12.3.

Q1. $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$

$= 1\times 4 + 2\times 3 + 1\times 5$

$= 15.$

Q13. $\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle$

$= 1\times 1 + 1\times(-2) + 1\times(-2)$

$= -3 \neq 0$

$\therefore$ They're not orthogonal.

$\cos\theta = \dfrac{-3}{\sqrt{1+1+1}\cdot\sqrt{1^2+(-2)^2+(-2)^2}}$

$= \dfrac{-3}{3\sqrt{3}}$

$= -\dfrac{1}{\sqrt{3}} < 0.$

$\therefore$ It's obtuse.

Q21. Assume they're $\langle 1,1,0\rangle$ and $\langle 0,1,2\rangle$

$\cos\theta = \dfrac{\langle 1,1,0\rangle\cdot\langle 0,1,2\rangle}{\sqrt{1^2+1^2}+\sqrt{1^2+2^2}}$

$= \dfrac{1+1}{\sqrt{10}}$

$= \dfrac{\sqrt{10}}{5}$

Q29 (a) $b+3b+2=0$

$4b=-2$

$b=-\frac{1}{2}$

(b) $\cancel{4b^2-2b+7=0}$

$\cancel{b=\frac{2\pm\sqrt{4-}}{8}}$

$4b^2-2b=0.$

$2b(2b-1)=0.$

$\begin{cases} b_1=0 \\ b_2=\frac{1}{2} \end{cases}$

Q31. Assume the vectors are

$A=\langle a_1, b_1, c_1\rangle$ and

$B=\langle a_2, b_2, c_2\rangle$

$2a_1-3c_1=0$

$2a_2-3c_2=0.$

$\because A\neq kB,\ k\in R$

$\therefore \frac{a_1}{b_1}\neq\frac{a_2}{b_2}$

We can make $a_1=a_2=3$ and $c_1=c_2=2$

$\therefore b_1\neq b_2;\ b_1, b_2\in R$

$A=\langle 3, 114514, 2\rangle$

$B=\langle 3, 1919810, 2\rangle.$

Q57

Assume the projection is w.

$$\cancel{w = \left|\frac{u\cdot v}{|v|}\right| = \left|\frac{\langle 5,7,-4\rangle\cdot\langle 0,0,1\rangle}{1}\right| =}$$

$$w = \frac{u\cdot v}{|v|^2}v = \frac{\langle 5,7,-4\rangle\cdot\langle 0,0,1\rangle}{1^2}\langle 0,0,1\rangle$$

$$= -4\langle 0,0,1\rangle$$

$$= \langle 0,0,-4\rangle.$$

Q63.

$$\overline{OP} = \left|\frac{u\cdot v}{|v|}\right| = \left|\frac{\langle 3,5\rangle\cdot\langle 8,2\rangle}{\sqrt{8^2+2^2}}\right|$$

$$= \frac{24+10}{2\sqrt{17}}$$

$$= \sqrt{17}.$$

12.4.

Q1. $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 1\times 3 - 2\times 4 = -5.$

Q5. $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$

$$= -3 - 2\times 4 + 3$$

$$= -8$$

Q13 $(i+j)\times k = i\times k + j\times k$

Q21.

$$(u-2v)\times(u+2v)$$

$$= (u-2v)\times u + (u-2v)\times 2v$$

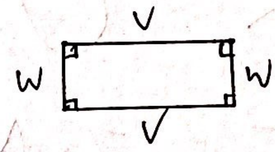
$$= u^2 - 2vu + 2vu - 4v^2$$

$$= u^2 - 4v^2.$$

Q25.

Based on the right-handed system. From v to w is ~~co~~ clockwise. So it's $-u$.

Q27



$\cancel{\cos\theta =}$ $v\cdot w = 0.$

$\therefore$ They're orthogonal.

$u = |v|\cdot|w| = 3\sqrt{2}$

~~Q39~~ $\cancel{u\times v\times w =}$

$$\cancel{|u\times v\times w|}$$

$$\cancel{= |\langle 0,0,2\rangle\times\langle 1,1,2\rangle|}$$

$$\cancel{= |\langle -2,2,0\rangle|}$$

$$\cancel{= \sqrt{2^2+(-2)^2}}$$

$$\cancel{= 2\sqrt{2}}$$

Q39 $(u\times v)\cdot w$

$$= \langle 0,0,2\rangle\cdot\langle 1,1,2\rangle$$

$$= 4$$

Q41

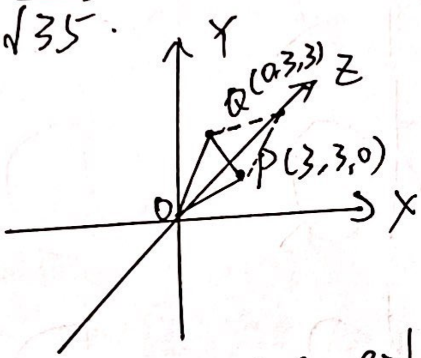
$|u \times v| = |\langle 1,0,3\rangle \times \langle 2,1,1\rangle|$

$= |\langle -3,5,1\rangle|$

$= \sqrt{(-3)^2+5^2+1^2}$

$= \sqrt{35}$.

Q43



$S = \frac{|Q \times P|}{2} = \frac{|\langle -9,9,-9\rangle|}{2}$

$= \frac{\sqrt{9^2+9^2+9^2}}{2}$

$= \frac{9\sqrt{3}}{2}$

Q45. Assume the three points are A, B, C.

$\overrightarrow{AB} = \cancel{\langle 2,2\rangle} \langle 2,2,0\rangle$

$\overrightarrow{BC} = \cancel{\langle -5,-2\rangle} \langle -5,-2,0\rangle$

$\cancel{\overrightarrow{CA} = \langle 3,0\rangle}$.

$S = \frac{|\overrightarrow{AB} \times \overrightarrow{BC}|}{2} = \frac{2\times(-2)+2\times 5}{2} = 3$

Q5.

$\cancel{\langle 1,0,0\rangle \times}$

$\langle 1,0,0\rangle \cdot \langle x-3, y-1, z+9\rangle = 0$

$(x-3) = 0$.

$x = 3$.

Q9 Assume $n = \langle a, b, c\rangle$

$\langle a,b,c\rangle \cdot \langle x,y,z\rangle = 0$.

$ax+by+cz=0$.

Q11. B and C.

Q13 $\langle 9,-4,-11\rangle$

Q15 $\langle 3,-8,11\rangle$.

Q17 $\overrightarrow{PQ} = \langle -1,2,-3\rangle$

$\overrightarrow{QR} = \langle 2,0,-3\rangle$

$n = \overrightarrow{PQ} \times \overrightarrow{QR} = \langle -6, \cancel{3}\,-9, -4\rangle$

$-6x-9y-4z=k$

pass $(2,-1,4)$

$(-6)\times 2+9-4\times 4=k$

$k=-19$

$\therefore$ the equation is $6x+9y+4z=19$

12.5.

$\langle x-4, y+1, z-1\rangle = 0$.

Q1. $\langle 1,3,2\rangle \cdot \langle x-4, y+1, z-1\rangle = 0$.

$(x-4)+3(y+1)+2(z-1)=0$.

Q19

$\overrightarrow{PQ} = \langle -1, 1, 1 \rangle$

$\overrightarrow{QR} = \langle 2, -1, 0 \rangle$

$n = \overrightarrow{PQ} \times \overrightarrow{QR} = \langle 1, 2, -1 \rangle$

$x + 2y - z = k$

pass $(1,0,0)$

$k = 1$

$\therefore$ the equation is $x + 2y - z = 1$

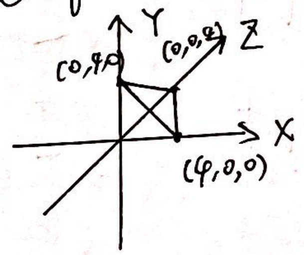
Q25.

$x + z = k$

$k = -2 + 5 = 3.$

$\therefore$ the equation $x + z = 3$

Q31



Q53 $\begin{cases} y = 0 \\ 3x + 2z - 5 = 0 \end{cases}$

$Ay + B(3x + 2z - 5) = 0$

$(A, B \in R)$

13.1

Q5 $\cdot \langle 3+3t, -5, 7+t \rangle.$

Q17 $r = 9$

center: $(0,0)$

plane: $z = 0.$

13.2.

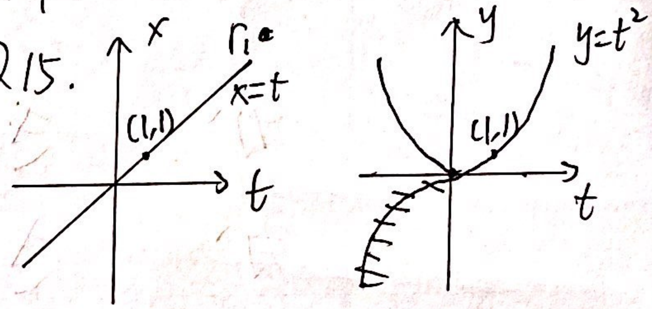
$\ln(1)j$

Q3. $e^0 i + \cancel{\ln(0)j} + 4k = \langle 1, 0, 4 \rangle$

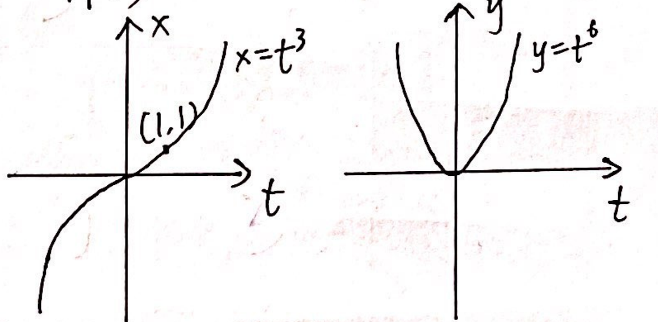
Q5. $r'(t) = \langle -t^{-2}, \cos t, 0 \rangle.$

Q7 $r'(t) = \langle 1, 2t, 3t^2 \rangle$

Q15.



$r_1'(1) = \langle 1, 2\times 1 \rangle = \langle 1, 2 \rangle.$



$r_2'(1) = \langle 3\times 1^2, 6\times 1^5 \rangle = \langle 3, 6 \rangle$

Q31 $\cancel{r'(2)} = r(2) = \langle -3, 10, 16 \rangle$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

$r'(2) = \langle -4, 5, 24 \rangle.$

$\therefore$ the tangent line is $\cancel{\langle -3-4t, 10+5t, 16+24t \rangle}$

$\langle -3-4t, 10+5t, 16+24t \rangle$

Q33

$r(s) = \langle 4s^{-1}, 0, -\frac{8}{3}s^{-3} \rangle$

$r'(s) = \langle -4s^{-2}, 0, 8s^{-4} \rangle$

$r'(2) = \langle -1, 0, \frac{1}{2} \rangle$.

$r(2) = \langle 2, 0, -\frac{1}{3} \rangle$.

$\therefore$ the tangent line is $\langle 2-t, 0, -\frac{1}{3}+\frac{1}{2}t \rangle$

Q41. $\left[\frac{u^4}{4} i\right]_{-2}^{2} + \left[\frac{u^6}{6} j\right]_{-2}^{2}$

$= 4i - 4i + \frac{32}{3} j - \frac{32}{3} j$

$= 0$.

Q49

$r(t) = \frac{t^3}{3} i + \frac{5t^2}{2} j + tk + C$.

$r(1) = \frac{1}{3} i + \frac{5}{2} j + k + C$

$= j + 2k$

$C = -\frac{1}{3} i - \frac{3}{2} j + k$

$r(t) = \frac{t^3-1}{3} i + \frac{5t^2-3}{2} j + (t+1) k$