12.3

1. $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$

$= 1\times4 + 2\times3 + 1\times5$

$= 4+6+5$

$= 15$

13. $\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle$

$= 1\times1 - 1\times2 - 1\times2$

$= 1 - 2 - 2$

$= -1 - 2$ the angle

$= -3 < 0$ is obtuse

21. $i+j = \langle 1,1,0\rangle$

$j+2k = \langle 0,1,2\rangle$

$a\cdot b = ab\cos\theta$

$\cos\theta = \dfrac{1\times0+1\times1+0\times2}{\sqrt{2}\cdot\sqrt{5}} = \dfrac{\sqrt{10}}{10}$

29. $\langle b,3,2\rangle \cdot \langle 1,b,1\rangle$

$= 1\times b + 3\times b + 2\times1$

$= b + 3b + 2$

$= 4b+2$

when the vectors are orthogonal, the dot product need to be 0.

$4b+2 = 0$

$b = -\frac{1}{2}$

31. $\langle x,y,z\rangle \cdot \langle 2,0,-3\rangle$

$= 0$

$2x = 3z$

$x=0$ or $z=0$

$x=3$ or $z=2$

$\langle 0,1,0\rangle$

$\langle 3,2,2\rangle$

57. $u = 5i+7j-4k$

$v = k$

$u = \langle 5,7,-4\rangle$

$v = \langle 0,0,1\rangle$

$\dfrac{u\cdot v}{|v|^2}\cdot v$

$= \dfrac{0+0-4}{(\sqrt{1})^2}\cdot k$

$= \dfrac{-4}{1}\cdot k = -4k$

63. $u\cdot v = 3\times8+5\times2 = 34$

$\text{OP} = \dfrac{u\cdot v}{|B|}$

$= \dfrac{34}{\sqrt{68}}$

$= \sqrt{17}$

12.4

1. $1\times3-4\times2=-5$

5. $1\times(-3-0)-2\times(4-0)+1\times(0+3)$

$=1\times(-3)-2\times4+1\times3$

$=-3-8+3$

$=-8$

13. $(i+j)\times k$

$\langle1,0,0\rangle+\langle0,1,0\rangle$

$=\langle1,1,0\rangle$

$k=\langle0,0,1\rangle$

$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1i-1j+(0-0)k$

$=i-j$

21. $(u-2v)\times(u+2v)$

$=(u-2v)\times u+(u-2v)\times 2v$

$=u\times u-\cancel{2\times u+2\times v}-4v\times v$

$2(v\times u)+2(u\times v)$

$=\cancel{8u\times u-4\times v\times v}$

$2(u\times v)+2(u\times v)$

$=4(u\times v)$

$=4\langle1,1,0\rangle$

$=\langle4,4,0\rangle$

25.

$-u$ is equal to $v\times w$

27.

$v\times w=\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$=(0-0)i-(-3-0)j+(3-0)k$

$=0i+3j+3k$

$=\langle0,3,3\rangle$

39. $(u\times v)\cdot w$

$u\times v=\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix}$

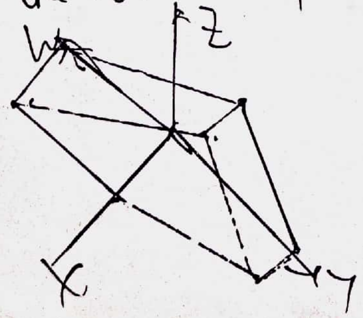
$=i(0-0)-j(0-0)+k(2-0)$

$=\langle0,0,2\rangle$

$\langle0,0,2\rangle\cdot\langle1,1,2\rangle$

$=0+0+4=4$

the volume is 4



41. $u\times v$

$u\times v=\begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix}$

$=i(0-3)-j(1-6)+k(1-0)$

$=-3i+5j+k$

$=\langle-3,5,1\rangle$

$|u\times v|=\sqrt{9+25+1}$

$=\sqrt{35}$

43. $P\times Q$

~~$P\times Q=\begin{vmatrix} i & j & k \\ 1 & 1 & 5 \end{vmatrix}$~~

$P\times Q=\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix}$

$=i(9-0)-j(9-0)+k(9-0)$

$=9i-9j+9k$

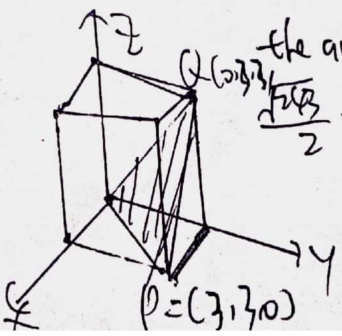
$=\langle9,-9,9\rangle$

$|P\times Q|=\sqrt{81+81+81}$

$=\sqrt{243}$

the area of triangle is

$\frac{\sqrt{243}}{2}=\frac{9\sqrt{3}}{2}$



45.

$$\vec{a} = \langle 3,4\rangle - \langle 1,2\rangle$$

$$= \langle 2,2\rangle$$

$$\vec{b} = \langle -2,2\rangle - \langle 1,2\rangle$$

$$= \langle -3,0\rangle$$

$$a \times b = \begin{vmatrix} 2 & 2 \\ -3 & 0 \end{vmatrix}$$

$$= 0+6$$

$$= 6$$

$$\text{Area} = \frac{6}{2} = 3$$

12 · 5

1. $x + 3y + 2z = d$

$4 - 3 + 2 = d$

$d = 3$

the equation of the plane is

$x + 3y + 2z = 3$

5. $n = i$

$= \langle 1, 0, 0 \rangle$

$ax + by + cz = d$

$1 \times x + 0y + 0z = d$

$x = d$

$3 \times 1 + 1 \times 0 + (-9 \times 0) = d$

$d = 3 \quad \therefore x = 3$

9. $x = 0$

11.

~~(a)~~ (b) (d)

13. $9x - 4y - 11z = 2$

$\langle 9, -4, -11 \rangle$

15. $3(x-4) - 8(y-1) + 11z = 0$

$3x - 12 - 8y + 8 + 11z = 0$

$3x - 8y + 11z = 4$

$\langle 3, -8, 11 \rangle$

17.

$u = \overline{PQ} = Q - P = \langle -1, 2, -3 \rangle$

$v = \overline{PR} = R - P = \langle 1, 2, -6 \rangle$

$$u \times v = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix}$$

$= i(-12+6) - j(6+3) + k(-2-2)$

$= -6i - 9j - 4k$

$= \langle -6, -9, -4 \rangle$

pick $(2, -1, 4)$ as the point

$-6(x-2) - 9(y+1) - 4(z-4) = 0$

$-6x + 12 - 9y - 9 - 4z + 16 = 0$

$-6x - 9y - 4z = -19$

$6x + 9y + 4z = 19$

19. $u = \overline{PQ} = Q - P = \langle -1, 1, 1 \rangle$

$v = \overline{PR} = R - P = \langle 1, 0, 1 \rangle$

$$u \times v = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$= i(1-0) - j(-1-1) + k(0-1)$

$= i + 2j - k = \langle 1, 2, -1 \rangle$

pick $(1, 0, 0)$ as the point

$1(x-1) + 2(y-0) - 1(z-0) = 0$

$x - 1 + 2y - 0 - z + 0 = 0$

$x + 2y - z = 1$

25. normal vector i+k

$= \langle 1, 0, 1 \rangle$

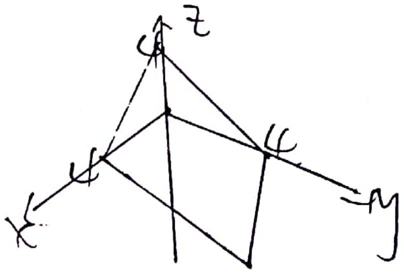
$(-2, -3, 5)$ is the point

$1(x+2) + 0(y+3) + 1(z-5) = 0$

$x+2 + 0 + z - 5 = 0$

$x + z = 3.$

31.



53.

$(3\lambda)x + by + (2\lambda)z = 5\lambda;\ \lambda \neq 0.$

13.1

$P = (3, -5, 7)$

5. $\langle 3, -5, 7 \rangle + t\langle 3, 0, 1 \rangle$

$\langle 3, -5, 7 \rangle + \langle 3t, 0, t \rangle$

$\langle 3+3t, -5, 7+t \rangle$

$r(t) = (3+3t)i - 5j + (7+t)k$

17. $r(t) = (9\cos t)i + (9\sin t)j$

$x = 9\cos t$

$y = 9\sin t$

$x = 9 \quad y = 0$

$t = \frac{\pi}{2} \quad x = 0 \quad y = 9$

$t = \pi \quad x = -9 \quad y = 0$

$t = \frac{3}{2}\pi \quad x = 0 \quad y = -9$

$t = 2\pi \quad x = 9 \quad y = 0$

$(x, y) = (0, 9);\ (9, 0);\ (0, -9);\ (-9, 0)$

the radius is 9.

the center is (0,0) and is in xy plane.

13.2

3. $\lim_{t\to 0} e^{2t} i + \ln(t+1) j + 4k$

$= e^{2\times 0} i + \ln(0+1) j + 4k$

$= e^{0} i + \ln 1 j + 4k$

$= i + 0j + 4k$

$= i + 4k$

7. $r(t) = \langle t, t^2, t^3 \rangle$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

31. $r(t) = \langle 1-t^2, 5t, 2t^3 \rangle$ $t=2$

$r(2) = \langle -3, 10, 16 \rangle$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

$r'(2) = \langle -4, 5, 24 \rangle$

$x = -3 + (-4)t$

$y = 10 + (5)t$

$z = 16 + 24t$

$l(t) = \langle -3-4t, 10+5t, 16+24t \rangle$

33. $r(s) = 4s^{-1} i - \frac{8}{3} s^{-3} k$ $s=2$

$r'(s) = \langle -4s^{-2}, 0, 8s^{-4} \rangle$

$r(2) = \langle 2, 0, -\frac{1}{3} \rangle$

$r'(2) = \langle -1, 0, \frac{1}{2} \rangle$

$x = 2 - 1t$

$y = 0$

$z = -\frac{1}{3} + \frac{1}{2}t$

$l(t) = \langle 2-1t, 0, -\frac{1}{3} + \frac{1}{2}t \rangle$

5. $r(t) = \langle t^{-1}, \sin t, 4 \rangle$

$r'(t) = \langle -t^{-2}, \cos t, 0 \rangle$

$r'(t) = \lim_{h\to 0} \frac{r(t+h) - r(t)}{h}$

$\lim_{h\to 0} \frac{r(t+h) - r(t)}{h} = \langle -t^{-2}, \cos t, 0 \rangle$

15. $r_1(t) = \langle t, t^2 \rangle$ $t = 1$

$= \langle 1, 1 \rangle$

$r_1'(t) = \langle 1, 2t \rangle = \langle 1, 2 \rangle$

$\langle 1, 1 \rangle + t\langle 1, 2 \rangle = \langle 1+t, 1+2t \rangle$.

$x = 1+t$, $y = 1+2t$

$t = x-1$ $y = 1+2(x-1) = 1+2x-2 = 2x-1$

$r_2(t) = \langle t^3, t^6 \rangle$ $t=1$

$= \langle 1, 1 \rangle$

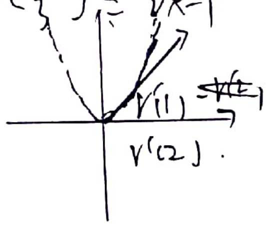
$r_2'(t) = \langle 3t^2, 6t^5 \rangle = \langle 3, 6 \rangle$

$\langle 1, 1 \rangle + t\langle 3, 6 \rangle = \langle 1+3t, 1+6t \rangle$

$x = 1+3t$ $y = 1+6t$

$t = \frac{x-1}{3}$ $y = 1+6\left(\frac{x-1}{3}\right) = 2x-1$

they're the same line



41. $\int_{-2}^{2} (u^3 i + u^5 j)\, du$

$\frac{u^4}{4} i + \frac{u^6}{6} j \Big|_{-2}^{2}$

$\left(\frac{2^4}{4} i + \frac{2^6}{6} j\right) - \left(\frac{(-2)^4}{4} i + \frac{(-2)^6}{6} j\right)$

$= 0$.

$\langle 0, 0 \rangle$

49. $r'(t) = t^2 i + 5t j + k$

$r(t) = \int r'(t) = \frac{t^3}{3} i + \frac{5}{2} t^2 j + tk + c$.

$r(1) = \frac{1}{3} i + \frac{5}{2} j + k + c = j + 2k$

$c = -\frac{1}{3} i - \frac{3}{2} j + k$

$r(t) = \frac{t^3}{3} i + \frac{5}{2} t^2 j + tk - \frac{1}{3} i - \frac{3}{2} j + k$

$= (\frac{t^3}{3} - \frac{1}{3}) i + (\frac{5}{2} t^2 - \frac{3}{2}) j + (t+1) k$