No. Due. 9.20

Date

Qu Zixin (sammy) #ok to post.

Exercise 12.3

Q1. $(1,2,1)\cdot(4,3,5)$

$= 1\times4+2\times3+1\times5$

$= 4+6+5 = 15$

Q13. $\overset{a}{(1,1,1)} \times \overset{b}{(1,-2,-2)}$

$C = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -2 & -2 \end{vmatrix}$

$= (-2+2)i - (-2-1)j + (-2-1)k$

$= 3j - 3k$

$C = (0,3,-3)$

$\because a \cdot c = (1,1,1)\cdot(0,3,-3)$

$= 0+3-3 = 0$

$\therefore$ this is orthogonal.

Q21. $i+j,\ j+2k$

$u = i+j = (1,1,0)$

$v = j+2k = (0,1,2)$

$u\cdot v = 0+1+0 = 1$

$\|u\| = \sqrt{1^2+1^2} = \sqrt{2}$

$\|v\| = \sqrt{1^2+2^2} = \sqrt{5}$

$\therefore \cos\theta = \frac{1}{\sqrt{2}\times\sqrt{5}}$

$= \frac{1}{\sqrt{10}}$

Q29. Find all the value of b which the vectors are orthogonal.

(a): $(b,3,2),\ (1,b,1)$

$u = (b,3,2)$

$v = (1,b,1)$

$u\times v = \begin{vmatrix} i & j & k \\ b & 3 & 2 \\ 1 & b & 1 \end{vmatrix}$

$= (3-2b)i - (b-2)j + (b^2-3)k$

$w = (3-2b, -b+2, b^2-3)$

$\because u \times w = 0$ (it is orthogonal)

$\therefore (3-2b, -b+2, b^2-3)\cdot(b,3,2)$

$= (3b-2b^2) + (-b+2)3 + (2b^2-6)$

$= 3b - 2b^2 - 3b + 6 + 2b^2 - 6$

$= 0$

$\therefore u \times w = 0$

it means two vectors are orthogonal.

$\therefore u\cdot v = b+3b+2$

$0 = 4b+2$

$\therefore b = -\frac{1}{2}$

the value of b is $-\frac{1}{2}$.

(b) $(4,-2,7)\ (b^2,b,0)$

$u = (4,-2,7)\quad v = (b^2,b,0)$

$u\cdot v = 4b^2 + (-2b) + 0$

$= 4b^2 - 2b$

$= 2b(2b-1)$

$b = 0$ or $\frac{1}{2}$

KOKUYO

Q31. set a vector as $(a, b, c)$

Q63. Find the length of $\overline{OP}$

$\therefore (a, b, c) \cdot (2, 0, -3)$

$= (2a, 0, -3c)$

$2a + 0 + (-3c) = 0$

$2a = +3c$

$a = \frac{3}{2}c$

$\therefore a, b, c = \frac{3}{2}c, b, c$

we set $c = 0, b = 1$

$\therefore (a, b, c) = (0, 1, 0)$

set $c = 2, b = 2$

$(a, b, c) = (3, 2, 2)$

so the answer should be $(0, 1, 0)$

and $(3, 2, 2)$

Q57. $u = 5i + 7j - 4k$

$v = k$

$\therefore u = (5, 7, -4)$

$v = (0, 0, 1)$

$$\therefore w_1 = \frac{u \cdot v}{\|v\|^2} \cdot v$$

$$= \frac{-4}{1} \cdot (0, 0, 1)$$

$= -4k$

$\therefore w_1 = -4k$

the answer is $-4k$.

# Exercise 12.4

Q1. ~~$\begin{vmatrix} -5 & 1 \\ 4 & 0 \end{vmatrix}$~~

$$\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$$

$= 1\times3 - 2\times4$

$= -5$

Q5. $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

$= 1\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$

$= -3 - 8 + 3 = -8$

Q13. $(i+j)\times k$

$= (1,1,0)\times(0,0,1)$

$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= i - j$

Q21. $(u-2v)\times(u+2v)$

$= u\times u + u\times(2v) - 2v\times u - 2v\times 2v$

$\because v\times v = 0$, $u\times u = 0$

$= 0 + 2(u\times v) - 2(v\times u) - 4(0)$

$= 2(u\times v) + 2(u\times v)$

$= 4(u\times v)$

$= 4(1,1,0)$

$= (4,4,0)$

Q~~25~~27. $u = v\times w$

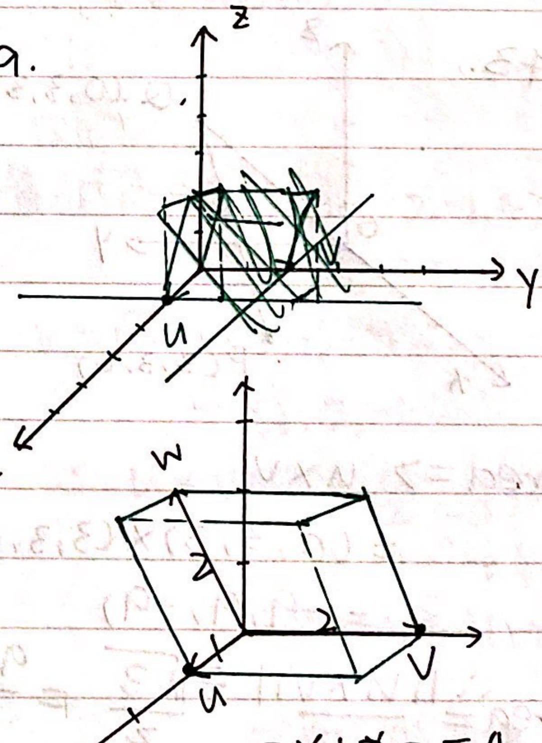
$= (3,0,0)\times(0,1,-1)$

$= (0,3,3)$

~~Q27~~. Q25.

$v\times w = -u$.

Q39.




$2\times1\times2 = 4$

the volume is 4.

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Q41. $u = (1, 0, 3)$

$v = (2, 1, 1)$

Area $\Rightarrow u \times v$

$$= \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix}$$

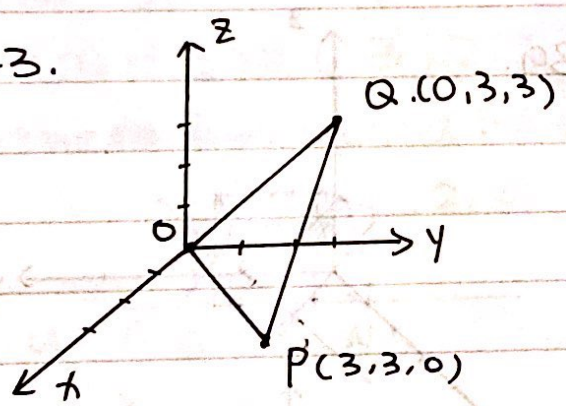
$$= -3i + 5j + k$$

$$\text{area} = ||u \times v||$$

$$= \sqrt{(-3)^2 + 5^2 + 1^2}$$

$$= \sqrt{35}$$

Q43.



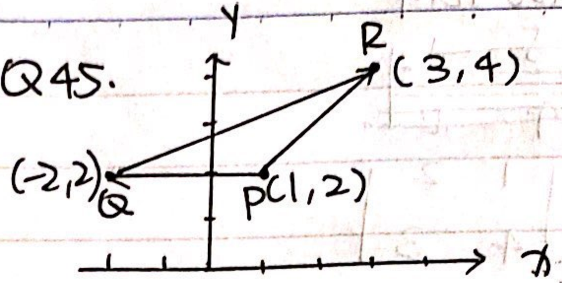

Area $\Rightarrow u \times v$

$$= (0,3,3) \times (3,3,0)$$

$$= (-9, 9, -9)$$

$$\therefore \text{area} = \frac{||u \times v||}{2} = \frac{9\sqrt{3}}{2} = \frac{9\sqrt{3}}{2}$$

Q45.




$\vec{PQ} = u = (-3, 0)$

$\vec{PR} = v = (2, 2)$

$\therefore u \times v = (-3, 0) \times (2, 2)$

$= -6$

$$\text{area} = \frac{||u \times v||}{2} = \frac{\sqrt{(-6)^2}}{2} = \frac{6}{2}$$

$$= 3$$

So the area is equal to 3.

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Qu Zixin (Sammy).

Exercise 12.5

Q1. $n=(1,3,2), (4,-1,1)$

$P_0=(4,-1,1)$

$d=n\cdot\overrightarrow{OP_0}$

$\overrightarrow{OP_0}=P_0-0=(4,-1,1)$

$n\cdot\overrightarrow{OP_0}=(1,3,2)\cdot(4,-1,1)$

$=4-3+2=3$

$\therefore ax+by+cz=d$

$1x+3y+2z=3$

Q5. $n=i \quad (3,1,-9)$

$n=(1,0,0)$

$\therefore \overrightarrow{OP_0}=(3,1,-9)$

$n\cdot\overrightarrow{OP_0}=(1,0,0)\cdot(3,1,-9)$

$=3$

$\therefore x=3$

Q9. pass through the origins means $d=0$

$\therefore ax+by+cz=0$

Q11. $\because$ yz plane means $n=(1,0,0)$ so b is true.

and $ax+by+cz=d$

$1x+0y+0z=d$

$x=d$

$\therefore$ b and d are true.

Q13. $\because 9x-4y-11z=2$

$\therefore n=(9,-4,-11)$

$d=2$

the vector is $(9,-4,-11)$

Q15. $3(x-4)-8(y-1)+11z=0$

$\therefore a(x-x_0)+b(y-y_0)+c(z-z_0)=d$

$a=3,\ b=-8,\ c=11\ \ d=0$

$(3,-8,11)$

Q17.

$u=\overrightarrow{PQ}=(1,1,1)-(2,-1,4)$

$=(-1,2,-3)$

$v=\overrightarrow{PR}=(3,1,-2)-(2,-1,4)$

$=(1,2,-6)$

$\therefore u\times v=\begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix}$

$=i(-12+6)-j(6+3)+k(-2-2)$

$=-6i-9j-4k$

$=(-6,-9,-4)$

pick Q as point.

$\therefore -6\cdot1+(-9)\cdot1+(-4\cdot1)=-19$

$\therefore -6x+(-9y)+(-4z)=-19$

$6x+9y+4z=19$

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Q19. $\vec{PQ} = (0,1,1)-(1,0,0) = (-1,1,1)$

$\vec{PR} = (2,0,1)-(1,0,0) = (1,0,1)$

~~$(-1,1,1)\times(1,0,1) = (-1,0,1)$~~

~~$(-1)\cdot 0 + 0\cdot 1 + 1\cdot 1 = 1$~~

~~$\therefore d = 1$~~

~~$x+z=1$~~

$$(-1,1,1)\times(1,0,1) = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$= i - j(-1-1) + (-1)k$

$= i + 2j - k$

$= (1,+2,-1)$

$\therefore 1\cdot 0 + (+2)\cdot 1 + (-1)\cdot 1 = 1$

$d = 1$

$\therefore x + 2y - z = +1$

Q31. $x+y+z=4$

$a=1,\ b=1,\ c=1\ d=4$

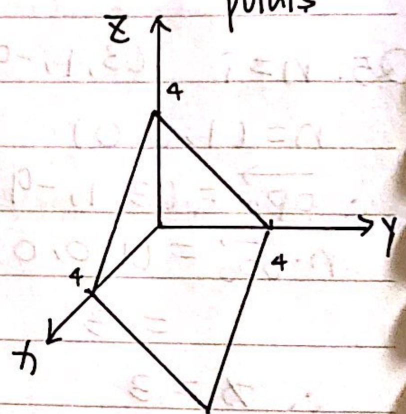
normal vector is $(1,1,1)$

$\because n\cdot\vec{OP_0} = 4 \quad n=(1,1,1)$

$\therefore \vec{OP_0} = P_0 = (x_0, y_0, z_0)$

$(x_0,y_0,z_0)\cdot(1,1,1) = 4$

$\therefore (4,0,0),\ (0,4,0),\ (0,0,4)$ are ~~given~~ points




Q25. normal vector is $i+k = (1,0,1)$

$n = (1,0,1) \quad P_0 = (-2,-3,5)$

$\therefore n\cdot\vec{OP_0} = (1,0,1)\cdot(-2,-3,5)$

$= -2+5 = 3$

$\therefore d = 3$

$\therefore a=1,\ b=0,\ c=1$

$x+z=3$

Q53. $\because$ xz-plane, so $y=1$

~~normal is $(0,1,0)$ vector~~

$ax+by+cz=d$

$ax+cz=d$

$\therefore$ ~~$3x+2z=$~~

$3x+by+2z=5$

b anything

Campus

## Exercise 13.1

Q5. $P=(3,-5,7)$

$v=(3,0,1)$

$r(t) = P + $ ~~[crossed out]~~ $t^* v$

$= (3,-5,7)+(3,0,1)^* t$

$= (3+3t)i - 5j + (7+t)k$

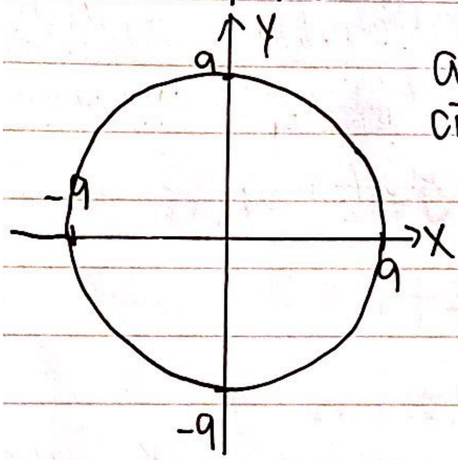
Q17. $r(t)=(9\cos t)i+(9\sin t)j$

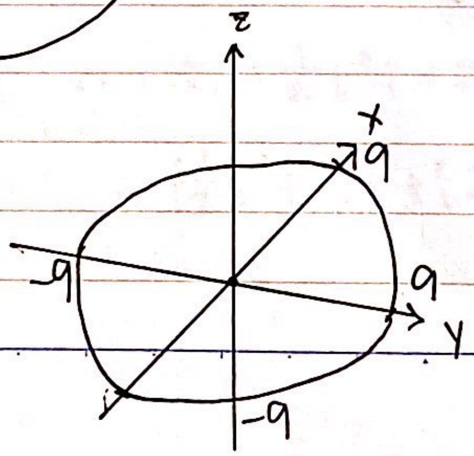
$\therefore x = 9\cos t$

$y = 9\sin t$

| t | x | y |
|---|---|---|
| 0 | 9 | 0 |
| $\frac{\pi}{2}$ | 0 | 9 |
| $\pi$ | -9 | 0 |
| $\frac{3\pi}{2}$ | 0 | -9 |
| $2\pi$ | 9 | 0 |

$\therefore$ radius is 9 and this circle is on xy plane.




## Exercise 13.2

Q3. $\lim_{t\to 0}\left(e^{2t} i + \ln(t+1) j + 4k\right)$

$= \lim_{t\to 0}(e^{2t})i + \lim_{t\to 0}\ln(t+1)j + 4k$

$= 1i + 0j + 4k$

$= i + 4k$

Q5. $\lim_{h\to 0} \frac{r(t+h)-r(t)}{h}$

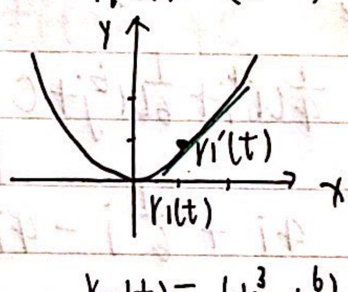
$= r'(t) = (-\frac{1}{t^2}, \cos t, 0)$

Q7. $r(t) = (t, t^2, t^3)$

$= (1, 2t, 3t^2)$

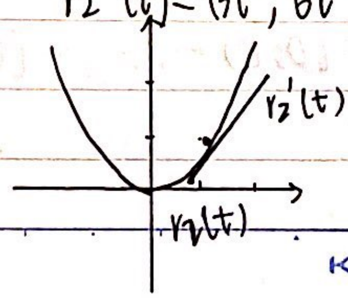
Q15. $r_1(t)$ at $t=1$ $= (1,1)$

$r_1'(t) = (1, 2t) = (1,2)$




$r_2(t) = (t^3, t^6) = (1,1)$

$r_2'(t) = (3t^2, 6t^5) = (3,6)$



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Q31. $r(t) = (1-t^2, 5t, 2t^3)$ $t=2$

$r(2) = (-3, 10, 16)$

$r'(t) = (-2t, 5, 6t^2)$

$r'(2) = (-4, 5, 24)$

$r(2) + r'(2) * t$

$l(t) = (-3, 10, 16) + (-4t, 5t, 24t)$

$l(t) = (-3-4t, 10+5t, 16+24t)$

Q33. $r(s) = 4s^{-1}i - \frac{8}{3}s^{-3}k$, $s=2$

$r(2) = (2, 0, ~~0.5~~ -\frac{1}{3})$

$r'(s) = -4s^{-2}i + 8s^{-4}k$

$r'(2) = -1i ~~325~~ + 0.5k$

$r(2) + r'(2)t$

$= (2, 0, -\frac{1}{3}) + (-1, 0, 0.5)t$

$= (2-t, 0, -\frac{1}{3} + \frac{1}{2}t)$

Q41. $\int_{-2}^{2} (u^3 i + u^5 j) du$

$= \frac{1}{4}u^4 i + \frac{1}{6}u^6 j + C \Big|_{-2}^{2}$

$= 4i + \frac{64}{6}j - 4i - \frac{64}{6}j$

$= 0i + 0j$

$= (0, 0)$

Q49. $r'(t) = t^2 i + 5tj + k$

$r(1) = j + 2k$

$r(t) = \int r'(t)\, dt$

$= \frac{1}{3}t^3 i + \frac{5}{2}t^2 j + tk + C$

$r(1) = \frac{1}{3}i + \frac{5}{2}j + 1k + C$

$C = -\frac{3}{2}j + k - \frac{1}{3}i$

$\therefore r(t) = (\frac{1}{3}t^3 - \frac{1}{3})i + (\frac{5}{2}t^2 - \frac{3}{2})j + (t+1)k$