12.3 : 1,13,21,29,31,52,57,63

Krithika Patrachari

1) $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$

$= 1(4)+2(3)+1(5)$

$= 4+6+5 = 15$

13) orthogonal? if no, angle?

$\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle$

$= 1(1)+1(-2)+1(-2)$

$= 1-2-2 = -3$

No, the vectors are not orthogonal

$\cos\theta = \dfrac{A\cdot B}{|A||B|}$

$|A| = \sqrt{1^2+1^2+1^2} = 1$

$|B| = \sqrt{1^2+(-2)^2+(-2)^2} = 3$

$\cos\theta = \dfrac{-3}{3} = -1$

$\cos^{-1}(-1) = \theta = \pi$

obtuse

21) find $\cos\theta$ between vectors

$i+j \quad , \quad j+2k \qquad \cos\theta = \dfrac{A\cdot B}{|A||B|}$

$\downarrow \qquad \downarrow$

$\langle 1,1,0\rangle \cdot \langle 0,1,2\rangle = 1$

$\cos\theta = \dfrac{\langle 1,1,0\rangle \cdot \langle 0,1,2\rangle}{\sqrt{2}\,(\sqrt{5})} = \dfrac{1}{\sqrt{10}}$

$|A| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

$|B| = \sqrt{0^2+1^2+2^2} = \sqrt{5}$

29) find all values of b for which the vectors are orthogonal

a. $\langle b,3,2\rangle \cdot \langle 1,b,1\rangle = 0$

$b+3b+2 = 0$

$4b = -2 \qquad b = -\frac{1}{2}$

b. $\langle 4,-2,7\rangle \cdot \langle b^2,b,0\rangle = 0$

$4b^2 - 2b + 0 = 0$

$2b(2b-1) = 0 \qquad b=0, \; b=\frac{1}{2}$

31) Find 2 vectors orthogonal to $\langle 2,0,-3\rangle$

$\langle a,0,2\rangle \cdot \langle 2,0,-3\rangle = 0$

$2a+0-6 = 0$

$2a = 6 \quad a = 3$

$\langle 6,0,2\rangle$

$\langle 4,0,b\rangle \cdot \langle 2,0,-3\rangle = 0$

$8+0-3b = 0$

$-3b = -8 \qquad b = \frac{8}{3}$

$\langle 4,0,\frac{8}{3}\rangle$

52) Let u and v be 2 nonzero vectors

a. Yes, it is possible for the component of u along v to have the opposite sign from the component of v and u because the vector v determines the direction in one instance and u determines in the other.

b. If either of the components were 0 then the vectors would be parallel or anti-parallel.

57) Find the projection of u along v

$u = 5i+7j-4k \quad , \quad v = k$

$\langle 5,7,-4\rangle \qquad \langle 0,0,1\rangle$

$\text{comp}_v u = \dfrac{u\cdot v}{|v|} = \dfrac{\langle 5,7,-4\rangle \cdot \langle 0,0,1\rangle}{\sqrt{0^2+0^2+1^2}}$

$= \dfrac{-4}{1} = -4k$

63)

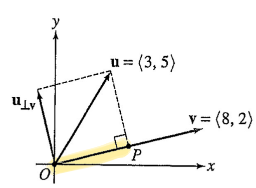


FIGURE 15

Find the length of $\overline{OP}$

$|u| = \sqrt{9+25} = \sqrt{36} = 6$

$u_{\perp v} = \dfrac{u\cdot v}{|v|} = \dfrac{24+10}{\sqrt{68}} = \dfrac{34}{\sqrt{68}}$

$OP = \sqrt{6^2 - \left(\frac{34}{\sqrt{68}}\right)^2}$

$= \sqrt{36-17} = \sqrt{19}$

12.4 : 1, 5, 13, 21, 25, 27, 39, 41, 43, 45

1) $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 1(3) - 2(4)$
$= 3 - 8 = -5$

5) $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1(-3(1) - 0(0)) - 2(4(1) - 0(1))$
$+ 1(4(0) - (-3)(1))$
$= -3 - 8 + 3 = -8$

13) $(i + j) \times k$

$\langle 1, 1, 0 \rangle \times \langle 0, 0, 1 \rangle$

$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = i(1-0) - j(1-0)$
$+ k(0-0)$
$= i - j$

21) $(u - 2v) \times (u + 2v)$?

$u \times v = \langle 1, 1, 0 \rangle$
$u \times w = \langle 0, 3, 1 \rangle$
$v \times w = \langle 2, -1, 1 \rangle$

$w \times u + w \times v =$
$-u \times w + -v \times w =$
$-\langle 0, 3, 1 \rangle - \langle 2, -1, 1 \rangle =$
$\langle 0, -3, -1 \rangle - \langle 2, -1, 1 \rangle$
$= \langle -2, -2, -2 \rangle$

$4x - 9y + z = 0$

25)

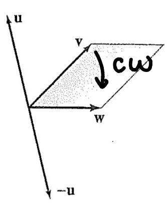


FIGURE 17

which of u and $-u$ is equal to $v \times w$?

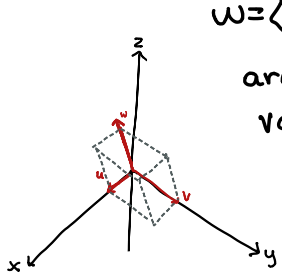
$-u$

27) $v = \langle 3, 0, 0 \rangle$ $w = \langle 0, 1, -1 \rangle$

$u = v \times w$

$\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = i(0-0) - j(-3-0)$
$+ k(3-0)$
$= \langle 0, 3, 3 \rangle$

39) $u = \langle 1, 0, 0 \rangle$, $v = \langle 0, 2, 0 \rangle$,
$w = \langle 1, 1, 2 \rangle$



area $= |u||v|\sin\theta = |u \times v|$

volume = area • h

$= \sqrt{6} \cdot |w| \cos\theta$ $\quad |w| = \sqrt{6}$

$= \sqrt{6} \cdot \sqrt{6} \cos\left(\frac{\sqrt{5}}{3}\right)$

$= \sqrt{6} \cdot \frac{2}{3}\sqrt{6} = 4$

41) calculate area of parallelogram spanned by $u = \langle 1, 0, 3 \rangle$ $v = \langle 2, 1, 1 \rangle$

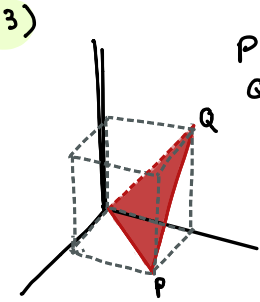
area $= |u||v|\sin\theta = |u \times v|$

$w = u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = i(0-3) - j(1-6) + k(1-0)$

$\langle -3, -5, 1 \rangle$

$|w| = \sqrt{9 + 25 + 1} = \sqrt{35}$

43)



$P = (3, 3, 0)$ $\quad |P \times Q|/2 = $ area
$Q = (0, 3, 3)$

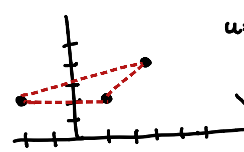
$P \times Q = \begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix}$

$= i(9-0) - j(9-0) + k(9-0)$

$\langle 9, 9, 9 \rangle$ $\sqrt{243}$

$= \frac{9\sqrt{3}}{2}$

45) Find area of Δ in xy-plane defined by $(1,2), (3,4), (-2,2)$



$u = PQ = \langle 3-1, 4-2 \rangle$
$= \langle 2, 2 \rangle$

$v = PR = \langle -2-3, 2-4 \rangle$
$= \langle -5, -2 \rangle$

$A = \frac{|u \times v|}{2}$ $\quad \begin{vmatrix} 2 & 2 \\ -5 & -2 \end{vmatrix} = -4 - (-10)$
$= \frac{6}{2} = 3$

12.5 : 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

1) equation of plane passing through $(4,-1,1)$ and normal to $n=\langle 1,3,2\rangle$

$1(x-4)+3(y+1)+2(z-1)=0$

$x-4+3y+3+2z-2=0$

$x+3y+2z=3$

5) equation of plane passing through $(3,1,-9)$ and normal to $n=i$

$\langle 1,0,0\rangle$

$1(x-3)+0(y-1)+0(z+9)=0$

$x-3=0 \rightarrow x=3$

9) equation of any plane through origin

$(0,0,0)$

$y=0$

11) plane parallel to the yz-plane

$\langle 0,1,1\rangle$

b) $n=\langle 1,0,0\rangle$ is a normal vector

d) equation form: $x=d$

13) $9x-4y-11z=2 \rightarrow$ find vector normal to this

$\langle 9,-4,-11\rangle$

15) $3(x-4)-8(y-1)+11z=0 \rightarrow$ find vector normal to this

$3x-12-8y+8+11z=0$

$3x-8y+11z=4$

$\langle 3,-8,11\rangle$

17) equation of plane passing through 3 points:

$P=(2,-1,4),\ Q=(1,1,1),\ R=(3,1,-2)$

$u = PQ=\langle 1-2,1+1,1-4\rangle=\langle -1,2,-3\rangle$

$v = PR=\langle 3-2,1+1,-2-4\rangle=\langle 1,2,-6\rangle$

$$u\times v=\begin{vmatrix} i & j & k\\ -1 & 2 & -3\\ 1 & 2 & -6\end{vmatrix}=i(-12-(-6))-j(6-(-3))+k(-2-2)$$

$=-6i-9j-4k$

$\langle -6,-9,-4\rangle$

$-6(x-1)+-9(y-1)+-4(z-1)=0$

$-6x+6-9y+9-4z+4=0$

$-6x-9y-4z=-19$

19) equation of plane passing through:

$P=(1,0,0),\ Q=(0,1,1),\ R=(2,0,1)$

$u = PQ=\langle 0-1,1-0,1-0\rangle=\langle -1,1,1\rangle$

$v = PR=\langle 2-1,0-0,1-0\rangle=\langle 1,0,1\rangle$

$$u\times v=\begin{vmatrix} i & j & k\\ -1 & 1 & 1\\ 1 & 0 & 1\end{vmatrix}=i(1-0)-j(-1-1)+k(0-1)$$

$=i+2j-k \rightarrow \langle 1,2,-1\rangle$

$1(x-1)+2(y-0)-1(z-0)=0$

$x-1+2y-z=0$

$x-2y-z=1$

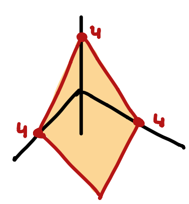
25) equation of plane that passes through $(-2,-3,5)$ and has a normal vector $i+k$

$\langle 1,0,1\rangle$

$1(x+2)+0(y+3)+1(z-5)=0$

$x+2+z-5=0 \quad x+z=3$

31) $x+y+z=4 \rightarrow \langle 1,1,1\rangle$ Radius: 4



53) Find all planes in R^3 whose intersection with the xz-plane is the line with equation $3x+2z=5$

$\langle 1,0,1\rangle$

?

$\langle 3,0,2\rangle$

$3tx+by-2tz=5t$

13.1 : 5, 17

5) Find vector parametrization of the line through $P=(3,-5,7)$ the direction $v=\langle 3,0,1\rangle$

$r(t)=\langle 3,-5,7\rangle + t\langle 3,0,1\rangle$

$r(t)=(3+3t)i-5j+(7+t)k$

17) Find radius, center, plane

$r(t)=(9\cos t)i+(9\sin t)j$

$r(t)=\langle 0,0,0\rangle+9\langle \cos t,\sin t,0\rangle$

radius = 9
center = $\langle 0,0,0\rangle$
plane = xy-plane

13.2 : 3, 5, 7, 15, 31, 33, 41, 49

3) $\lim_{t\to 0} e^{2t}i+\ln(t+1)j+4k$

$=e^0 i+\ln(1)j+4k$

$=i+4k$

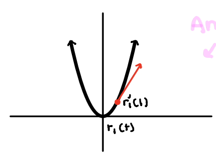
5) $\lim_{h\to 0}\frac{r(t+h)-r(t)}{h}$ for $r(t)=\langle t^{-1},\sin t,4\rangle$

$r'(t)=\langle -t^{-2},\cos t,0\rangle$

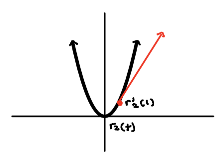
7) $r(t)=\langle t,t^2,t^3\rangle$

$r'(t)=\langle 1,2t,3t^2\rangle$

15) sketch curve : $r_1(t)=\langle t,t^2\rangle$ with its tangent vector at $t=1$; do same for $r_2(t)=\langle t^3,t^6\rangle$



Answers



31) find a parametrization of tangent line

$r(t)=\langle 1-t^2,5t,2t^3\rangle$, $t=2$

$r(2)=\langle -3,10,16\rangle$

$r'(t)=\langle -2t,5,6t^2\rangle$ $\ell(t)=\langle -3-4t,10+5t,16+24t\rangle$

$r'(2)=\langle -4,5,24\rangle$

33) find a parametrization of tangent line

$r(s)=4s^{-1}i-\frac{8}{3}s^{-3}k$, $s=2$

$r(2)=\langle 2,0,-\frac{1}{3}\rangle$ $\ell(t)=\langle 2-t,0,-\frac{1}{3}+\frac{1}{2}t\rangle$

$r'(s)=\langle -4s^{-2},0,8s^{-4}\rangle$

$r'(2)=\langle -1,0,\frac{1}{2}\rangle$

41) $\int_{-2}^{2}(u^3i+u^5j)d$

$\langle 0,0\rangle$

$3u^2i+5u^4j\Big|_{-2}^{2}$

$(3(2)^2i+5(2)^4j)-(3(-2)^2i+5(-2)^4j)$

$(12i+80j)-(12i+80j)=0i+0j$

49) Find both the general solution of the differential equation and the solution with the given initial condition

$r'(t)=t^2i+5tj+k$, $r(1)=j+2k$

$\int r'(t)=r(t)=\frac{1}{3}t^3i+\frac{5}{2}t^2j+tk+C$

$r(1)=\frac{1}{3}i+\frac{5}{2}j+k+C=j+2k \rightarrow C=-\frac{1}{3}i-\frac{3}{2}j+k$

$r(t)=\frac{1}{3}t^3i+\frac{5}{2}t^2j+tk-\frac{1}{3}i-\frac{3}{2}j+k=(\frac{1}{3}t^3-\frac{1}{3})i+(\frac{5}{2}t^2-\frac{3}{2})j+(t+1)k$