $\lim_{(x,y)\to(2,5)} \left[e^{f(x,y)^2 - g(x,y)}\right] = e^2$

15. $f(x,y) = \dfrac{x^3+y^3}{xy^2}$, $y = mx$

$f(x,mx) = \dfrac{x^3+(mx)^3}{x(mx)^2} = \dfrac{1+m^3}{m^2}$ → since m is in the denominator, $m \neq 0$.

therefore $\lim_{(x,y)\to(0,0)} f(x,y) = \text{DNE}$.

21. $\lim_{(x,y)\to(0,0)} \dfrac{xy}{3x^2+2y^2} = \dfrac{0}{0} = \text{DNE}$

23. $\lim_{(x,y,z)} \dfrac{x+y+z}{x^2+y^2+z^2} = \dfrac{0}{0} = \text{DNE}$

27. $\lim_{(z,w)\to(-2,1)} \dfrac{z^4\cos(\pi w)}{e^{z+w}} = \dfrac{-2^4\cos(\pi)}{e^{-2+1}} = -16e$

31. $\lim_{(x,y)\to(3,4)} \dfrac{1}{\sqrt{x^2+y^2}} = \dfrac{1}{\sqrt{9+16}} = \dfrac{1}{5}$

35. $\lim_{(x,y)\to(-3,2)} (x^2y^3+4xy) = (9(-8)+4(-3)(-2)) = -72+24 = -48$

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