12.3

SHUBIN XIE
RUID: 203002353
section 22.

1, $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$
$= 1\times4+3\times2+1\times5$
$=4+6+5$
$=15$

13, $\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle$
$= 1\times1+(-2)\times1+(-2)\times1$
$= 1-2-2$
$= -3$
~~not orthogonal~~

$\cos\theta = \dfrac{-3}{\sqrt{3}\,\sqrt{1+4+4}} = \dfrac{-3}{\sqrt{3}\cdot\sqrt{9}} = \dfrac{-3}{3\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

$\theta \approx 54.7$

The angle is ~~acute~~ obtuse.

21, $i+j$, $j+2k$
$\langle 1,1,0\rangle$, $\langle 0,1,2\rangle$

$\cos\theta = \dfrac{\langle 1,1,0\rangle\cdot\langle 0,1,2\rangle}{\sqrt{1^2+1^2}\times\sqrt{1^2+2^2}}$
$= \dfrac{0+1+0}{\sqrt{2}\times\sqrt{5}}$
$= \dfrac{1}{\sqrt{10}}$
$= \dfrac{\sqrt{10}}{10}$

29, (a) $\langle b,3,2\rangle$, $\langle 1,b,1\rangle$
$b\times1+3\times b+2=0$
$4b+2=0$
$4b=-2$
$b=-\dfrac{1}{2}$

(b) $\langle 4,-2,7\rangle$, $\langle b^2,b,0\rangle$
$b^2\times4+b\times(-2)+7\times0=0$
$4b^2-2b=0$
$2b(2b-1)=0$
$b_1=0 \quad b_2=\dfrac{1}{2}$

31, ~~①<3,~~
① $\langle 3,0,2\rangle$ $\langle 3,2,2\rangle$

② ~~$\langle -3,0,-2\rangle$~~ $\langle -3,1,-2\rangle$
$\langle 0,1,0\rangle$

57, $u=5i+7j-4k$, $v=k$

$v=\langle 0,0,1\rangle$
$u=\langle 5,7,-4\rangle$

~~$\dfrac{u\cdot v}{|u||v|}$~~

$\dfrac{u\cdot v}{|v|^2}\,v$
$= \dfrac{-4}{1}\cdot\langle 0,0,1\rangle$
$= \langle 0,0,-4\rangle$

projection: $\langle 0,0,-4\rangle$.
$-4k$

63,
$\overrightarrow{OP} = \dfrac{u\cdot v}{|u|\cdot|v|}\cdot|u|$
$= \dfrac{24+10}{\sqrt{64+4}}$
$= \dfrac{34}{\sqrt{68}}$
$= \dfrac{34}{2\sqrt{17}}$
$= \dfrac{17}{\sqrt{17}}$
$= \dfrac{17\sqrt{17}}{17}$
$= \sqrt{17}$.

12.4

SHUBIN XIE
RUID:203002353
section22

1、 $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$

$3-8=-5$.

5、 $\begin{vmatrix} 1 & 2 & [redacted] \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

$(-3-0)\times 1-(4-0)\times 2+(0-(-3))\times 1$

$=-3-8+3$

$=-8$

13、 $(i+j)\times k$

$=i\times k+j\times k$

$-j+i$

?21、 $(u-2v)\times(u+2v)$

$u\times v=\langle 1,1,0\rangle$, $u\times w=\langle 0,3,1\rangle$, $v\times w=\langle 3,-1,1\rangle$

$\langle 4,4,0\rangle$

25、 $-u$ equal to $v\times w$.

27、 $u=v\times w$

$v=\langle 3,0,0\rangle$ $w=\langle 0,1,-1\rangle$

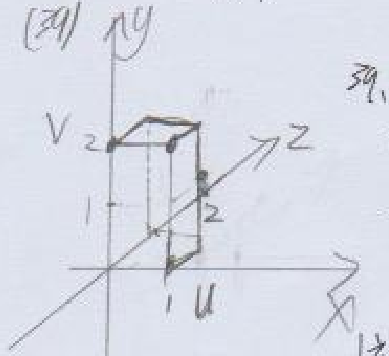
$\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}=0\times i-(-3-0)j+(3-0)k$

$=3j+3k$ $\langle 0,3,3\rangle$

$=\sqrt{9+9}$

$=3\sqrt{2}$

(39)



39、

~~$(u\times v)\cdot w$~~

$(u\times v)\cdot w=2k\cdot w$

$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix}$

$=\langle 0,0,2\rangle\cdot w$

$=\langle 0,0,2\rangle\cdot\langle 1,1,2\rangle$

$=4$

The volume is 4.

41、 $|u\times v|=(0-3)i-(1-6)j+(1-0)k$

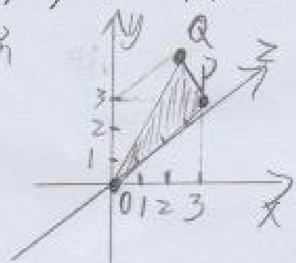
$u=\langle 1,0,3\rangle$ $=-3i+5j+k$

$v=\langle 2,1,1\rangle$ $=\sqrt{9+25+1}$

$\begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix}$ $=\sqrt{35}$

Area: $\sqrt{35}$.

43、



43、 $\overrightarrow{OP}=(3,3,0)$

$\overrightarrow{OQ}=(0,3,3)$

$\text{Area}=\frac{1}{2}|OP\times OQ|=\frac{1}{2}[(9-0)i-(9-0)j+(9-0)k]$

$\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix}$

$=\frac{1}{2}[9i-9j+9k]$

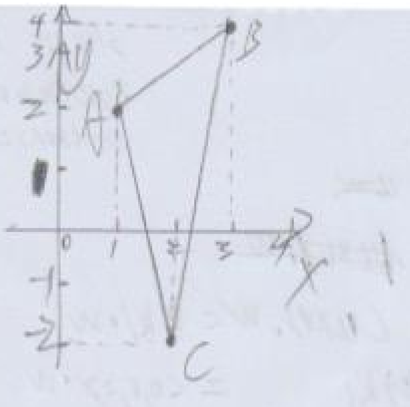
$=\frac{1}{2}\langle 9,-9,9\rangle$

$=\frac{1}{2}\times\sqrt{81+81+81}$

$=\frac{1}{2}\times 9\sqrt{3}$

$=\frac{9}{2}\sqrt{3}$

45.



$\overrightarrow{AB} = (3,4) - (1,2) = (2,2)$

$\overrightarrow{AC} = (-2,2) - (1,2) = (-3,0)$

$\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\cancel{\ldots}\sqrt{(0+6)^2}$

$= \frac{1}{2} \times \cancel{6}\,3$

$\frac{1}{2}\cdot\begin{vmatrix} \vec{i} & \vec{j} \\ 2 & 2 \\ -3 & 0 \end{vmatrix} \qquad = 3$

Area: 3

12.5

SHUBIN XIE
RUID: 203002353
section22

1. $n=\langle 1,3,2\rangle$, $\langle 4,-1,1\rangle$

$(x-4)\times 1+(y+1)\times 3+(z-1)\times 2=0$

$x-4+3y+3+2z-2=0$

$x+3y+2z=3$

5. $n=i$ $(3,1,-9)$

$(x-3)\times 1+(y-1)\times 0+(z+9)\times 0=0$

$x-3=0$

$x=3$.

9. $x=0$

11. which of the following statements are true of a plane that is parallel to the ~~x-y~~ yz-plane? ~~(a)~~ b, d

(a) $n=\langle 0,0,1\rangle$ is a normal vector.

(b) $n=\langle 1,0,0\rangle$ is a normal vector.

(c) The equation has the form $ay+bz=d$

(d) the equation has the form $x=d$

13. $\langle 9,4,-11\rangle$

15. $\langle 3,-8,11\rangle$

17. $P=(2,-1,4)$ $Q=(1,1,1)$ $R=(3,1,-2)$

$\overrightarrow{PQ}=[(1,1,1)-(2,-1,4)]=\langle -1,2,-3\rangle$

$\overrightarrow{PR}=[(3,1,-2)-(2,-1,4)]=\langle 1,2,-6\rangle$

$\overrightarrow{PQ}\times\overrightarrow{PR}=(-12+6)i-(6+3)j+(-2-2)k$

$\begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix}$ $=-6i-9j-4k$

$=\langle -6,-9,-4\rangle$

~~[illegible]~~ choose $Q(1,1,1)$

$(x-1)\times(-6)+(y-1)\times(-9)+(z-1)\times(-4)=0$

$-6x+6-9y+9-4z+4=0$

$\boxed{-6x-9y-4z=-19}$

$6x+9y+4z=19$

19. $P=(1,0,0)$ $Q=(0,1,1)$ $R=(2,0,1)$

$\overrightarrow{PQ}=[(0,1,1)-(1,0,0)]=\langle -1,1,1\rangle$

$\overrightarrow{PR}=[(2,0,1)-(1,0,0)]=\langle 1,0,1\rangle$

$\overrightarrow{PQ}\times\overrightarrow{PR}=(1-0)i-(-1-1)j+(0-1)k$

$\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$ $=i+2j-k$

$=\langle 1,2,-1\rangle$

choose $P(1,0,0)$

$(x-1)\times 1+(y-0)\times 2+(z-0)\times(-1)=0$

$x-1+2y-z=0$

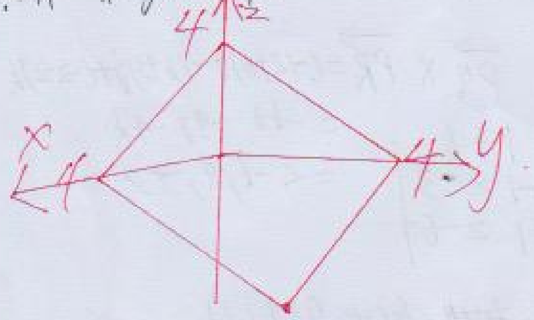
$\boxed{x+2y-z=1}$

25、 $(x+2)\times 1+(y+3)\times 0+(z-5)\times 1=0$

$x+2+0+z-5=0$

$\boxed{x+z=3}$

73、 $x+y+z=4$.



75、 find all planes in R^3 whose intersection with the xz-plane is the ~~line~~ with equation $3x+2z=5$

$(3\lambda)x+by+(2\lambda)z=5\lambda,\ \lambda\neq 0.$

13.1

SHUBIN XIE
RUID 1203002353
section 22

5. $\langle 3,-5,7\rangle + t\langle 3,0,1\rangle$
$= \langle 3+3t, -5, t+7\rangle$

15. (A) - (ii)
(B) - (iii)
(C) - (i)

17. $r(t) = (9\cos t)i + (9\sin t)j$
$= \langle 1,1,0\rangle + 9\langle\cos t, \sin t, 0\rangle$
center: $(1,1,0)$ $(0,0,0)$
radius: 9
plane: $z=0$ ~~x-y pla~~ x-y plane

13.2

3. $\lim_{t\to 0} e^{2t} i + \ln(t+1) j + 4k$
$= i + 4k$

5. Evaluate $\lim_{h\to 0} \frac{r(t+h)-r(t)}{h}$ for $r(t) = \langle t^{-1}, \sin t, 4\rangle$
~~$r(t) = \langle \ln t, \cos t, 4t\rangle$~~
$r'(t) = \langle -2t^{-2}, \cos t, 0\rangle$ $\langle -t^{-2}, \cos t, 0\rangle$
$r'(h) = \langle 0, 1, 0\rangle$

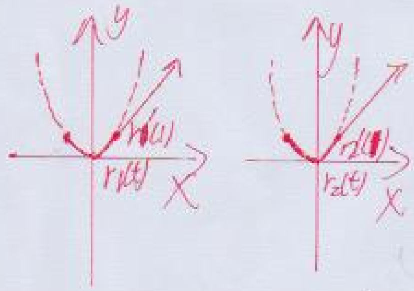
7. $r(t) = \langle t, t^2, t^3 \rangle$

~~$r(t) = \langle 1, t$~~

~~$r'(t) = \langle 1, t, 2t^2$~~

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

?15.



31. $r(t) = \langle 1-t^2, 5t, 2t^3 \rangle \quad t=2.$

$r(2) = \langle 1-4, 10, 2\times 8 \rangle$

$= \langle \cancel{-5}\ -3, 10, 16 \rangle$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

$r'(2) = \langle -4, 5, 24 \rangle$

$\langle -3, 10, 16 \rangle + t \langle -4, 5, 24 \rangle$

$= \langle -3-4t, 10+5t, 16+24t \rangle$

$x = -3-4t \quad y = 10+5t \quad z = 16+24t$

That is Ans.

33. $r(s) = 4s^{-1} i - \frac{8}{3} s^{-3} k, \quad s=2$

$= \langle 4s^{-1}, 0, -\frac{8}{3} s^{-3} \rangle$

$r(2) = \langle 2, 0, -\frac{1}{3} \rangle$

$r'(s) = \langle -4s^{-2}, 0, 8s^{-4} \rangle$

$r'(2) = \langle -1, 0, \frac{1}{2} \rangle$

$\langle 2, 0, -\frac{1}{3} \rangle + t \langle -1, 0, \frac{1}{2} \rangle$

$= \langle 2-t, 0, -\frac{1}{3} + \frac{1}{2}t \rangle$

41. $\int_{-2}^{2} (u^3 i + u^5 j)\, du$

$= \left(\frac{1}{4} u^4 i + \frac{1}{6} u^6 j \right) \Big|_{-2}^{2}$

~~$= \left(\frac{1}{4} \times (16) \ldots + \left(\frac{1}{6} \times (64) \right) j \right)$~~

$= 0$

$\langle 0, 0 \rangle$

49. ~~$r'(t) = t^3$~~

$r(t) = \int r'(t) = \frac{1}{3} t^3 i + \frac{5}{2} t^2 j + tk + c$

$r(1) = \frac{1}{3} i + \frac{5}{2} j + k + c = j + 2k$

$C = -\frac{1}{3} i - \frac{3}{2} j + k$

initial conditions!

$r(t) = \left(\frac{1}{3} t^3 - \frac{1}{3} \right) i + \left(\frac{5}{2} t^2 - \frac{3}{2} \right) j + (t+1) k$