Sai Embar
Section 23.

12.3 - 13.2 (Sept. 20th)

12.3: # 1, 13, 21, 29, 31, 57, 63

12.4: # 1, 5, 13, 21, 25, 27, 39, 41, 43, 45

12.5: # 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

13.1: # 5, 17

13.2: # 3, 5, 7, 15, 31, 33, 41, 49

12.3

1) $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 4 + 6 + 5 = \boxed{15}$

13) $\overset{u}{\langle 1,1,1\rangle} \cdot \overset{v}{\langle 1,-2,-2\rangle} = 1 + (-2) + (-2) = -3$

$|\vec{u}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$|\vec{v}| = \sqrt{1^2+(-2)^2+(-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{-3}{\sqrt{3}(3)} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{3}\right) = 125.26°$ The vectors are not orthogonal and they are obtuse

21) $\overset{u}{i+j}, \overset{v}{j+2k} = \langle 1,1,0\rangle \cdot \langle 0,1,2\rangle = 0+1+0 = 1$

$|u| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$, $|v| = \sqrt{0^2+1^2+2^2} = \sqrt{5}$

$\cos\theta = \frac{1}{\sqrt{2}(\sqrt{5})} = \frac{1}{\sqrt{10}}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$ $\frac{1}{\sqrt{10}}$ is the angle

29) a) $\langle b,3,2\rangle \cdot \langle 1,b,1\rangle = b + 3b + 2 = 0$

$4b+2=0$, $4b = -2$, $\boxed{b = -\frac{1}{2}}$

b) $\langle 4,-2,7\rangle, \langle b^2,b,0\rangle = 4b^2 - 2b + 0 = 0$

$2b(2b-1) = 0$

$\boxed{b = 0, \frac{1}{2}}$

31) $\langle a,b,c\rangle \cdot \langle 2,0,-3\rangle = 0$

$2a - 3c = 0$ $\boxed{\langle 3,2,2\rangle \text{ and } \langle 0,1,0\rangle}$

$a=3, c=2$
or
$a=0, c=0$

57) $\overset{u}{\langle 5,7,-4\rangle}$ $\quad\overset{v}{\langle 0,0,1\rangle}$

$\langle 5,7,-4\rangle \cdot \langle 0,0,1\rangle = 0+0-4 = -4$

$\text{proj}_{\vec v}\,\vec u = \left(\frac{\vec u \cdot \vec v}{|\vec v|^2}\right) = \frac{-4}{1} = -4$

$\langle 0(-4), 0(-4), 1(-4)\rangle = \boxed{-4k}$

63) $\langle 3,5\rangle \cdot \langle 8,2\rangle = 24+10 = 34$, $|\vec v| = \sqrt{8^2+2^2} = \sqrt{68}$

$\text{Proj}_{\vec v}\,\vec u = \left(\frac{34}{\sqrt{68}}\right) = 4.123 = \boxed{\sqrt{17}}$

12.4: # 1, 5, 13, 21, 25, 27, 39, 41, 43, 45

1) determinant: ad - bc

$\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 1(3) - 2(4) = 3-8 = \boxed{-5}$

5) $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1(-3(1) - 0(0)) - 2(4(1) - 0(1)) + 1(4(0) - (-3)(1))$

$= 1(-3) - 1(0) - 2(4) + 2(0) + 1(4) - 1(-3) =$

$= -3 - 0 - 8 + 0 + 4 + 3 = \boxed{-8}$

13) $(i+j)\times k$

$i = \langle 1,0,0\rangle$, $j = \langle 0,1,0\rangle$ $i+j = \langle 1,1,0\rangle$

$k = \langle 0,0,1\rangle$

$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = i(1(1) - 0(0)) - j(1(1) - 0(0)) + k(1(0) - 1(0))$

$= i(1-0) - j(1-0) + k(0-0)$

$= i - j = \boxed{-j+i}$

21) $(u\times u) + (u\times 2v) + (-2v\times u) + (-2v\times 2u)$

$u\times(2v+u) + (-2v)\times(u+2u)$

$= 2(u\times v) - 2(v\times u) - 4(0)$

$= 2(u\times v) + 2(u\times v) = 4(u\times v) = 4\langle 1,1,0\rangle \boxed{\langle 4,4,0\rangle}$

25) $\boxed{1-u}$

27) $v = \langle 3,0,0\rangle \quad w = \langle 0,1,-1\rangle$

$$\vec{v}\cdot\vec{w} = |v||w|\cos\theta$$

$$\langle 3,0,0\rangle \cdot \langle 0,1,-1\rangle = |\vec{v}||\vec{w}|\cos\theta$$

$$0 = \sqrt{3^2+0^2+0^2}\sqrt{0^2+1^2+1^2}\cos\theta$$

$$0 = |3||\sqrt{2}|\cos\theta$$

$$\theta = 90^\circ$$

$$\vec{u} = \vec{v}\times\vec{w}$$

$$= |\vec{v}||\vec{w}|\sin\theta$$

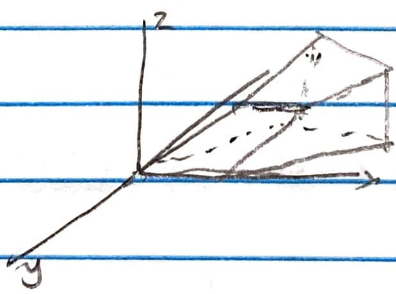
$$\vec{v}\times\vec{w} = 3\sqrt{2}$$

$$\boxed{\vec{u} = \langle 0,0,3}$$

39)

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 1(2(2)-0(1)) - (0)(0(2)-0(1)) + (0)(0(1)-2(1))$$

$$= 4 - 0 + 0 = \boxed{4 \text{ units}^3}$$



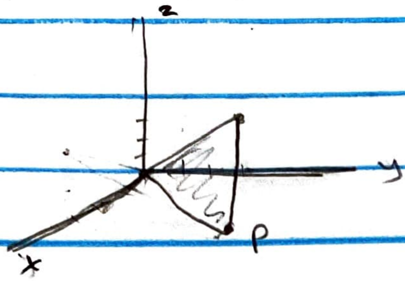
41) $\langle 1,0,3\rangle \quad \langle 2,1,1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = i(0(1)-3(1)) - j(1(1)-2(3)) + k(1(1)-0(2)) = i(-3) - j(-5) + k(1)$$

$$= -3i + 5j + k$$

$$= \sqrt{(-3)^2+5^2+1^2} = \boxed{\sqrt{35}}$$

43)

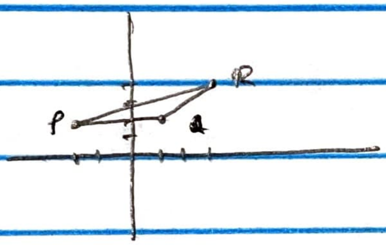


$\langle 3,3,0\rangle \times \langle 0,3,3\rangle$

$$\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} = 9i - 9j + 9k$$

$$\sqrt{9^2+9^2+9^2} = \frac{15.59}{2} = \boxed{7.79}$$

45)



$v = \vec{PR}$

$v = \langle 2,2,0\rangle$

$u = \vec{PQ}$

$u = \langle -3,0,0\rangle$

$$\vec{v}\times\vec{u} = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ -3 & 0 & 0 \end{vmatrix} = 0i + 0j + 6k$$

$$\text{Area} = \frac{1}{2}|\vec{v}\times\vec{u}| = \frac{1}{2}\sqrt{6^2} = \boxed{3 \text{ units}^2}$$

12.5 #1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

1) $n: \langle 1,3,2\rangle \cdot \langle 4,-1,1\rangle = 4 - 3 + 2 = 3$

$\boxed{1x + 3y + 2z = 3}$

5) $n = \langle 1,0,0\rangle\ \langle 3,1,-9\rangle = 3 - 0 - 0 = 3$

$1x = 3,\ \boxed{x = 3}$

9) $\boxed{x=0}$

11) $\boxed{\text{b and d}}$

13) $9x-4y-11z=2$

$\boxed{(9,-4,-11)}$

15) $3x-12-8y+8+11z=0$

$3x-8y+11z=4$

$\boxed{\langle 3,-8,11\rangle}$

17) $P=\langle 2,-1,4\rangle$

$Q=\langle 1,1,1\rangle$

$R=\langle 3,1,-2\rangle$

$\overrightarrow{PQ}=\langle -1,2,-3\rangle$

$PR=\langle 1,2,-6\rangle$

$$\vec{n}=\begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix}=(-12+6)i-(6+3)j+(-2-2)k$$

$$=\langle -6,-9,-4\rangle$$

$-6(x-2)-9(y+1)-4(z-4)=0$

$\boxed{6x+9y+4z=19}$

19) $P=\langle 1,0,0\rangle$

$Q=\langle 0,1,0\rangle$

$R=\langle 2,0,1\rangle$

$\overrightarrow{PQ}=\langle -1,1,1\rangle$

$\overrightarrow{PR}=\langle 1,0,1\rangle$

$$\vec{n}=\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}=1i-2j-1k=\langle 1,2,-1\rangle$$

$1(x-1)+2(y)+(-1)(z)=0$

$x-1+2y-z=0$, $\boxed{x+2y-z=1}$

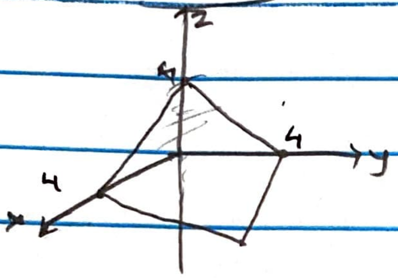
25) $(-2,-3,5)$ and has normal vector $i+k = \langle 1,0,1\rangle$

$1(x+2)+0(y+3)+1(z-5)=0$

$x+z-3=0$

$\boxed{x+z=3}$

31)



53) $3x+ky+2z=-5$

k is any real number

13.1: # 5, 17

5) $r(t)=P+t\cdot v$

$r(t)=(3i+5j+7k)+t(3i+0j+1k)$

$\boxed{r(t)=(3+3t)i+5j+(7+1t)k}$

17) $r(t)=(9\cos t)i+(9\sin t)j$

$x=9\cos t$ $\quad$ $x+y=9\cos t+9\sin t$

$y=9\sin t$ $\quad$ $x+y=9(\cos t+\sin t)$

$x+y=9$

$\boxed{r=9, \text{ centered at origin in } x\text{-}y \text{ plane}}$

13.2: # 3,5,7,15,31,33,41,49

3) $e^{2(0)}i+\ln(0+1)j+4k=\boxed{i+4k}$

5) $r(t)=\langle t^{-4},\sin t,4\rangle$

$\lim_{h\to 0}\frac{[(t+h)^{-4}-t^{-4}, \sin(t+h)-\sin t, 0]}{h}$

$=\lim_{h\to 0}\left[\frac{[(t+h)]^{-4}}{h}, 2\sin\left(\frac{h}{2}\right)\cos\left(\frac{2t+h}{2}\right), 0\right]$

$=\lim_{h\to 0}\left[\frac{-4(t+h)^{-2}}{1}, \cos t, 0\right.$

$=-4t^{-2}, \cos t, 0\rangle = \boxed{\langle -\frac{1}{t^2}, \cos t, 0\rangle}$

7) $r(t) = \langle t, t^2, t^3 \rangle$

$\boxed{\dfrac{dr}{dt} = \langle 1, 2t, 3t^2 \rangle}$

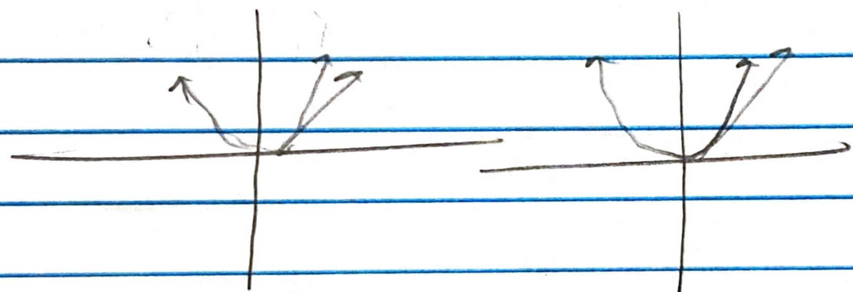
15) $x = t \quad y = t^2$

$y = x^2$

$r'(t) = \langle 1, 2t \rangle$

$|r'(t)| = \sqrt{1 + 4t^2}$

$T(t) = \dfrac{r'(t)}{|r'(t)|} = \left\langle \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}} \right\rangle$



31) $r(t) = \langle 1-t^2, 5t, 2t^3 \rangle, \; t = 2$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

$r'(2) = \langle -4, 5, 24 \rangle$

$\boxed{d(t) = \langle -3-4t, 10+5t, 16+24t \rangle}$

33) $r(s) = 4s^{-1}i - \frac{8}{3}s^{-3}k, \; s = 2$

$r'(s) = \langle 4\ln|s|, \frac{16}{3}s^{-2} \rangle$

$r'(2) = \langle 4\ln 2, 0, \frac{4}{3} \rangle$

$d(t) = \langle 2-t, 0, -\frac{1}{3} + \frac{1}{2}t \rangle$

41)

$\int_{-2}^{2} (u^3 i + u^5 j)\, du$

$\left(\frac{u^4}{4} i + \frac{u^6}{6} j\right)\Big|_{-2}^{2}$

$\left(4i + \frac{64}{6} j\right) - \left(4i + \frac{64}{6} j\right) = \boxed{\langle 0, 0 \rangle}$

49) $r'(t) = t^2 i + 5t j + k, \; r(1) = j + 2k$

$\frac{dx}{dt} = t^2, \quad x = \frac{t^3}{3} + c$

$\frac{dy}{dt} = 5t, \quad y = \frac{5}{2}t^2 + d$

$\frac{dz}{dt} = 1, \quad z = t + f$

$r(t) = \frac{1}{3}t^3 i + \frac{5t^2}{2} j + tk + C$

$\boxed{r(t) = \left(\frac{1}{3}t^3 - \frac{1}{3}\right)i + \left(\frac{5}{2}t^2 - \frac{3}{2}\right)j + (t+1)k}$