HW due 9/20/20

12.3: 1, 13, 21, 29, 31, 57, 63

12.4: 1, 5, 13, 21, 25, 27, 39, 41, 43, 45

12.5: 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

13.1: 5, 17

13.2: 3, 5, 7, 15, 31, 33, 41, 49

12.3

1. $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$

$= (1)(4)+(2)(3)+(1)(5) = 4+6+5 = \boxed{15}$

13. $u = \langle 1,1,1\rangle \quad w = \langle 1,-2,-2\rangle$

$\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle$

$= (1)(1)+(1)(-2)+(1)(-2) = 1-2-2 = -3$

$\boxed{\text{vectors NOT orthogonal}}$

$\|u\| = \sqrt{1^2+1^2+1^2} = \sqrt{3} \quad \|w\| = \sqrt{1^2+(-2)^2+(-2)^2} = 3$

$\cos\theta = \frac{-3}{3\sqrt{3}} \quad \boxed{\theta \text{ is obtuse}}$

21. $v = \langle 1,1,0\rangle \quad w = \langle 0,1,2\rangle$

$v\cdot w = (1)(0)+(1)(1)+(0)(2) = 1$

$\|v\| = \sqrt{1^2+1^2+0^2} = \sqrt{2} \quad \|w\| = \sqrt{0^2+1^2+2^2} = \sqrt{5}$

$\cos\theta = \boxed{\frac{1}{\sqrt{10}}}$

29a. $\langle b,3,2\rangle \cdot \langle 1,b,1\rangle$

$= (b)(1)+(3)(b)+(2)(1) = 4b+2$

$\boxed{b = -\frac{1}{2}}$

b. $\langle 4,-2,7\rangle \cdot \langle b^2, b, 0\rangle$

$= (4)(b^2)+(-2)(b)+(7)(0) = 4b^2-2b$

$4b^2 = 2b \quad \boxed{b = \frac{1}{2} \text{ or } 0}$

31. $v = \langle 2,0,-3\rangle \quad w = \langle a,b,c\rangle$

$v\cdot w = (2)(a)+(0)(b)+(-3)(c) = 2a-3c = 0$

$a=3,\ c=2,\ b=0 \quad w_1 = \langle 3,0,2\rangle$

$a=0,\ c=0,\ b=2 \quad w_2 = \langle 0,2,0\rangle$

57. $u = \langle 5,7,-4\rangle \quad v = \langle 0,0,1\rangle$

$\text{Proj}_v u = \left(\frac{u\cdot v}{v\cdot v}\right) v$

$u\cdot v = (5)(0)+(7)(0)+(-4)(1) = -4$

$v\cdot v = (0)(0)+(0)(0)+(1)(1) = 1$

$= \left(\frac{-4}{1}\right) v = -4v = \boxed{-4k}$

63. $u\cdot v = (3)(8)+(5)(2) = 34$

$\|v\| = \sqrt{8^2+2^2} = \sqrt{68}$

$\overrightarrow{OP} = \frac{34}{\sqrt{68}} = \boxed{\sqrt{17}}$

12.4

1. $\det = (1)(3)-(2)(4) = 3-8 = \boxed{-5}$

5. $\det = 1\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$

$= 1(-3) - 2(4) + 1(3) = -3-8+3 = \boxed{-8}$

13. $(i+j)\times k = i\times k + j\times k = -(k\times i) + i\times k$

$= \boxed{-j+i}$

21. $(u-2v)\times(u+2v)$

$= (u-2v)\times u + (u-2v)\times 2v$

$= u\times u - 2(v\times u) + 2(u\times v) - 4(v\times v)$

$= 0 - 2(-(u\times v)) + 2(u\times v) - 4(0)$

$= 4(u\times v) = 4\langle 1,1,0\rangle = \boxed{\langle 4,4,0\rangle}$

25. $v\times w = -u$

27. $\|v\| = \sqrt{9+0+0} = 3 \quad \|w\| = \sqrt{1+1+0} = \sqrt{2}$

$\theta = \frac{\pi}{2} \quad v\cdot w = 0$

$\|u\| = \sqrt{b^2+b^2} = b\sqrt{2}$

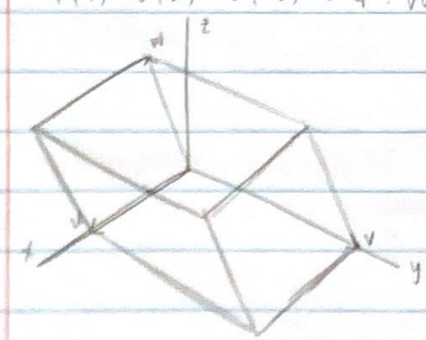
$\|u\| = \|v\|\cdot\|w\|\sin\frac{\pi}{2} = 3\sqrt{2}$

$b = 3 \rightarrow \boxed{u = \langle 0,3,3\rangle}$

39. $V = \det\begin{pmatrix} u \\ v \\ w\end{pmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 2\end{vmatrix}$

$= 1\begin{vmatrix} 2 & 0 \\ 1 & 2\end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 1 & 2\end{vmatrix} + 0\begin{vmatrix} 0 & 2 \\ 1 & 1\end{vmatrix}$

$= 1(4) - 0(0) + 0(-2) = 4 = \text{volume}$



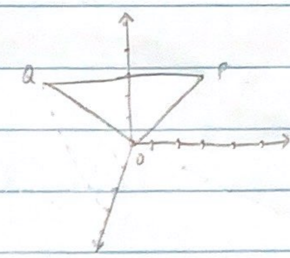
41 Area $= \|u\times v\| = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1\end{vmatrix}$

$= i(0-3) - j(1-6) + k(1-0)$

$= -3i + 5j + k$

$\|u\times v\| = \sqrt{(-3)^2+5^2+1^2} = \sqrt{9+25+1} \quad \boxed{\sqrt{35}}$

43.



Area $= \frac{1}{2}\|\overline{OP}\times\overline{OQ}\|$

$\overline{OP} = \langle 3,3,0\rangle \quad \overline{OQ} = \langle 0,3,3\rangle$

$\overline{OP}\times\overline{OQ} = \begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3\end{vmatrix} = i(9-0) - j(9-0) + k(9-0)$

$= 9i - 9j + 9k = \langle 9,9,9\rangle$

$\|\overline{OP}\times\overline{OQ}\| = \sqrt{9^2+9^2+9^2} = 9\sqrt{3}$

$A = \frac{1}{2}(9\sqrt{3}) = \boxed{\frac{9}{2}\sqrt{3}}$

45. $\overline{PQ} = \langle 3-1, 4-2\rangle = \langle 2,2\rangle$

$\overline{PR} = \langle -2-1, 2-2\rangle = \langle -3,0\rangle$

$\overline{PQ}\times\overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ -3 & 0 & 0\end{vmatrix} = i\begin{vmatrix} 2 & 0 \\ 0 & 0\end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ -3 & 0\end{vmatrix} + k\begin{vmatrix} 2 & 2 \\ -3 & 0\end{vmatrix}$

$= i(0) - j(0) + k(0+6) = 6k$

Area $= \frac{\|6k\|}{2} = \frac{6}{2} = \boxed{3}$

12.5

1. $n = \langle a,b,c\rangle = \langle 1,3,2\rangle \quad \langle x_0,y_0,z_0\rangle = \langle 4,-1,1\rangle$

$ax+by+cz = ax_0+by_0+cz_0$

$x+3y+2z = (1)(4)+(3)(-1)+(2)(1)$

$x+3y+2z = 4-3+2 \rightarrow \boxed{x+3y+2z = 3}$

5. $n = \langle a,b,c\rangle = \langle 1,0,0\rangle \quad \langle x_0,y_0,z_0\rangle = \langle 3,1,-9\rangle$

$ax+by+cz = ax_0+by_0+cz_0$

$\boxed{x = 3}$

9. $ax+by+cz = 0$

↑ equation of any plane through origin

11. B

13. $9x-4y-11z=2$

$\langle 9,-4,-11\rangle \cdot \langle x,y,z\rangle = 2$

$n = \boxed{\langle 9,-4,-11\rangle}$

15. $3(x-4)-8(y-1)+11z=0$

$3x-12-8y+8+11z=0$

$3x-8y+11z=4$

$n=\boxed{\langle 3,-8,11\rangle}$

17. $P=(2,-1,4)$ $R=(3,1,-2)$ $Q=(1,1,1)$

$a=(1,1,1)-(2,-1,4)=\langle -1,2,-3\rangle$

$b=(3,1,-2)-(2,-1,4)=\langle 1,2,-6\rangle$

$n=a\times b=\begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix} = i(-12+6)-j(6+3)+k(-2-2)$
$= -6i-9j-4k$

$P_0=(1,1,1)$

$(-6,-9,-4)\langle x,y,z\rangle = (-6,-9,-4)(1,1,1)$

$= -6x-9y-4z=-6-9-4$

$= \boxed{6x+9y+4z=19}$

19. $P=(1,0,0)$ $R=(2,0,1)$ $Q=(0,1,1)$

$a=(0,1,1)-(1,0,0)=\langle -1,1,1\rangle$

$b=(2,0,1)-(1,0,0)=\langle 1,0,1\rangle$

$n=a\times b=\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = i(1)-j(-1-1)+k(0-1)$
$= i+2j-k$

P_0 $(1,0,0)$

$\langle 1,2,-1\rangle\langle x,y,z\rangle = \langle 1,2,-1\rangle(1,0,0)$

$\boxed{x+2y-z=1}$

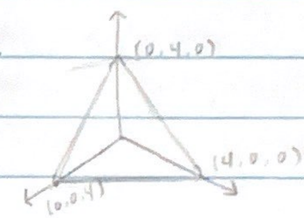
25. $P_0=(-2,-3,5)$ $n=\langle 1,0,1\rangle$

$\langle 1,0,1\rangle\langle x,y,z\rangle = (-2,-3,5)\langle 1,0,1\rangle$

$x+z=-2+5=3$

$\boxed{x+z=3}$

31.



53. $3x+2z=5$

$3x+ky+2z=r$

$3\lambda x+k\lambda y+2\lambda z=5\lambda$ $\lambda\neq 0$

$\boxed{3\lambda x+by+2\lambda z=5\lambda \quad \lambda\neq 0}$

13.1

5. $P=(3,-5,7)$ $v=\langle 3,0,1\rangle$

$r(t)=\langle x_0,y_0,z_0\rangle + tv$

$=\langle x_0,y_0,z_0\rangle + t\langle a,b,c\rangle$

$=\langle x_0+ta,\ y_0+tb,\ z_0+tc\rangle$

$=\langle 3+3t,\ -5,\ 7+t\rangle$

$=\boxed{(3+3t)i-5j+(7+t)k}$

17. $r(t)=(9\cos t)i+(9\sin t)j=\langle 9\cos t, 9\sin t\rangle$

radius $=9$ center $=(0,0,0)$

plane is on x,y - plane

13.2

3. $\lim_{t\to 0} e^{2t}i+\ln(t+1)j+4k$

$=\lim_{t\to 0} e^{2t}\hat{i}+\lim_{t\to 0}\ln(t+1)\hat{j}+\lim_{t\to 0}4\hat{k} = \boxed{\hat{i}+4\hat{k}}$

5. $\lim_{h\to 0} \frac{r(t+h)-r(t)}{h}$ $\quad r(t) = \langle t^{-1}, \sin t, 4\rangle$

$\lim_{h\to 0}\left\langle \frac{x(t+h)-x(t)}{h}, \frac{y(t+h)-y(t)}{h}, \frac{z(t+h)-z(t)}{h}\right\rangle$

$=\lim_{h\to 0}\left\langle \frac{\frac{1}{t+h}-\frac{1}{t}}{h}, \frac{\sin(t+h)-\sin(t)}{h}, \frac{4-4}{h}\right\rangle$

$=\lim_{h\to 0}\left\langle \frac{\frac{t-t-h}{t(t+h)}}{h}, \frac{2\cos\left(\frac{t+h+t}{2}\right)\sin\left(\frac{t+h-t}{2}\right)}{h}, 0\right\rangle$

$=\lim_{h\to 0}\left\langle \frac{-1}{t(t+h)}, \frac{2\cos\left(\frac{2t+h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}\cdot 2}, 0\right\rangle$

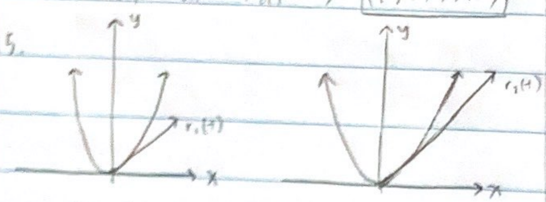
$=\left\langle \frac{-1}{t(t+0)}, \cos\left(\frac{2t+0}{2}\right), \lim_{h\to 0} 0\right\rangle$

$=\boxed{\left\langle -\frac{1}{t^2}, \cos t, 0\right\rangle}$

7. $\vec{r}(t) = \langle t, t^2, t^3\rangle$

$\frac{d\vec{r}}{dt} = \left\langle \frac{d}{dt}t, \frac{d}{dt}t^2, \frac{d}{dt}t^3\right\rangle = \boxed{\langle 1, 2t, 3t^2\rangle}$

15.



31. $r(t) = \langle 1-t^2, 5t, 2t^3\rangle$ $\quad t=2$

$r(2) = \langle 1-(2)^2, 5(2), 2(2)^3\rangle$

$= \langle 1-4, 10, 16\rangle = \langle -3, 10, 16\rangle$

$r'(t) = \langle -2t, 5, 6t^2\rangle$

$r'(2) = \langle -4, 5, 24\rangle$

$L(t) = r(2) + t\,r'(2)$

$= \langle -3, 10, 16\rangle + t\langle -4, 5, 24\rangle$

$= \boxed{\langle -3-4t, 10+5t, 16+24t\rangle}$

33. $r(s) = 4s^{-1} i - \frac{8}{3}s^{-3} k$ $\quad s=2$

$r(2) = \langle 4(2)^{-1}, 0, -\frac{8}{3}(2)^{-3}\rangle$

$= \langle 2, 0, -\frac{1}{3}\rangle$

$r'(s) = \langle -4s^{-2}, 0, 8s^{-4}\rangle$

$r'(2) = \langle -1, 0, \frac{1}{2}\rangle$

$L(s) = \langle 2-s, 0, -\frac{1}{3}+\frac{1}{2}s\rangle$

41. $\int_{-2}^{2}(u^3 i + u^5 j)\,du$

$= \left\langle \int_{-2}^{2} u^3\,du, \int_{-2}^{2} u^5\,du\right\rangle$

$= \left\langle \left(\frac{u^4}{4}\right)_{-2}^{2}, \left(\frac{u^6}{6}\right)_{-2}^{2}\right\rangle$

$= \left\langle \frac{2^4}{4} - \frac{(-2)^4}{4}, \frac{2^6}{6} - \frac{(-2)^6}{6}\right\rangle = \left\langle \frac{16}{4} - \frac{16}{4}, \frac{64}{6} - \frac{64}{6}\right\rangle$

$= \boxed{\langle 0, 0\rangle}$

49. $r'(t) = t^2 i + 5t j + k$ $\quad r(1) = j + 2k$

$r(t) = \int r'(t)\,dt$

$= \int \langle t^2, 5t, 1\rangle\,dt$

$= \left\langle \int t^2\,dt, \int 5t\,dt, \int 1\,dt\right\rangle = \left\langle \frac{t^3}{3}, \frac{5t^2}{2}, t\right\rangle + c$

general solution: $r(t) = \frac{t^3}{3} i + \frac{5t^2}{2} j + tk + c$

$r(1) = \langle 0, 1, 2\rangle$

$r(1) = \langle \frac{1}{3}, \frac{5}{2}, 1\rangle + c$

$c = \langle 0, 1, 2\rangle - \langle \frac{1}{3}, \frac{5}{2}, 1\rangle$

$= \langle 0 - \frac{1}{3}, 1 - \frac{5}{2}, 2 - 1\rangle = \langle -\frac{1}{3}, -\frac{3}{2}, 1\rangle$

$r(t) = \left\langle \frac{t^3}{3}, \frac{5t^2}{2}, t\right\rangle + \left\langle -\frac{1}{3}, \frac{-3}{2}, 1\right\rangle$

$= \left\langle \frac{t^3-1}{3}, \frac{5t^2-3}{2}, t+1\right\rangle$

$\boxed{r(t) = \left(\frac{t^3-1}{3}\right) i + \left(\frac{5t^2-3}{2}\right) j + (t+1) k}$