Math 251 Shaun Goda Section 23

12.3 :

1) $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 4+6+5 = \boxed{15}$

13) when $a = \langle 1,1,1\rangle$ and $b = \langle 1,-2,-2\rangle$

$$a\cdot b = 1-2-2 = -3$$

$$\|a\| = \sqrt{3} \quad \|b\| = 3$$

$$\cos\theta = \frac{a\cdot b}{\|a\|\cdot\|b\|} = \frac{-3}{3\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{-3}{3\sqrt{3}}\right) = 125.26^\circ$$

$\boxed{\text{Not orthogonal}}$

$\boxed{\text{Obtuse}}$

21) $$\cos\theta = \frac{(i+j)\cdot(j+2k)}{\|i+j\|\cdot\|j+2k\|}$$

$$= \frac{\langle 1,1,0\rangle\cdot\langle 0,1,2\rangle}{\sqrt{2}\cdot\sqrt{5}} = \frac{1}{\sqrt{10}}$$

$$\theta = \arccos(10^{-\frac{1}{2}}) = 71.57^\circ$$

$$\boxed{\cos\theta = 10^{-\frac{1}{2}}}$$

29) a) $A = \langle b,3,2\rangle,\ B = \langle 1,b,1\rangle$

$$A\cdot B = b + 3b + 2 = 2+4b$$

$$\frac{2+4b}{\sqrt{b^2+13}\cdot\sqrt{b^2+1}} = \frac{2+4b}{\sqrt{(b^2+13)(b^2+1)}}$$

$\boxed{b = -\frac{1}{2}}$

$\cancel{2+4b = \sqrt{(b^2+13)(b^2+1)}}$

~~graph~~ $\cancel{f(x) = \sqrt{(x^2+13)(x^2+1)} - 4x - 2}$

$\cancel{x = 0.532,\ x = 2.61}$

b) $A = \langle 4,-2,7\rangle \quad B = \langle b^2, b, 0\rangle$

$$A\cdot B = 4b^2 - 2b = 2b(b-1)$$

$$\boxed{b = 0,\ b = 1}$$

31) when $A = \langle 2, 0, -3 \rangle$, $B_{4} = \langle X, Y, Z \rangle$

$A \cdot B = 2X - 3Z$

$B_1 = 0 = 2X - 3Z$

$x = 3,\ z = 2 \Rightarrow \boxed{\langle 3, 0, 2, \rangle}$

$B_2 = 0 = 2X - 3Z$

$x = 0,\ z = 0 \Rightarrow \boxed{\langle 0, 0, 0 \rangle}$

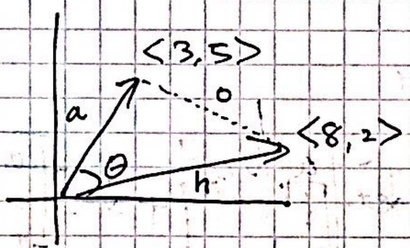
57) projection of u along v

$= \frac{u \cdot v}{\|v\|^2} \cdot v \qquad u \cdot v = -4 \qquad \|v\|^2 = 1$

$u = 5i + 7j - 4k = \langle 5, 7, -4 \rangle \qquad \frac{-4}{1} \cdot k = \boxed{-4k}$

$v = k = \langle 0, 0, 1 \rangle$

63)



$\|u\| = \sqrt{34} \qquad \|v\| = \sqrt{68}$

$\theta = \arccos\left(\frac{\sqrt{34}}{\sqrt{68}}\right) = 45°$

$\frac{\|u\|}{\sqrt{2}} = \frac{\sqrt{34}}{\sqrt{2}} = \boxed{\sqrt{17} = \overline{OP}}$

12.4:

1) $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 3 - 8 = \boxed{-5}$

$-3 - 0$

5) $1 \cdot (\cancel{\ \ }) - 2(4 - 0) + 1(0 + 3) = -3 - 8 + 3 = \boxed{-8}$

13) $(i + j) \times k = (\langle 1, 0, 0 \rangle + \langle 0, 1, 0 \rangle) \times \langle 0, 0, 1 \rangle$

$= \langle 1, 1, 0 \rangle \times \langle 0, 0, 1 \rangle$

$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = i(1-0) - j(1-0) + k(0-0)$

$= i - j + 0$

$\boxed{= i - j}$

21) $(u-2v) \times (u+2v) w$

$= ((u \times w) - (-2v \times w)) \times ((u \times w) + (2v \times w))$

$= (\langle 0,3,1\rangle + 2\langle 2,-1,1\rangle) \times (\langle 0,3,1\rangle + 2\langle 2,-1,1\rangle)$

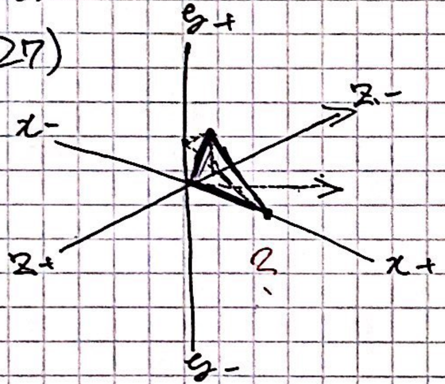
$= (\langle 0,3,1\rangle + \langle 4,-2,2\rangle) \times (\langle 4,1,3\rangle)$

$= \langle 4,1,3\rangle \times$

$= \boxed{0}$

25) $-u$

27)



Not so sure of this question.

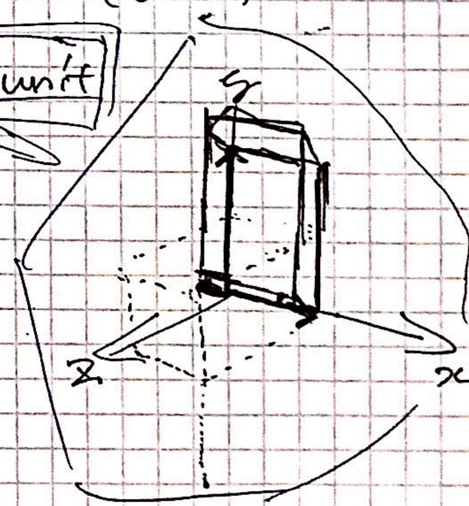
$$\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = i(0-0) - j(-3-0) + k(3-0)$$

$= 3j + 3k$

$= \boxed{\langle 0, 3, 3\rangle}$

39) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 1(4-0) - 0(0-0) + 0(0-2)$

$= 4$ $\boxed{4 \text{ cubic unit}}$



41) $\langle 1,0,3\rangle \times \langle 2,1,1\rangle$

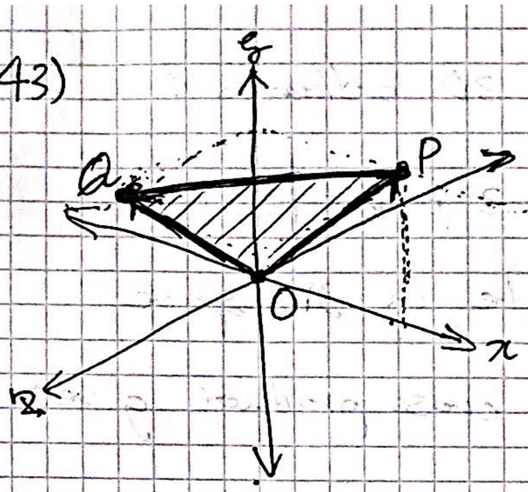
$= \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix}$

$= i(0-3) - j(1-6) + k(1-0)$

$= -3i + 5j + k = \langle -3, 5, 1\rangle$

$|\langle -3,5,1\rangle| = \sqrt{9+25+1} = \boxed{\sqrt{35}}$

43)



$$\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} = i(9-0) - j(9-0) + k(9-0)$$

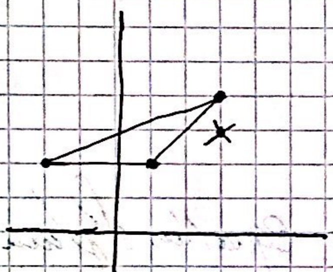
$$= 9i - 9j + 9k$$

$$= \langle 9, -9, 9 \rangle$$

$$\text{Area} = \frac{1}{2}\sqrt{9^2 + 9^2 + 9^2}$$

$$= \boxed{\frac{9\sqrt{3}}{2}}$$

45)



$$\text{Area} = 3 \times 2 \div 2 = \boxed{3}$$

How do i use cross product for solving this problem?

12.5:

1) $1(x-4) + 3(y+1) + 2(z-1) = d$

$(x-4) + (3y+3) + (2z-2) = d$

$\boxed{x + 3y + 2z = 3}$

5) $i = \langle 1, 0, 0 \rangle$

$1(x-3) + 0(y-1) + 0(z+9) = d$

$\boxed{x = 3}$

9) $\boxed{x = 0}$

11) $\boxed{\text{b and d}}$

13) $\boxed{\langle 9, -4, -11 \rangle}$

17) $\vec{PQ} = \langle -1, 2, -3 \rangle$ $\vec{QR} = \langle 2, 0, -3 \rangle$

$$\vec{PQ} \times \vec{QR} = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 2 & 0 & -3 \end{vmatrix} = i(-6-0) - j(3+6) + k(0-4)$$
$$= -6i - 9j - 4k = d$$

$$\begin{vmatrix} 2 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{vmatrix} = 2(-2-1) + 1(-2-3) + 4(1-3) = -19$$

$$\boxed{-6x - 9y - 4z = -19}$$

19) $\vec{PQ} = \langle -1, 1, 1 \rangle$ $\vec{QR} = \langle 2, -1, 0 \rangle$

$$\vec{PQ} \times \vec{QR} = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = i(0+1) - j(0-2) + k(1-2)$$
$$= i + 2j - k = d$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = 1(1-0) - 0 + 0 = 1$$

$$\boxed{x + 2y - z = 1}$$

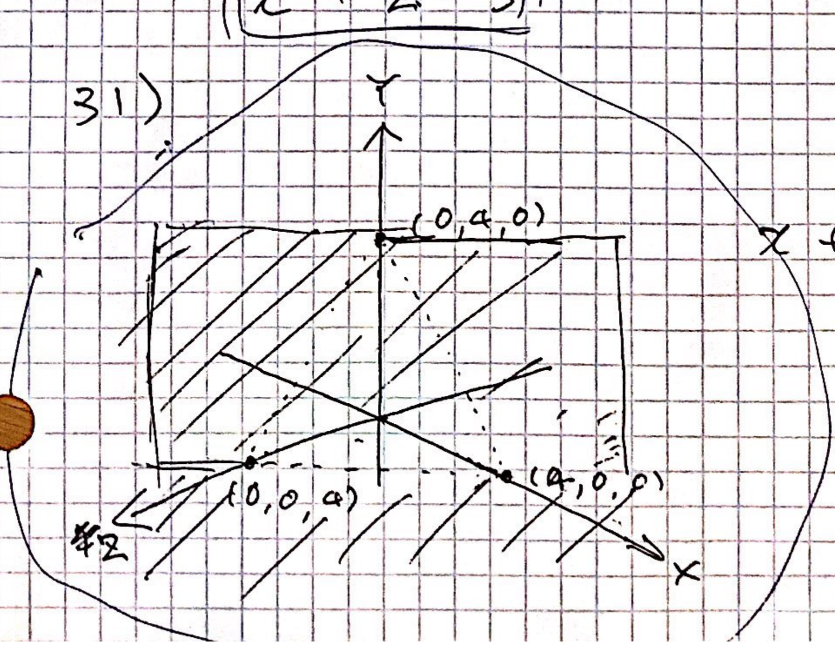
25) $i + k = \langle 1, 0, 1 \rangle$

$1(x+2) + 0(y+3) + 1(z-5) = 0$

$(x+2) + (z-5) = 0$

$$\boxed{x + z = 3}$$

31)



$x + y + z = 4$

53) $\boxed{(3\lambda)x + 6y + (2\lambda)z = 5\lambda}$
where λ is not 0

13.1:

5) vector form: $r = (3,-5,7) + t\langle 3,0,1\rangle$

$x = 3t+3$

$y = -5$

$z = t+7$

$\boxed{r(t) = (3t+3)i + (-5)j + (t+7)k}$

17) centered at (0,0), the radius is 9 and the location is xy plane

13.2:

3) $\lim_{t\to 0} e^{2t}i + \ln(t+1)j + 4k$

$= 1i + 0j + 4k = \boxed{\langle 1, 0, 4\rangle}$

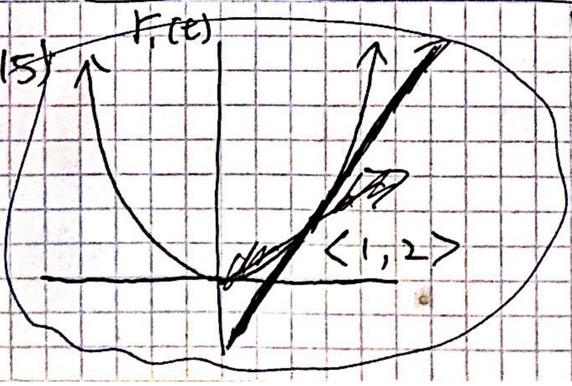
5) $\lim_{h\to 0} \frac{r(t+h)-r(t)}{h}$ for $r(t) = \langle t^{-1}, \sin t, 4\rangle$

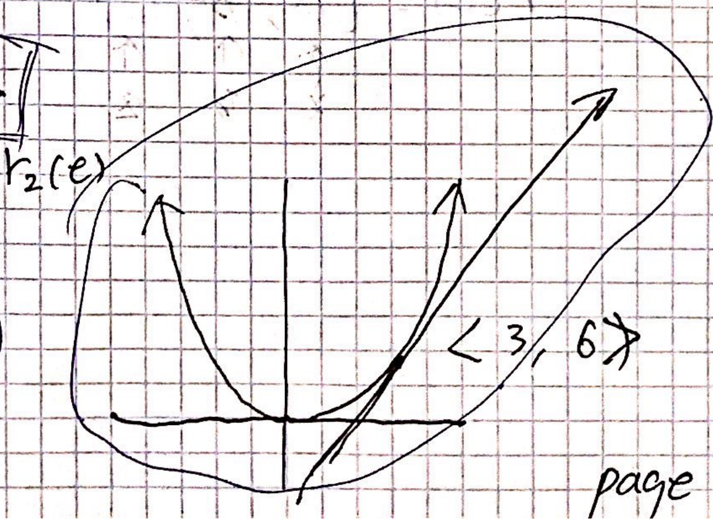
is equal to $\frac{d}{dt}r(t) = \frac{d}{dt}\langle t^{-1}, \sin t, 4\rangle$

$\boxed{r'(t) = \langle -t^{-2}, \cos t, 0\rangle}$

7) $\boxed{r'(t) = \langle 1, 2t, 3t^2\rangle}$

15)





$\cancel{13}.31)$ $r(t) = \langle 1-t^2, 5t, 2t^3 \rangle \quad t=-2$

$r(-2) = \langle \underset{-3}{\cancel{\quad}}, -10, -16 \rangle$

$\frac{d\,r(t)}{dt} = \langle -2t, 5, 6t^2 \rangle$

$r'(-2) = \langle 4, 5, 24 \rangle$

$l(t) = \langle -3, -10, -16 \rangle + t\langle 4, 5, 24 \rangle$

$\boxed{l(t) = \langle -3-4t, \cancel{-}10+5t, \cancel{-}16+24t \rangle}$

33) $r(s) = 4s^{-1}i - \frac{8}{3}s^{-3}k$, $s=2$

$r(2) = 2i - \frac{1}{3}k = \langle 2, 0, -\frac{1}{3} \rangle$

$r'(s) = -4s^{-2}i + 8s^{-4}k$

$r'(2) = -i + \frac{1}{2}k = \langle -1, 0, \frac{1}{2} \rangle$

$l(s) = \langle 2, 0, \frac{1}{3} \rangle + s\langle -1, 0, \frac{1}{2} \rangle$

$\boxed{l(s) = \langle 2-s, 0, -\frac{1}{3}+\frac{1}{2}s \rangle}$

41) $\int_{-2}^{2} (u^3 i + u^5 j)\,du = \Big|_{-2}^{2} \frac{u^4}{4}i + \frac{u^6}{6}j$

$\left(\frac{2^4}{4}i + \frac{2^6}{6}j\right) - \left(\frac{(-2)^4}{4}i + \frac{(-2)^6}{6}j\right) = 0i + 0j = \boxed{\langle 0, 0 \rangle}$

49) $\int r'(t)\,dt = r(t) - c$

$r(t) = \int (t^2 i + 5t j + k)\,dt = \boxed{\frac{t^3}{3}i + \frac{5t^2}{2}j + tk + c}$

$r(1) = \frac{1^3}{3}i + \frac{5 \cdot 1^2}{2}j + 1k + c = j + 2k$

$= \frac{1}{3}i + \frac{5}{2}j + k + c = j + 2k \quad c = -\frac{1}{3}i - \frac{3}{2}j + k$

$\boxed{r(t) = \frac{t^3}{3}i + \frac{5t^2}{2}j + tk - \frac{1}{3}i - \frac{3}{2}j + k}$