12.3 HW Calc 251

1. $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 1(4) + 2(3) + 1(5)$
$= 4 + 6 + 5$
$\boxed{\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 15}$

13. $u = \langle 1,1,1\rangle$, $v = \langle 1,-2,-2\rangle$

$\cos\theta = \frac{u \cdot v}{|u||v|}$

$u \cdot v = 1(1) + 1(-2) + 1(-2) = 1-2-2 = -3$

$|u| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$|v| = \sqrt{1^2+(-2)^2+(-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$\cos\theta = -\frac{3}{3\sqrt{3}}$

$\cos\theta = -\frac{1}{\sqrt{3}}$

$\boxed{\theta = 125.26°\text{, angle is obtuse}}$

21. $i + j$, $j + 2k$

$\underset{u}{\langle 1,1\rangle}$, $\underset{v}{\langle 1,2\rangle}$

$u \cdot v = 1(1) + 1(2) = 1 + 2 = 3$

$|u| = \sqrt{1^2+1^2} = \sqrt{2}$

$|v| = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$

$\boxed{\cos\theta = \frac{3}{\sqrt{10}}}$

29. (a). $\underset{u}{\langle b,3,2\rangle}, \underset{v}{\langle 1,b,1\rangle}$

set $u \cdot v = 0$

$u \cdot v = 1(b) + 3(b) + 2(1) = 0$

$b + 3b + 2 = 0$

$4b + 2 = 0$

$4b = -2$

$\boxed{b = -\frac{1}{2}}$

31. $u = \langle 2, 0, -3\rangle$, $v = \langle a, b, c\rangle$

$u \cdot v = 0$

$u \cdot v = 2a + \cancel{0b} - 3c = 0$

$2a - 3c = 0$

$2a = 3c$

$a = \frac{3c}{2}$

$v = \langle \frac{3c}{2}, b, c\rangle$ b and c can be any number, $b = 6$, $c = 2$ or $b = 7$, $c = 6$

$$\boxed{\begin{array}{l} v_1 = \langle 3, 6, 3\rangle \\ v_2 = \langle 9, 7, 6\rangle \end{array}}$$

57. $u_{\parallel v} = \frac{u \cdot v}{\|v\|^2}\vec{v}$ $\quad u = 5i + 7j - 4k$, $v = k$

$u = \langle 5, 7, -4\rangle$, $v = \langle 0, 0, 1\rangle$

$u \cdot v = 5(0) + 7(0) - 4(1) = -4$

$\|v\| = \sqrt{0^2 + 0^2 + 1^2} = \sqrt{1^2} = 1$

$\boxed{u_{\parallel v} = -4k}$

Scanned with CamScanner

63. $u = \langle 3, 5 \rangle$, $v = \langle 8, 2 \rangle$

$u \cdot v = 3(8) + 5(2) = 24 + 10 = 34$

$\|v\| = \sqrt{8^2 + 2^2} = \sqrt{64 + 4} = \sqrt{68}$

$\text{comp}_v u = \dfrac{u \cdot v}{\|v\|} = \dfrac{34}{\sqrt{68}} = \dfrac{34}{\sqrt{34 \cdot 2}} \cdot \dfrac{\sqrt{34}}{\sqrt{34}}$

$\text{comp}_v u = \dfrac{\cancel{34}\sqrt{34}}{\cancel{34}\sqrt{2}} = \sqrt{\dfrac{34}{2}} = \sqrt{17}$

$\boxed{\text{comp}_v u = \sqrt{17}}$

Calc 251 HW: 12.4

1. $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$ $1(3) - 4(2) = 3 - 8 = -5$

$\boxed{\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = -5}$

5. $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix}$ $i = 1,\ j = 2,\ k = 1$

$\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix}(1) - \begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix}(2) + \begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}(1) = 8$

$(1(-3) - 0(0))(1) - (4(1) - 0(1))(2) + (0(4) - 1(-3))(1)$

$(-3)(1) + (4)(2) - (3)(1) = -3 - 8 + 3 = \boxed{-8}$

13. $(i + j) \times k = i \times k + j \times k$

$k \times i = j$ $\qquad j \times k = i$

$i \times k = -j$

$\boxed{-j + i}$

21. $(u - 2v) \times (u + 2v) = u \times (u + 2v) - 2v \times (u + 2v)$

$= u \times u + u \times 2v - 2v \times u - 2v \times 2v$

$= 0 + 2(u \times v) - 2(v \times u) - 0$

$= 2\langle 1, 1, 0 \rangle - 2\langle -1, -1, 0 \rangle$

$= \langle 2, 2, 0 \rangle + \langle 2, 2, 0 \rangle$

$= \boxed{\langle 4, 4, 0 \rangle}$

25. $v \times w = -u$ using the right hand rule.

27. $v = \langle 3,0,0\rangle$, $w = \langle 0,1,-1\rangle$, $u = v \times w$

$u \perp v$, $u \perp w$ if $u = v \times w$. If v is on the x-axis the u lies on the yz axis
$u = \langle 0, b, c\rangle$. $u \cdot w = \langle 0, b, c\rangle \cdot \langle 0,1,-1\rangle = b - c = 0$.
$b = c$, so $u = \langle 0, b, b\rangle$.

$\|v\| = \sqrt{3^2+0^2+0^2} = 3$
$\|w\| = \sqrt{0^2+1^2+(-1)^2} = \sqrt{2}$
$\|u\| = \sqrt{0^2+b^2+b^2} = |b|\sqrt{2}$

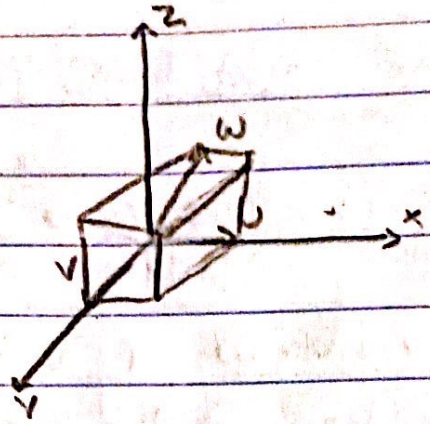
$\|v\|\|w\|\sin\frac{\pi}{2} = \|u\|$
$3\sqrt{2} = |b|\sqrt{2}$
$b = \pm 3$
Using the right hand rule u is in the z direction, thus $u = \langle 0,3,3\rangle$

39. $Vol_p = |u \cdot (v \times w)|$ $u = \langle 1,0,0\rangle$, $v = \langle 0,2,0\rangle$, $w = \langle 1,1,2\rangle$

$$\begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} i - \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} j + \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} k$$
$(4-0)i - (0-0)j + (0-2)k$
$\langle 4, 0, -2\rangle$

$\langle 1,0,0\rangle \cdot \langle 4,0,-2\rangle = 4(1) + 0(0) + 0(-2) = 4$

Scanned with CamScanner



41. $\text{Area}_p = \|u \times v\|$, $u = \langle 1,0,3\rangle$, $v = \langle 2,1,1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} i - \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} j + \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} k$$

$$(0-3)i - (1-6)j + (1-0)k$$

$$-3i + 5j - 1k$$

$$\langle -3, 5, -1\rangle$$

$\|v \times w\| = \sqrt{(-3)^2 + 5^2 + (-1)^2} = \sqrt{9+25+1} = \sqrt{35}$

$\boxed{\|v \times w\| = \sqrt{35}}$

43. $\text{Area}_T = \dfrac{\|v \times w\|}{2}$

$O = (0,0,0)$ $\quad \overrightarrow{OP} = \langle 3-0, 3-0, 0-0\rangle = \langle 3,3,0\rangle$

$P = (3,3,0)$ $\quad \overrightarrow{OQ} = \langle 0-0, 3-0, 3-0\rangle = \langle 0,3,3\rangle$

$Q = (0,3,3)$

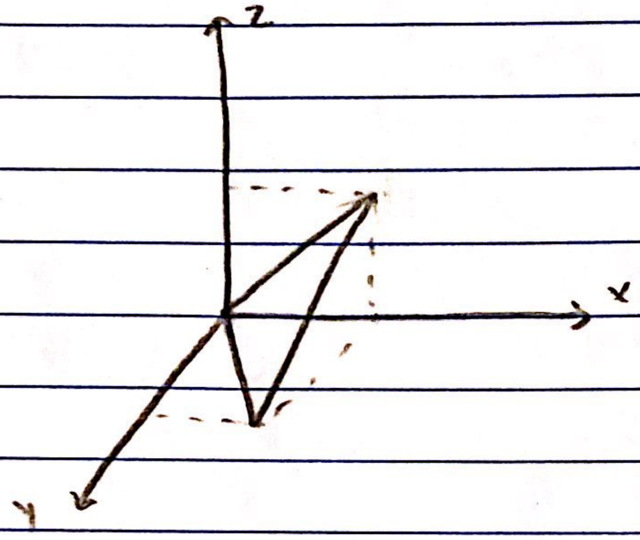
$$\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 3 & 3 \end{vmatrix} i - \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} j + \begin{vmatrix} 3 & 3 \\ 3 & 0 \end{vmatrix} k$$

$$(9-0)i - (9-0)j + (0-9)k$$

$$(9i - 9j - 9k)$$

$\|v \times w\| = \sqrt{9^2 + (-9)^2 + (-9)^2} = \sqrt{243} = 9\sqrt{3}$

$\boxed{\text{Area}_T = \frac{9\sqrt{3}}{2}}$



45. $(1,2)$, $(3,4)$, $(-2,2)$ $\quad \text{Area}_T = \frac{\|v \times w\|}{2}$

$(1,2,0)$, $(3,4,0)$, $(-2,2,0)$
$\quad P \qquad Q \qquad R$

$\overrightarrow{PQ} = \langle -2, -2, 0 \rangle$

$\overrightarrow{PR} = \langle 3, 0, 0 \rangle$

$$\begin{vmatrix} i & j & k \\ -2 & -2 & 0 \\ 3 & 0 & 0 \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & 0 \end{vmatrix} i - \begin{vmatrix} -2 & 0 \\ 3 & 0 \end{vmatrix} j + \begin{vmatrix} -2 & -2 \\ 3 & 0 \end{vmatrix} k$$

$$(0-0)i - (0-0)j + (0+6)k$$

$$\langle 0, 0, 6 \rangle$$

$\|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \sqrt{0^2 + 0^2 + 6^2} = 6$

$\boxed{\text{Area}_T = 3}$

Scanned with CamScanner

Calc 251 HW: 12.5

1. $n = \langle 1,3,2 \rangle$, $(4,-1,1)$

$d = \langle 1,3,2 \rangle \cdot \langle 4,-1,1 \rangle = 4-3+2 = 3$

$\boxed{x + 3y + 2z = 3}$

5. $n = i$, $(3,1,-9)$

$d = \langle 1,0,0 \rangle \cdot \langle 3,1,-9 \rangle = 3+0+0 = 3$

$\boxed{x = 3}$

9. $n = \langle 6,6,6 \rangle$, $(0,0,0)$

$d = \langle 6,6,6 \rangle \cdot \langle 0,0,0 \rangle = 0$

$\boxed{6x + 6y + 6z = 0}$

11. (b) and (d)

13. $n = \langle 9,-4,-11 \rangle$, $d = 2$

$\boxed{\langle 9,-4,-11 \rangle}$

15. $3(x-4) - 8(y-1) + 11z = 0$

$3x - 12 - 8y + 8 + 11z = 0$

$3x - 8y + 11z = 4$

$\boxed{\langle 3,-8,11 \rangle}$

17. $P=(2,-1,4)$, $Q=(1,1,1)$, $R=(3,1,-2)$

$PQ=(2,-1,4)-(1,1,1)=\langle 1,-2,3\rangle$

$PR=(2,-1,4)-(3,1,-2)=\langle -1,-2,6\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ -1 & -2 & 6 \end{vmatrix} = \begin{vmatrix} -2 & 3 \\ -2 & 6 \end{vmatrix} i - \begin{vmatrix} 1 & 3 \\ -1 & 6 \end{vmatrix} j + \begin{vmatrix} -1 & +2 \\ -1 & -2 \end{vmatrix} k$$

$(-12+6)i-(6+3)j+(-2-2)k$

$-6i-9j-4k$

$d=\langle 2,-1,4\rangle\cdot\langle 6,-9,-4\rangle = 12+9-16=5$

$\boxed{6x-9y-4z=5}$

19. $P=(1,0,0)$, $Q=(0,1,1)$, $R=(2,0,1)$

$PQ=(1,0,0)-(0,1,1)=\langle 1,-1,-1\rangle$

$PR=(1,0,0)-(2,0,1)=\langle -1,0,-1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ -1 & 0 & -1 \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix} i - \begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} j + \begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix} k$$

$(1-0)i-(-1-1)j+(-1-0)k$

$1i+2j-k$

$d=\langle 1,0,0\rangle\cdot\langle 1,2,-1\rangle = 1+0+0=1$

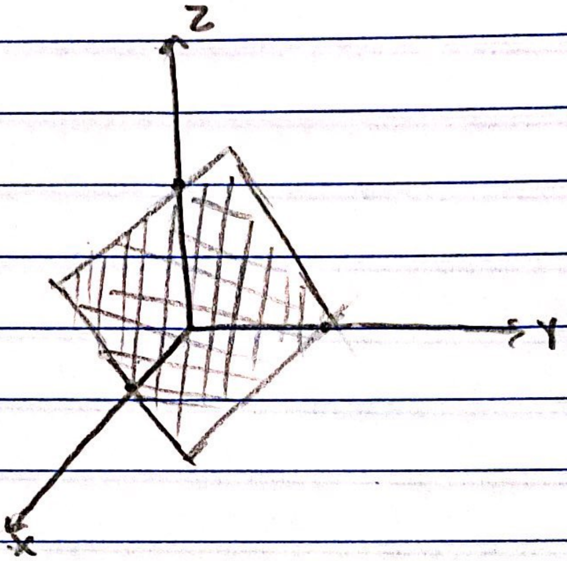
$\boxed{x+2y-z=1}$

Scanned with CamScanner

25. $P = (-2,-3,5)$ $n = i + k \to \langle 1,0,1 \rangle$

$d = \langle -2,-3,5 \rangle \cdot \langle 1,0,1 \rangle = -2 + 0 + 5 = 3$

$\boxed{x + z = 3}$

31. 

53. $3x + by + 2z = 5$, $b =$ Anything

Calc 251 HW: 13.1

5. $P = (3,-5,7)$, $v = \langle 3,0,1\rangle$

$\langle 3,-5,7\rangle + t\langle 3,0,1\rangle$

$\langle 3,-5,7\rangle + \langle 3t,0,t\rangle$

$\boxed{\langle 3+3t,-5,7+t\rangle}$

17. $r(t) = (9\cos t)i + (9\sin t)j$

$r(t) = \langle 9\cos t, 9\sin t, 0\rangle$

$\langle 0,0,0\rangle + 9\langle \cos t, \sin t, 0\rangle$

Center = $(0,0,0)$

radius = 9

Plane: $y = 9\sin t$

Calc 251 HW: 13.2

3. $e^0 i + \ln(1) j + 4k$

$i + 0j + 4k$

$\boxed{i + 4k}$

5. $\boxed{r'(t) = \langle -t^{-2}, \cos t, 0\rangle}$

7. $\boxed{r'(t) = \langle 1, 2t, 3t^2\rangle}$

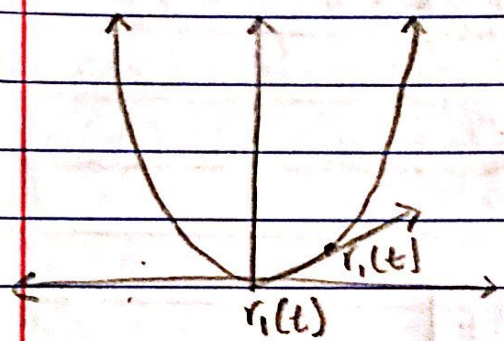
Scanned with CamScanner

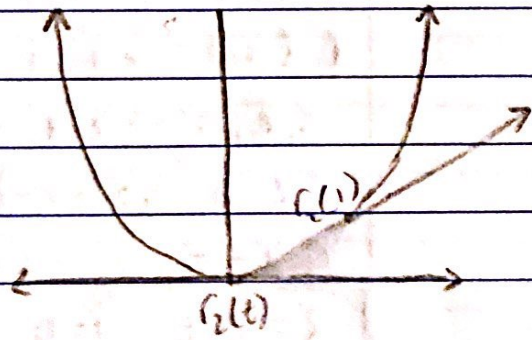
15. $r_1(t) = \langle t, t^2 \rangle$, $t=1$

$r_1'(t) = \langle 1, 2t \rangle = r_1(1) = \langle 1, 2 \rangle$

$r_2(t) = \langle t^3, t^6 \rangle$, $t=1$

$r_2'(t) = \langle 3t^2, 6t^5 \rangle \to \langle 3, 6 \rangle$





31. $r(t) = \langle 1-t^2, 5t, 2t^3 \rangle$, $t=2$

$r(2) = \langle 1-4, 5(2), 2(2)^3 \rangle = \langle -3, 10, 16 \rangle$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

$r'(2) = \langle -4, 5, 24 \rangle$

$\langle -3, 10, 16 \rangle + t \langle -4, 5, 24 \rangle$

$\langle -3, 10, 16 \rangle + \langle -4t, 5t, 24t \rangle$

$\langle -3-4t, 10+5t, 16+24t \rangle$

$\boxed{x = -3-4t,\ y = 10+5t,\ z = 16+24t}$

33. $r(s) = (4s^{-1})i - (\frac{8}{3}s^{-3})k, \quad s = 2$

$r(2) = (\frac{4}{2})i - (\frac{8}{3(8)})k = 2i - \frac{1}{3}k$

$r(2) = \langle 2, 0, -\frac{1}{3} \rangle$

$r'(s) = (-4s^{-2})i + (8s^{-4})k$

$r'(2) = (-4(2)^{-2})i + (8(2)^{-4})k$

$r'(2) = -i + \frac{1}{2}k$

$\langle 2, 0, -\frac{1}{3} \rangle + t\langle -1, 0, \frac{1}{2} \rangle$

$\langle 2, 0, -\frac{1}{3} \rangle + \langle -t, 0, \frac{1}{2}t \rangle$

$\langle 2 - t, 0, -\frac{1}{3} + \frac{1}{2}t \rangle$

$\boxed{x = 2 - t, \ y = 0, \ z = -\frac{1}{3} + \frac{1}{2}t}$

41. $\int_{-2}^{2} (u^3 i + u^5 j)\, du$

$\int_{-2}^{2} (u^3 i)\, du + \int_{-2}^{2} (u^5 j)\, du$

$\left[\frac{u^4}{4}\right]_{-2}^{2} i + \left[\frac{u^6}{6}\right]_{-2}^{2} j$

$\left[\frac{2^4}{4} - \frac{(-2)^4}{4}\right] i + \left[\frac{2^5}{5} - \frac{2^5}{5}\right] j$

$[4 - 4]\, i + \left[\frac{32}{5} - \frac{32}{5}\right] j$

$0i + 0j$

$\boxed{\langle 0, 0 \rangle}$

49. $r'(t) = t^2 i + 5t j + k, \quad r(1) = j + 2k$

$\boxed{r(t) = \frac{t^3}{3} i + \frac{5t^2}{2} j + t k + C}$

$r(1) = \frac{1}{3} i + \frac{5}{2} j + k + C = j + 2k$

$r(1) \Rightarrow \frac{1}{3} i + \frac{5}{2} j - j + k - 2k = -C$

$-\frac{1}{3} i - \frac{5}{2} j + j - k + 2k = C$

$-\frac{1}{3} i - \frac{3}{2} j + k = C$

$r(t) = \frac{t^3}{3} i + \frac{5t^2}{2} + t k - \frac{1}{3} i - \frac{3}{2} j + k$

$\boxed{r(t) = \left(\frac{t^3}{3} - \frac{1}{3}\right) i + \left(\frac{5t^2}{2} - \frac{3}{2}\right) j + (t+1) k}$