13.3 HW

3) $r'(t) = \langle 2, \frac{1}{t}, 2t \rangle$ $\quad \|r'(t)\| = \sqrt{2^2 + (\frac{1}{t})^2 + (2t)^2} = \frac{2t^2+1}{t}$

$s = \int_1^4 \frac{2t^2+1}{t}\,dt = (t^2 + \ln|t|)\Big|_1^4$ $\quad \boxed{s = 15 + \ln 4}$

9) $r'(t) = \langle 2t, 4t, 3t^2 \rangle$, $a = 0$ $\quad \int_a^t$

$\|r'(t)\| = t\sqrt{20 + 9t^2}$ $\quad \int_0^t t\sqrt{20+9t^2} = \boxed{\frac{1}{27}\left[(20+9t^2)^{3/2} - 40\sqrt{5}\right]}$

11) $r(t) = (2t+3, 4t-3, 5-t)$, $t = 4$

$v(t) = (2, 4, -1)$ $\quad |v| = \sqrt{4+16+1} = \sqrt{21}$ $\quad \boxed{v(4) = \sqrt{21}}$

13) $r(t) = \langle t, \ln t, (\ln t)^2 \rangle$, $t = 1$

$v(t) = \langle 1, \frac{1}{t}, \frac{2}{t} \rangle$ $\quad |v| = \sqrt{1 + \frac{1}{t^2} + \frac{4}{t^2}} = \sqrt{\frac{t^2+5}{t^2}}$, $t = 1$: $\sqrt{\frac{6}{1}} = \boxed{\sqrt{6}}$

15) $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle$, $t = \pi/2$

$v(t) = \langle 3\cos 3t, -4\sin 4t, 5\sin 5t \rangle$, $t = \pi/2$

$|v(t)| = \sqrt{9\cos^2 3t + 16\sin^2 4t + 25\sin^2 5t}$ $\quad v(\pi/2) = \sqrt{9(0) + 16(0) + 25(1)} = \boxed{5}$

13.4 HW

1. $r'(t) = \langle 8t, 9 \rangle$ $T(t) = \frac{\langle 8t, 9\rangle}{\sqrt{64t^2+81}}$ $T(1) = \langle \frac{8}{\sqrt{145}}, \frac{9}{\sqrt{145}} \rangle$

5. $r'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$, $T(t) = \frac{1}{\sqrt{\pi^2+1}} \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$ $T(1) = \langle 0, \frac{-\pi}{\sqrt{\pi^2+1}}, \frac{1}{\sqrt{\pi^2+1}} \rangle$

7. $r(t) = \langle 1, e^t, t \rangle$ $r'(t) = \langle 0, e^t, 1 \rangle$ $\kappa(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$

$r''(t) = \langle 0, e^t, 0 \rangle$

$\kappa(t) = \frac{e^t}{\left(\sqrt{0^2+(e^t)^2+(1)}\right)^3} = \boxed{\frac{e^t}{(e^{2t}+1)^{3/2}}}$

11. $r(t) = \langle 1/t, 1/t^2, t^2 \rangle$, $t = -1$ $\kappa(t) = \boxed{\frac{2\sqrt{74}}{27}}$

17. $y = t^4$, $t = 2$ $y' = 4t^3$ $\kappa(t) = \frac{12t^2}{(1+16t^6)^{3/2}}$ $\boxed{\kappa(t) = \frac{48}{(1025)^{3/2}}}$

$y'' = 12t^2$

21. $r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$ $\kappa(t) = \operatorname{sech}(t)$

$r'(t) = \langle 1 - \operatorname{sech}^2 t, -\operatorname{sech} t \tanh t \rangle$

$r''(t) = \langle 2 \tanh t \operatorname{sech}^2 t, \operatorname{sech} t (1 - 2\operatorname{sech}^2 t) \rangle$

$\kappa(t) = \frac{\tanh^2 t \operatorname{sech} t}{\left(\sqrt{\tanh^4 t + \operatorname{sech}^2 t \tanh^2 t}\right)^3}$ $\kappa(t) = \frac{\tanh^2 t \operatorname{sech} t}{(\tanh t)^3} = \boxed{\operatorname{csch} t}$

13.5 HW

3. $v(t) = \langle 3t^2, -1, 8t\rangle$ $\quad v(1) = \langle 3, -1, 8\rangle$
$a(t) = \langle 6t, 0, 8\rangle$ $\quad a(1) = \langle 6, 0, 8\rangle$

5. $v(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta\rangle$ $\quad v(\pi/3) = \langle 1/2, -\sqrt{3}/2, 0\rangle$

$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta\rangle$ $\quad a(\pi/3) = \langle -\sqrt{3}/2, -1/2, 9\rangle$

15. $\int \langle t, 4\rangle = v(t) = \langle \frac{t^2}{2}, 4t\rangle + c$ $\quad v(t) = \langle \frac{t^2}{2} + 3, 4t - 2\rangle$

$r(t) = \int v(t) = \boxed{\langle t^3/6 + 3t, 2t^2 - 2t\rangle}$

17. $a(t) = tk$ $\quad v(0) = i$ $\quad r(0) = j$

$v(t) = \frac{t^2}{2}k + v_0$ $\quad v_0 = i$ $\quad \boxed{v(t) = t^2/2\, k + i}$

$\boxed{r(t) = ti + j + t^3/6\, k}$

31. $(\|v\|^2)' = (v \cdot v)' = 2v' \cdot v = 2av = 2\langle 2, 1, -3\rangle \cdot \langle 12, 20, 20\rangle$
$= 2(24 + 20 - 60)$
$= -32$

decreasing

33. $r(t) = \langle t, \cos t, \sin t\rangle$
$a_T = a \cdot T = \frac{a \cdot u}{\|u\|}$, $a_N = a \cdot N = \sqrt{\|a\|^2 - |a_T|^2}$

$v(t) = r'(t) = \langle 1, -\sin(t), \cos(t)\rangle$
$a(t) = v'(t) = \langle 0, -\cos(t), -\sin(t)\rangle$
$a \cdot v = \langle 0, -\cos(t), -\sin(t)\rangle \cdot \langle 1, -\sin(t), \cos(t)\rangle$
$a \cdot v = 0 + \cancel{\sin t\cos t} - \cancel{\sin t \cos t} = 0$
$\|v\| = \sqrt{1^2 + \sin^2(t) + \cos^2(t)} = \sqrt{2}$ $\quad a_T = 0$
$\|a\|^2 = (\sqrt{0 + \cos^2 t + \sin^2 t})^2 = 1$
$a_N = \sqrt{1 - 0} = 1$
$a_N = 1$

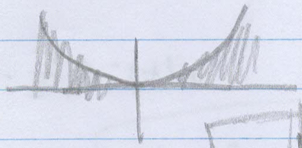
14.1 HW

1. $f(x,y) = x + yx^3$ $(2,2),(-1,4)$

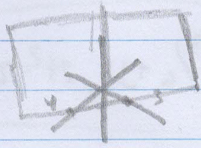
$f(2,2) = 18$
$f(-1,4) = -5$

3. $h(x,y,z) = xyz^{-2}$ $(3,8,2),(3,-2,-6)$ $h(3,8,2) = 6$ $h(3,-2,-6) = -1/6$

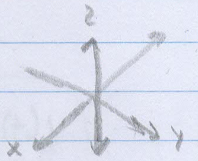
7. $f(x,y) = \ln(4x^2 - y)$



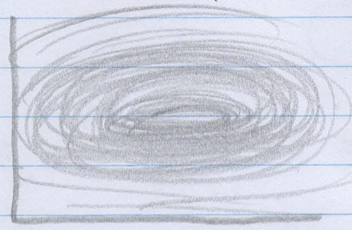
21. $f(x,y) = 12 - 3x - 4y$



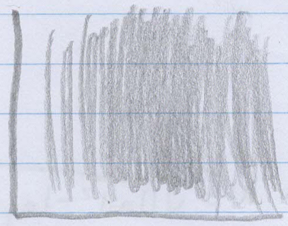
23. $f(x,y) = x^2 + 4y^2$



33. $f(x,y) = x^2 + 4y^2$



35 $f(x,y) = x^2$



14.2 HW

$f(x,y)=3 \quad g(x,y)=7$

9. $\lim_{(x,y)\to(2,5)} (g(x,y)-2f(x,y)) = 7-2(3)=1$

11. $e^{f(x,y)^2-g(x,y)} = e^{9-2} = e^7$

15. $f(x,y) = \frac{x^3+y^3}{xy^2} \quad \lim_{x\to 0} \frac{x^3+m^3x^3}{m^2x^3} = \frac{1+m^3}{m^2}$, DNE

21. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} \quad \lim_{x\to 0} \frac{mx^2}{3x^2+2m^2x^2} = \frac{m}{3+2m^2}$ DNE

23. $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+z}{x^2+y^2+z^2} \quad \lim_{x\to 0} \frac{x}{x^2} \quad \lim_{x\to 0} \frac{1}{x}$ DNE

27. $\lim_{(z,w)\to(-2,1)} \frac{z^4\cos(\pi w)}{e^{z+w}} = \boxed{-16e}$

31. $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{9+16}} = \boxed{\frac{1}{5}}$

35. $\lim_{(x,y)\to(-3,2)} (x^2y^3+4xy) = 9\cdot 8 + 24 = 72+24 = \boxed{96}$