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13.3 Homework

3. $r(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \le t \le 4$

$r'(t) = \langle 2, 1/t, 2t \rangle$

$|r'(t)| = \sqrt{(2)^2 + (1/t)^2 + (2t)^2} = \sqrt{4 + 1/t^2 + 4t^2}$

$= \sqrt{\dfrac{4t^2 + 1 + 4t^2}{t^2}} = \sqrt{\dfrac{(2t^2+1)^2}{t^2}} = \dfrac{2t^2+1}{t}$

$\displaystyle\int_1^4 \frac{2t^2+1}{t} = \int_1^4 2t + \frac{1}{t} = t^2 + \ln|t| \Big]_1^4$

$(16 + \ln|4|) - (1 + \ln|1|)$

$= \boxed{15 + \ln|4|}$

9. $r(t) = \langle t^2, 2t^2, t^3 \rangle \quad a = 0$

$r'(t) = \langle 2t, 4t, 3t^2 \rangle$

$||r'(t)|| = \sqrt{(2t)^2 + (4t)^2 + (3t^2)^2} = \sqrt{4t^2 + 16t^2 + 9t^4}$

$= \sqrt{20t^2 + 9t^4} = \sqrt{t^2(20 + 9t^2)} = t\sqrt{20 + 9t^2}$

$\displaystyle\int_0^t t\sqrt{20+9t^2}\,dt \qquad U = 20 + 9t^2 \qquad du = 18t\,dt \qquad dt = \frac{du}{18t}$

$\displaystyle\int_{20}^{20+9t^2} t\sqrt{U}\,\frac{du}{18t} \quad \frac{1}{18}\int \sqrt{U}\,du = \frac{1}{18} \cdot \frac{2U^{3/2}}{3} \Big]_0^{20+9t^2}$

$\dfrac{1}{27}\left[(20+9t^2)^{3/2} - (20)^{3/2}\right] = s(t)$

11. $r(t) = \langle 2t+3, 4t-5, 5-t \rangle \quad t=4$

$r'(t) = \langle 2, 4, -1 \rangle$

$r'(4) = \langle 2, 4, -1 \rangle$

$\|r'(4)\| = \sqrt{4+16+1} = \boxed{\sqrt{21}}$

13. $r(t) = \langle t, \ln t, (\ln t)^2 \rangle \quad t=1$

$r'(t) = \langle 1, 1/t, 2(\ln t)/t \rangle$

$r'(1) = \langle 1, 1, 0 \rangle$

$\|r'(1)\| = \sqrt{1+1} = \boxed{\sqrt{2}}$

15. $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle \quad t = \pi/2$

$r'(t) = \langle 3\cos 3t, -4\sin 4t, 5\sin 5t \rangle$

$r'(\pi/2) = \langle 0, 0, 5 \rangle$

$\|r'(\pi/2)\| = \sqrt{5^2} = \boxed{5}$

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13.4 Homework

1. $r(t) = \langle 4t^2, 9t \rangle$

$r'(t) = \langle 8t, 9 \rangle \qquad \|r'(t)\| = \sqrt{64t^2+81}$

$T(t) = \dfrac{\langle 8t, 9 \rangle}{\sqrt{64t^2+81}}$

$T(1) = \dfrac{\langle 8, 9 \rangle}{\sqrt{64+81}} \qquad T(1) = \langle 8/\sqrt{145}, 9/\sqrt{145} \rangle$

5. $r(t) = \langle \cos\pi t, \sin\pi t, t \rangle$

$r'(t) = \langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle \quad \|r'(t)\| = \sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1}$

$T(t) = \dfrac{\langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle}{\sqrt{(-\pi\sin\pi t)^2 + (\pi\cos\pi t)^2 + 1}}$

$T(1) = \left\langle 0, \dfrac{-\pi}{\sqrt{\pi^2+1}}, \dfrac{1}{\sqrt{\pi^2+1}} \right\rangle$

7. $r(t) = \langle 1, e^t, t \rangle \qquad r'(t) = \langle 0, e^t, 1 \rangle$

$r''(t) = \langle 0, e^t, 0 \rangle$

$r'(t) \times r''(t) = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & e^t & 1 \\ 0 & e^t & 0 \end{bmatrix}$

$\hat{i}[0 - e^t] - \hat{j}[0] + \hat{k}[0]$

$\langle -e^t, 0, 0 \rangle$

$\|r'(t) \times r''(t)\| = \sqrt{(-e^t)^2} = \sqrt{e^{2t}} = e^t$

$|r'(t)| = \sqrt{e^{2t}+1}$

$k(t) = \dfrac{e^t}{(\sqrt{e^{2t}+1})^3}$

11. $r(t) = \langle 1/t,\ 1/t^2,\ t^2 \rangle \quad t = -1$

$r'(t) = \langle -1/t^2, -1/2t^3, 2t \rangle \qquad r''(t) = \langle -1/2t^3, 1/6t^4, 2 \rangle$

$r'(-1) = \langle -1, 1/2, -2 \rangle \qquad r''(-1) = \langle 1/2, 1/6, 2 \rangle$

$r'(-1) \times r''(-1) = \begin{matrix} \hat{i} & \hat{j} & k \\ -1 & 1/2 & -2 \\ 1/2 & 1/6 & 2 \end{matrix}$

$\hat{i}[1 + 1/3] - \hat{j}[-1+1] + \hat{k}[-1/6 - 1/4]$

$4/3\hat{i} + (-1/6 - 1/4)\hat{k} \quad = 4/3\hat{i} - 10/6\hat{k}$

$\|r'(-1) \times r''(-1)\| = \sqrt{(4/3)^2 + (-10/6)^2} = \sqrt{16/9 + 100/36}$

$\|r'(-1)\| = \sqrt{1 + 1/4 + 4} = 4/4 + 1/4 + 16/4 = \sqrt{21/4}$

$K(1) = \dfrac{\sqrt{16/9 + 100/36}}{\left(\sqrt{21/4}\right)^3}$

17. $y = t^4, \quad t = 2$

$k(x) = \dfrac{|f''(x)|}{(1 + f'(x)^2)^{3/2}} \qquad y' = 4t^3 \quad y'' = 12t^2$

$y''(2) = 48$

$\boxed{K(2) = \dfrac{48}{(1025)^{3/2}}} \qquad y'(2) = 32$

21. $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$

$r'(t) = \langle 1 - \operatorname{sech}^2 t, -\operatorname{sech} t \tanh t \rangle = \langle \tanh^2 t, -\operatorname{sech} t \tanh t \rangle$

$r''(t) = \langle 2\tanh t \operatorname{sech}^2 t, \operatorname{sech} t(1 - 2\operatorname{sech}^2 t) \rangle$

$r'(t) \times r''(t) = \begin{matrix} \hat{i} & \hat{j} & k \\ \tanh^2 t & -\operatorname{sech} t^2 \tanh^2 & 0 \\ 2\tanh t \operatorname{sech}^2 t & \operatorname{sech} t(1 - 2\operatorname{sech}^2 t) & 0 \end{matrix}$

$\hat{i}[0] - k[\tanh^2 t \cdot \operatorname{sech} t(1 - 2\operatorname{sech}^2 t) - (-\operatorname{sech}^2 \tanh^2)($ [illegible] $)$

13.5 Homework

3. $r(t) = \langle t^3, 1-t, 4t^2 \rangle \quad t=1$

$r'(t) = v(t) = \langle 3t^2, -1, 8t \rangle$

$r''(t) = a(t) = \langle 6t, 0, 8 \rangle$

$v(1) = \langle 3, -1, 8 \rangle$

$a(1) = \langle 6, 0, 8 \rangle$

5. $r(\theta) = \langle \sin\theta, \cos\theta, \cos 3\theta \rangle \quad \theta = \pi/3$

$r'(\theta) = v(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$r''(\theta) = a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$v(\pi/3) = \langle 1/2, -\sqrt{3}/2, 0 \rangle$

$a(\pi/3) = \langle \sqrt{3}/2, -1/2, 9 \rangle$

15. $a(t) = \langle t, 4 \rangle = t\hat{\imath} + 4\hat{\jmath}$

$v(t) = \int t\hat{\imath} + 4\hat{\jmath}\, dt \quad \frac{t^2}{2}\hat{\imath} + 4t\hat{\jmath} + C$

$v(0) = C$

$C = 3\hat{\imath} - 2\hat{\jmath}$

$v(t) = \left(\frac{t^2}{2} + 3\right)\hat{\imath} + (4t-2)\hat{\jmath}$

$r(t) = \int \left(\frac{t^2}{2} + 3\right)\hat{\imath} + (4t-2)\hat{\jmath}\, dt$

$= \left(\frac{t^3}{6} + 3t\right)\hat{\imath} + (2t^2 - 2t)\hat{\jmath} + C$

$r(0) = C = 0$

$r(t) = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

17. $a(t) = t\hat{k}$

$v(t) = \int t\hat{k}\,dt = \left(\frac{t^2}{2}\right)\hat{k} + C$

$v(0) = C = \hat{\imath}$

$v(t) = \hat{\imath} + \left(\frac{t^2}{2}\right)\hat{k} = \langle 1, 0, t^2/2 \rangle$

$r(t) = \int \hat{\imath} + \left(\frac{t^2}{2}\right)\hat{k}\,dt = t\hat{\imath} + \left(\frac{t^3}{6}\right)\hat{k} + C$

$r(0) = C = \hat{\jmath}$

$r(t) = t\hat{\imath} + \hat{\jmath} + \left(\frac{t^3}{6}\right)\hat{k} = \langle t, 1, t^3/6 \rangle$

31. Not sure how to do this, but I saw online you take the dot product of a and v.

$a \cdot v = 24 + 20 - 60 < 0$ so it is slowing down

33. $r(t) = \langle t, \cos t, \sin t \rangle$

$r'(t) = \langle 1, -\sin t, \cos t \rangle = v(t)$

$r''(t) = \langle 0, -\cos t, -\sin t \rangle = a(t)$

$a_T = a \cdot \vec{T}(t)$ $\sqrt{t^2+1}$

$T(t) = \frac{r'(t)}{||r(t)||} = \frac{\langle 1, -\sin t, \cos t \rangle}{\sqrt{t^2+1}}$

$T(t) = \left\langle \frac{1}{\sqrt{t^2+1}}, \frac{-\sin t}{\sqrt{t^2+1}}, \frac{\cos t}{\sqrt{t^2+1}} \right.$

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14.1 Homework

1. $f(2,2) = 2 + 2(2)^3 = 18$

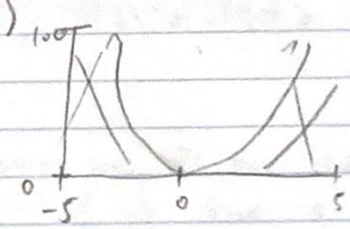
   $f(-1,4) = -1 + 4(-1)^3 = -5$

3. $h(3,8,2) = (3)(8)(2)^{-2} = (3)(8)\frac{1}{2^2} = \frac{24}{4} = 6$

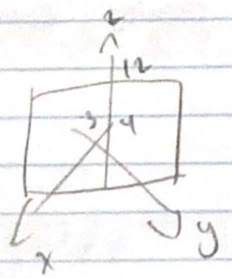
   $h(3,-2,-6) = (3)(-2)\frac{1}{36} = \frac{-6}{36} = \frac{-1}{6}$

7. $f(x,y) = \ln(4x^2 - y)$

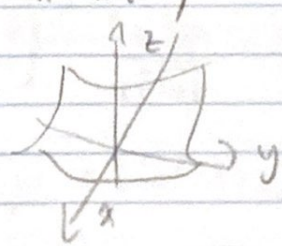
   $y < 4x^2$



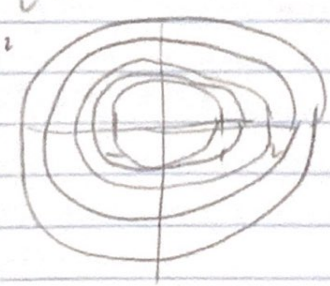
21. $3x+4y=12$



23. Parabola for x and y



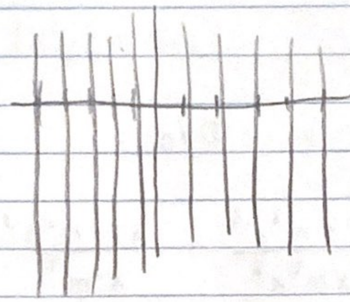
33. $f(x,y) = x^2 + 4y^2$



35. $f(x,y) = x^2$

x^2 can be 1, 2, 3, 4, 5

x can also be negative



I didn't really understand this section.

14.2 Homework

9. $7-2(3) = 7-6 = 1$

11. $e^{9-7} = e^2$

15. First, plugging $(0,0)$ gets vs $0/0$.

$y = mx \qquad \frac{x^3+(mx)^3}{x(mx)^2} = \frac{x^3+m^3x^3}{xm^2x^2} = \frac{x^3+m^3x^3}{x^3m^2}$

$= \frac{\cancel{x^3}(1+m^3)}{\cancel{x^3}m^2} = \frac{(1+m^3)}{m^2}$. $\therefore$ It depends on m, so the limit does not exist

21. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$ Plug. in $(0,0)$: $\frac{0}{0}$

$y = cx \qquad \frac{x \cdot cx}{3x^2+2(cx)^2} = \frac{cx^2}{3x^2+2c^2x^2} = \frac{cx^2}{3x^2+2x^2c^2}$

$= \frac{c\cancel{x^2}}{\cancel{x^2}(3+2c^2)} = \frac{c}{3+2c^2}$

Limit depends on c, so DNE

23. Approach on x-axis:

$\lim_{x\to 0} \frac{x}{x^2} = \frac{1}{x}$ DNE

27. $\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \frac{(-2)^4\cos(\pi)}{e^{-1}} = \frac{16\cdot -1}{e^{-1}}$

$= \boxed{-16e}$

31. $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{9+16}} = \frac{1}{\sqrt{25}} = \boxed{\frac{1}{5}}$

35. $\lim_{(x,y)\to(-3,-2)} (x^2y^3 + 4xy) = (-3)^2(-2)^3 + 4(-3)(-2)$

$= (9)(-8) + 4(6)$

$= -72 + 24 = \boxed{-48}$