12.3

1. $\langle 1,2,1\rangle * \langle 4,3,5\rangle = 1*4+2*3+1*5 = 15$

13. Because $\langle 1,1,1\rangle * \langle 1,-2,-2\rangle = 1*1+1*(-2)+1*(-2) = -3$ is negative, the angle between two vectors is obtuse.

21. $u = i + j + 0k$ and $v = 0i + j + 2k$, $cos\langle u,v\rangle = \frac{u*v}{|u|*|v|} = \frac{1}{\sqrt{2}*\sqrt{5}} = \frac{\sqrt{10}}{10}$

29. (a) If these two vectors are orthogonal, $\langle b,3,2\rangle * \langle 1,b,1\rangle = 0 \to b + 3*b+2 = 0$. And $b = -\frac{1}{2}$.

(b) $\langle 4,-2,7\rangle * \langle b^2,b,0\rangle = 4*b^2 - 2*b = 0$. We can solve that $b = \frac{1}{2}$ or 0.

31. $\langle 0,1,0\rangle$ and $\langle 3,0,2\rangle$.

57. $u = 5i + 7j - 4k$ and $v = k$ $V = \frac{u*v}{|v|^2} * v = -4k$

63. $\overrightarrow{OP}$ is $projection$ of u $along$ v, $\overrightarrow{OP} = \frac{u*v}{|v|^2} * v = \langle 4,1\rangle$.

12.4

1. $det\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix} = 1*3-2*4 = -5$

5. $det\begin{matrix}1 & 2 & 1\\ 4 & -3 & 0\\ 1 & 0 & 1\end{matrix} = \det\left(1*\begin{bmatrix}-3 & 0\\ 0 & 1\end{bmatrix}\right) - \det\left(2*\begin{bmatrix}4 & 0\\ 1 & 1\end{bmatrix}\right) + \det\left(1*\begin{bmatrix}4 & -3\\ 1 & 0\end{bmatrix}\right) = (-3)-(2*4)+3 = -8$

13. $(i+j)\times k = i\times k + j\times k$

21. $(u-2v)\times(u+2v) = u\times(u+2v) - 2v\times(u+2v) = u\times u + 2(u\times v) - 2(v\times u) - 4(v\times v) = 0 + 4*\langle 1,1,0\rangle - 0 = \langle 4,4,0\rangle$

25. According to right-hand rule, $v \times w = -u$.

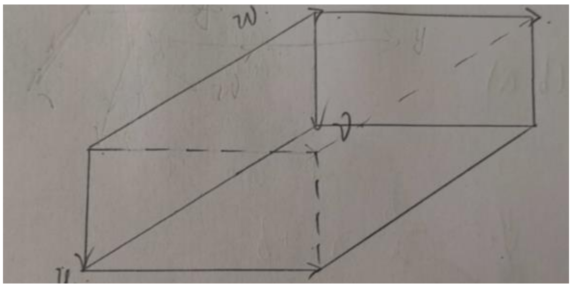
27. $v = (3,0,0)$ *and* $w = (0,1,-1)$

we assume that $u = (x, y, z)$ *and we can get* $\begin{cases} 3 * x = 0 \\ y - z = 0 \end{cases}$. *So*, $u = (0, a, -a)$.

We can get $\sin\langle v, w\rangle = 1$ *from* $\cos\langle v, w\rangle = \frac{v*w}{|v||w|} = 0$

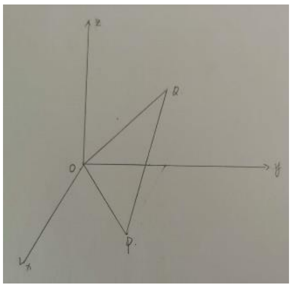
$|v \times w| = |v| * |w| * \sin\langle v, w\rangle = 3 * \sqrt{2}$,

so, $a = 3$ *and* $u = (0,3,-3)$.

39. 

Volume of this parallelepiped is $w * (u \times v) = 4$.

41. The area of the parallelogram is equal to $|u \times v| = det\begin{matrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{matrix} = \det\left(i * \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix}\right) - \det\left(j * \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}\right) + \det\left(k * \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\right) = |-3 * i + 5 * j + k| = \sqrt{35}$

43. 

The area of triangle $\overline{OPQ}$ is half of area of the parallelogram spanned by O, P, Q, which equal to $\overrightarrow{OQ} \times \overrightarrow{OP}$. So, the area of triangle is $\frac{1}{2}\left(\overrightarrow{OQ} \times \overrightarrow{OP}\right) = \frac{1}{2}\left(det\begin{matrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{matrix}\right) = \frac{9\sqrt{3}}{2}$.

45. We assume a= (1,2), b= (3,4), c= (-2,2).

$\overrightarrow{ab} = (2,2)\ and\ \overrightarrow{ac} = (-3,0)$. And the area is $\frac{|\overrightarrow{ab} \times \overrightarrow{ac}|}{2} = \frac{det\begin{bmatrix} 2 & 2 \\ -3 & 0 \end{bmatrix}}{2} = 3.$