~~Lecture 3~~ homework 2

12.3

1.) $\langle 1, 2, 1\rangle \cdot \langle 4, 3, 5\rangle$

$1\cdot 4 + 2\cdot 3 + 1\cdot 5 = 4+6+5 = \boxed{15}$

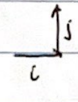
13.) $\langle 1, 1, 1\rangle, \langle 1, -2, -2\rangle$

$1\cdot 1 + 1\cdot -2 + 1\cdot -2 = 1-2-2 = -3 \rightarrow$ not orthogonal

$a\cdot b < 0 \rightarrow$ $\boxed{\text{obtuse!}}$

21.) $i+j, \; j+2K$

$\cos\alpha = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$



$\langle 1, 1, 0\rangle, \langle 0, 1, 2\rangle$

$0+1+0 = 1$

$|\vec{a}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

$|\vec{b}| = \sqrt{0^2+1^2+2^2} = \sqrt{5}$

$|\vec{a}|\cdot|\vec{b}| = \sqrt{10}$ $\quad \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \boxed{\dfrac{1}{\sqrt{10}}}$

29.) a.) $\langle b, 3, 2\rangle, \langle 1, b, 1\rangle$

$b\cdot 1 + 3\cdot b + 2\cdot 1 = b+3b+2 = 4b+2 = \boxed{b = -\frac{1}{2}}$

b.) $\langle 4, -2, 7\rangle, \langle b^2, b, 0\rangle$

$4b^2 + -2b + 7\cdot 0 = 4b^2 - 2b = 0 \rightarrow b^2 - b = 0 \quad b^2 = b \rightarrow \boxed{b=1}$

31.) $\langle 2, 0, 3\rangle$ a

$\langle 0, 0, 0\rangle$ b $\rightarrow \vec{a}\cdot\vec{b} = 2\cdot 0 + 0\cdot 0 + 3\cdot 0 = 0+0+0 = 0$

$\langle -3, 0, 2\rangle$ c $\rightarrow \vec{a}\cdot\vec{c} = 2\cdot -3 + 0\cdot 0 + 3\cdot 2 = -6+0+6 = 0$

57.) $u = 5i+7j-4K, \; v = K$

$\langle 5, 7, -4\rangle \quad \langle 0, 0, 1\rangle$

$u\cdot v = 0+0-4$

$\|v\| = (\sqrt{1})^2 = 1$

$\dfrac{-4}{1}K \rightarrow \boxed{-4K}$

63.) length of $\overline{OP}$

$v: \langle 8, 2\rangle \quad u\cdot v = 24+10 = \dfrac{34}{2} = \sqrt{17}$

$u: \langle 3, 5\rangle \quad 5-3 = 2$

12.4

1.) $\begin{vmatrix} \overset{a}{1} & \overset{b}{2} \\ \overset{c}{4} & \overset{d}{3} \end{vmatrix}$ = det = ad − bc

$1 \cdot 3 - 2 \cdot 4 = \cancel{3 - 8 = -3}$

$= 3 - 8 = \boxed{-5}$

5.) $\begin{vmatrix} \overset{a}{1} & \overset{b}{2} & \overset{c}{1} \\ \overset{d}{4} & \overset{e}{-3} & \overset{f}{0} \\ \overset{g}{1} & \overset{h}{0} & \overset{i}{1} \end{vmatrix}$ det $= a(ei - fh) - b(di - fg) + c(dh - eg)) =$

$1((-3 \cdot 1) - (0 \cdot 0)) - 2((4 \cdot 1) - (0 \cdot 1)) + 1((4 \cdot 0) - (-3 \cdot 1))$

$\cancel{1((-3 - 0) - 2(4 - 0) + 1(0 + 4)) =}$

$\cancel{1(-3) - 2(4) + 1(4) = -3 - 8 + 4 = -11 + 4 = -7}$

$\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix}$ $-3 \cdot 1 - 0 \cdot 0 = -3 - 0 = -3$ $-$ $\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix}$ $4 \cdot 1 - 0 \cdot 1 = 4 - 0 = 4$ $+$ $\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$ $4 \cdot 0 - -3 \cdot 1 = 0 - -3 = 3$

$1(-3) - 2(4) + 1(3)$

$-3 - 8 + 3 = \boxed{-8}$

13.) $(i + j) \times k$

$(v + w) \times u = v \times u + w \times u$

$(i + j) \times k = i \times k + j \times k$

$\langle 1, 0, 0 \rangle \langle 0, 0, 1 \rangle$

$0 - 0,\ 1 - 0,\ 0 - 0 = \langle 0, 1, 0 \rangle = -j$

$\langle 0, 1, 0 \rangle \langle 0, 0, 1 \rangle$

$1 - 0,\ 0 - 0,\ 0 - 0 = \langle 1, 0, 0 \rangle = i$

$\boxed{-j + i}$

21.) $(u - 2v)(u + 2v)$

$u \times v = \langle 1, 1, 0 \rangle$

$u_y v_z - u_z v_y = 1$
$u_x v_z - u_z v_x = 1$
$u_x v_y - u_y v_x = 0$

$\rightarrow u_y v_z - u_z v_y = u_x v_z - u_z v_x$

$u_y v_z - u_z v_y - u_x v_z - u_z v_x = 0$

$u_y v_z - u_z v_y - u_x v_z - u_z v_x = u_x v_y - u_y v_x$

$u_y v_z - u_x v_z - u_z v_y - u_z v_x = u_x v_y - u_y v_x$

$v_z(u_y - u_x) - u_z(v_y - v_x) = u_x v_y - u_y v_x$

25.) $-v$

27.) $v: \langle 3, 0, 0 \rangle$

$w: \langle 0, 1, -1 \rangle$

$0 \cdot 0, \ +3-0, \ +3 \cdot 0$

$\langle 0, 3, 3 \rangle$

39.) $V: \|a \times b\| \|c\| |\cos\theta| = |(a \times b) \cdot c|$

$\overset{u}{\langle 1, 0, 0 \rangle} \quad \overset{v}{\langle 0, 2, 0 \rangle} \quad \overset{w}{\langle 1, 1, 2 \rangle}$

$0-0, \ 0-0, \ 2-0 = \langle 0, 0, 2 \rangle - c$

$\langle 0, 0, 2 \rangle \cdot \langle 1, 1, 2 \rangle$

$\langle 0, 0, 4 \rangle$

$\boxed{V: 4}$

41.) $v: \langle 1, 0, 3 \rangle$

$v: \langle 2, 1, 1 \rangle$

$\sqrt{1^2+0^2+3^2} = \sqrt{1+9} = \sqrt{10}$

$\sqrt{2^2+1^2+1^2} = \sqrt{4+1+1} = \sqrt{6}$

$\sqrt{60}$? $\sin\theta$

$0-3, \ 1-6, \ 1-0 = \langle -3, -5, 1 \rangle$

$\sqrt{3^2+5^2+1^2} = \sqrt{9+25+1} = \boxed{\sqrt{35}}$

43.) $O(0,0,0)$ $\rangle OP = \langle 3, 3, 0 \rangle$

$P(3,3,0)$ $\rangle PQ = \langle -3, 0, 3 \rangle$ $\quad \overline{OP} \times \overline{PQ} = 9-0, \ 9-0, \ 0, -9 = 9, 9, -9$

$Q(0,3,3)$ $\quad \sqrt{9^2+9^2+9^2} = \sqrt{243}$ = parallelogram

$\frac{\sqrt{243}}{2} \approx \boxed{7.79}$

45.) $(1,2)$ $\rangle A = \langle 2, 2 \rangle \to \langle 2, 2, 0 \rangle$

$(3,4)$

$(-2,2)$ $\rangle B = \langle -5, -2 \rangle \to \langle -5, -2, 0 \rangle$ $\quad \vec{A} \times \vec{B} = 0-0, \ 0-0, \ -4+10 = \frac{6}{2} = \boxed{3}$

12.5

1.) $n: \langle 1,3,2\rangle$

$(4,-1,1)$

$\sqrt{16+1+1} = \sqrt{\frac{18}{2}} = 3$

$x+3y+2z=3$

5.) $n: i \to (1,0,0)$

$(3,1,-9)$

$\langle 3,0,0\rangle \to x=3$

9.) $x=0$

11.) b d

13.) $9x-4y-11z=2$

$(9,-4,-11)$

15.) $3(x-4)-8(y-1)+11z=0$

$\langle 3,-8,11\rangle$

17.) $P(2,-1,4)$, $\overrightarrow{PQ} = \langle -1,2,-3\rangle$ $\overrightarrow{PQ}\times\overrightarrow{QR} = \langle 2+0,-1+6,0-4\rangle$

$Q(1,1,1)$, $\overrightarrow{QR} = \langle 2,0,1\rangle$ $\langle 2,5,-4\rangle$

$R(3,1,2)$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$2(x-1)+5(y-1)+4(z-1)=0$

$2x-2+5y-5+4z+4=0$

$6x-9y+4z=19$

19.) $P(1,0,0)$, $\overrightarrow{PQ} = \langle -1,1,1\rangle$ $= \overrightarrow{PQ}\times\overrightarrow{QR} = \langle 0+1,0-2,1-2\rangle = \langle 1,-2,-1\rangle$

$Q(0,1,1)$, $\overrightarrow{QR} = \langle 2,-1,0\rangle$

$R(2,0,1)$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$1(x-1)+2(y-0)-1(z-0)=0$

$x-1+2y-z=0$

$\boxed{x+2y-z=1}$

25.) $(-2,-3,5)$

$\langle 1, 0, 1 \rangle$

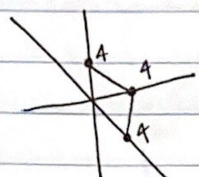
$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$1(x-2)+1(z-5)=0$

$x-2+z-5=0$

$\boxed{x+z=3}$

31.) $x+y+z=4$



53.) $3x+2z=5$

~~[illegible]~~

$(3\lambda)x+by+(2\lambda)z=(5\lambda)$

13.1

1.) $r(t)=e^{t}i+\frac{1}{t}j+(t+1)^{-3}k$

$D= t\neq 1,\ t\neq 0$ ignore

5.) $P(3,-5,7)$

$v=\langle 3,0,1\rangle$ $\boxed{(3+3t)i-5j-(7+t)k}$

$\cancel{3(x-3)+0(x+5)+1(x-7)=0}$

$\cancel{3x-9+x-7=0}$

$\cancel{4x=16}$

$\cancel{x=4}$

15.) A → ii

B → iii

C → i

17.) $r(t)=(9\cos t)i+(9\sin t)j$

center @ origin in x-y plane, radius = 9

13.2

3.) $\lim_{t\to 0} e^{2t} i + \cos t j + \tan 4t k = \boxed{i + 4t}$

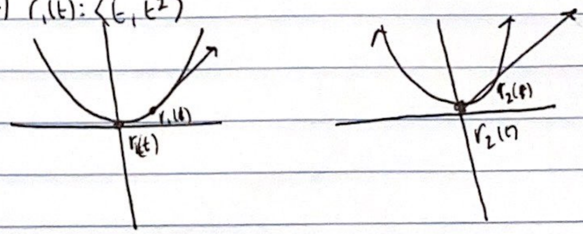
5.) $\lim_{h\to 0} \frac{r(t+h) - r(t)}{h}$ for $r(t) = \langle \frac{1}{t}, \sin t, 4 \rangle$:

$\langle \frac{-1}{t^2}, \cos t, 0 \rangle$

7.) $r(t) = \langle t, t^2, t^3 \rangle$

$\frac{dr}{dt} = \langle 1, 2t, 3t^2 \rangle$

15.) $r_1(t) = \langle t, t^2 \rangle$



31.) $r(t) = \langle 1 - t^2, 5t, 2t^3 \rangle$, $t = 2$

$1 - (2)^2, 5(2), 2(2)^3 = 1 - 4, 10, 2(8)$

$(-3, 10, 16)$

$l(t) = \langle -3 - 4t, 10 + 5t, 16 + 24t \rangle$

33.) $r(s) = 4s^{-1} i - \frac{8}{3} s^{-3} k$, $s = 2$

$4(2)^{-1} i, \frac{8}{3}(2)^{-3} k$

$4(\frac{1}{2})$ j $\cancel{\frac{8}{3}(\frac{1}{8})}$

$\cancel{1} \frac{1}{3}$ $2, 0, -\frac{1}{3}$

$l(t) = \langle 2 - t, 0, -\frac{1}{3} + \frac{1}{2} t \rangle$

41.) $\int_{-2}^{2} (u^3 i + u^5 j) \, du$

$\frac{u^4}{4} + \frac{u^6}{6}$ $\langle u^3, u^5 \rangle$

$\frac{16}{4} + \frac{64}{6}$ $(0, 0)$

49.) $r'(t) = t^2 i + 5t j + k$, $r(1) = j + 2k$

$\left(\frac{t^3}{3}\right) i + \left(\frac{5t}{2}\right) j + tk + c$

w/ i.c. $r(t) = \left(\frac{1}{3} t^3 - \frac{1}{3}\right) i + \left(\frac{5}{2} t^2 - \frac{3}{2}\right) j + (t + 1) k$