9-27 Homework 13.3-13.5, ~~14.1~~, 14.2

13.3

3. $L = \int_1^4 |r'(t)|\, dt$

$L = \int_1^4 |2, 1/t, 2t|$

$|r'(t)| = \sqrt{4 + 1/t^2 + 4t^2}$

$= \sqrt{\frac{4t^2 + 1 + 4t^4}{t^2}}$

$= \sqrt{\frac{(2t^2+1)^2}{t^2}} = \frac{2t^2+1}{t}$

$L = \int_1^4 \frac{2t^2+1}{t}\, dt$

$= \int_1^4 2t + 1/t$

$= t^2 + \ln(t)\Big|_1^4$

$= 16 + \ln(4) - 1 - \ln(1)$

$= 15 + \ln(4)$

9.

$$s(t) = r(a) + \int_a^t |r(u)|\,du$$

$$s(t) = 0 + \int_0^t |\langle 2t, 4t, 3t^2 \rangle|$$

$$|r'(t)| = \sqrt{4t^2 + 16t^2 + 9t^4}$$

$$|r'(t)| = \sqrt{20t^2 + 9t^4}$$

$$= t\sqrt{20 + 9t^2}$$

$$s(t) = \frac{1}{27}(9x^2 + 20)^{3/2} \Big|_0^t$$

$$\boxed{s(t) \quad \frac{1}{27}\left([9x^2 + 20]^{3/2} - 20^{3/2}\right)}$$

11. $s(t) = |v(t)| = |r'(t)|$

$r'(t) = \langle 2, 4, -1 \rangle$

$|r'(t)| = \sqrt{21}$

$\boxed{s(t) = \sqrt{21}}$

13. $r(t) = \langle t, \ln t, (\ln(t))^2 \rangle$

$r'(t) = \langle 1, 1/t, 2\ln(t)/t \rangle$

$|r'(t)| = \sqrt{1 + 1/t^2 + (2\ln(t)/t)^2}$

$s(1) = \sqrt{1+1/1} + 0$

$\boxed{s(2) = \sqrt{2}}$

15. $r'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$|r'(t)| = \sqrt{9\cos^2 3t + 16\sin^2 4t + 25\sin^2 5t}$

$s(\pi/2) = \sqrt{9(0) + 16(0) + 25(1)}$

$\boxed{s(\pi/2) = 5}$

13.4

1. $r(t) = \langle 8t, 9 \rangle$

$|r'(t)| = \sqrt{64t^2 + 81}$ $T(1) = \langle 8/\sqrt{145}, 0, 9/\sqrt{145} \rangle$

$T(t) = \dfrac{\langle 8t, 9 \rangle}{\sqrt{64t^2+1}}$

5. $r'(t) = \langle -12\sin 12t, 12\cos 12t, 1 \rangle$

$|r'(t)| = \sqrt{12^2 \sin^2 12t + 12^2 \cos^2 12t + 1}$

$= \sqrt{12^2 + 1}$

$T(t) = \dfrac{\langle -12\sin 12t, 12\cos 12t, 1 \rangle}{\sqrt{12^2+1}}$

$T(1) = \langle 0, -12/\sqrt{12^2+1}, 1/\sqrt{12^2+1} \rangle$

7.

$$K = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$r'(t) = \langle 0, e^t, 1 \rangle \quad r''(t) = \langle 0, e^t, 0 \rangle$

$|r'(t)| = \sqrt{e^{2t} + 1}$

$r'(t) \times r''(t) = \langle -e^t, 0, 0 \rangle$

$$\boxed{K = \frac{e^t}{(e^{2t}+1)^{3/2}}}$$

11.

$$K = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$r'(t) = \langle -1/t^2, -2/t^3, 2t \rangle$

$r''(t) = \langle 2/t^3, 6/t^4, 2 \rangle$

$|r'(t)| = \sqrt{1/t^4 + 4/t^6 + 4t^2}$

$|r'(t) \times r''(t)| = \sqrt{\frac{256t^6 + 36t^8 + 4}{t^{12}}}$

$$K = \frac{\sqrt{\frac{256t^6 + 36t^8 + 4}{t^{12}}}}{(1/t^4 + 4/t^6 + 4t^2)^{3/2}}$$

$\boxed{K(-1) = .6372}$

sin cos 1
tan 1 sec

17. $y = t^4 \quad t = 2$

$\langle 0, -t^4, 0 \rangle \qquad f'(x) = 4t^3$

$f''(x) = 12x^2$

$|\kappa(x)| = \dfrac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$

$K(x) = 48 \,|\, 32816.0117$

$x = 0014627$

21.

$K = \dfrac{T'(t)}{r'(t)} \qquad r(t) = \langle t - \tanh t, \operatorname{sech} t \rangle$

$T(t) = \dfrac{r'(t)}{|r'(t)|} = \dfrac{1}{\tanh t \sqrt{\tanh^2 t + \operatorname{sech}^2 t}} \langle \tanh^2(t), -\tanh x \operatorname{sech} y \rangle$

13.5

3. $v(t) = \langle 3t^2, -1, 8t \rangle$

$v(1) = \langle 3, -1, 8 \rangle \quad |v(1)| = \sqrt{74}$

$a(t) = \langle 6t, 0, 8 \rangle$

$a(1) = \langle 6, 0, 8 \rangle$

5. $v(\theta) = \langle \cos\theta, -\sin\theta, -3\sin 3\theta \rangle$

$v(\pi/3) = \langle 1/2, -\sqrt{3}/2, 0 \rangle \quad |v(\pi/3)| = 1$

$a(\theta) = \langle -\sin\theta, -\cos\theta, -9\cos 3\theta \rangle$

$a(\pi/3) = \langle -\sqrt{3}/2, -1/2, 9 \rangle$

15.

$$v(t) = v(0) + \int \langle t, 4 \rangle \, dt$$

$$v(t) = \langle t^2/2 + 3, 4t - 2 \rangle$$

$$r(t) = r(0) + \int v(t) \, dt$$

$$r(t) = \langle t^3/6 + 3t, 2t^2 - 2t \rangle$$

17.

$$v(t) = i + \int t k \, dt$$

$$v(t) = i + t^2/2 k$$

$$r(t) = j + \int v(t) \, dt$$

$$r(t) = ti + j + t^3/6 k$$

31. $a \cdot v = -16$

$-16 < 0$ Particle is slowing down

33.

$$a_N = \frac{|r' \times r''|}{|r'|} \qquad r' = \langle 1, -\sin t, \cos t \rangle$$

$$r'' = \langle 0, -\cos t, -\sin t \rangle$$

$$a_T = \frac{r' \cdot r''}{|r'|}$$

$$a_N = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$a_T = 0$$

$|r'| = \sqrt{1 + \sin^2 t + \cos^2 t}$

$= \sqrt{2}$

14.2

9. $7-6=1$

11. $e^{9-7} = e^2$

15.

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

$y = mx$

$$f(x,y) = \frac{x^3 + m^3x^3}{x\,m^2x^2}$$

$$\lim_{x\to 0} \frac{x^3(1+m^3)}{x^3\,m^2}$$

$$\lim_{x\to 0} \frac{1+m^3}{m^2} = \text{Dependent on } M; \text{ DNE}$$

21. $y = mx$

$$\lim_{x \to 0} \frac{mx^2}{3x^2 + 2x^2m^2} = \lim_{x \to 0} \frac{m}{3 + 2m^2}$$

DNE

23. $$\lim_{(x,y,z) \to (0,0,0)} \frac{x+y+z}{x^2+y^2+z^2}$$

$x=0$, $y=0$: $$\lim_{z \to 0} \frac{z}{z^2} = \frac{1}{z}$$ DNE

27. $$\lim_{z,w \to -2,1} \frac{2^7 \cos 2w}{e^{z+w}}$$

$$= \frac{16 \cos 2}{e^1} = \frac{16}{e}$$

37. $\lim_{x,y \to 3,4} \frac{1}{\sqrt{x^2+y^2}}$

$= \lim_{x,y \to 3,4} \frac{1}{\sqrt{25}} = \boxed{\frac{1}{5}}$

38. $\lim_{x,y \to -3,-2} (x^2 y^3 + 4xy)$

$= \lim_{x,y \to -3,-2} ((9)(-8) + 4(-3)(-2))$

$\boxed{= -48}$