12.3 Homework

#1, 13, 21, 29, 31, 57, 63

1) $\langle 1,2,1 \rangle \cdot \langle 4,3,5 \rangle$

$4 + 6 + 5$

$= 15$

13) $\langle 1,1,1 \rangle \cdot \langle 1,-2,-2 \rangle$

$1 - 2 - 2$

$= -3$

$$\frac{-3}{\sqrt{3}\sqrt{9}} = \frac{-1}{\sqrt{3}} = \cos\theta$$

$\theta = 125°$

Obtuse

21) $i + j, \quad j + 2k$

$\langle 1,1,0 \rangle \quad \langle 0,1,2 \rangle$

$$\frac{0 + 1 + 0}{\sqrt{2}\ \sqrt{5}} = \cos\theta$$

$$\frac{1}{\sqrt{2}\sqrt{5}} = \cos\theta$$

29) a) $\langle b, 3, 2\rangle \quad \langle 1, b, 1\rangle$

$b + 3b + 2 = 0$

$4b = -2$

$b = -\frac{1}{2}$

b) $\langle 4, -2, 7\rangle, \langle b^2, b, 0\rangle$

$4b^2 - 2b = 0$

$b(b-1 \quad b^2 - b = 0$

$b = 1$ or $b = 0$.

31) $\langle 2, 0, -3\rangle \cdot \langle a, b, c\rangle = 0$

$2a - 3c = 0$

$a = \frac{3c}{2}$

$\langle 3, 0, 2\rangle$ or $\langle 3, 10, 2\rangle$

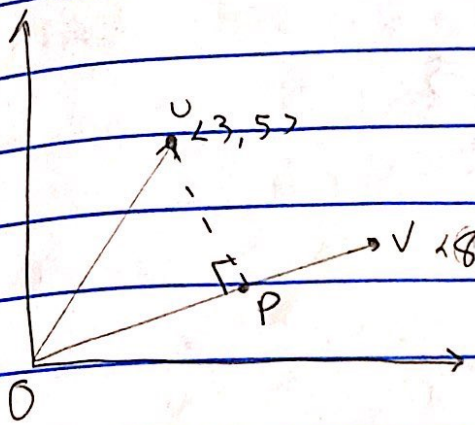
57) $v = 5i + 7j - 4k \quad v = k \quad \langle 5, 7, -4\rangle \quad \langle 0, 1, 0\rangle$

Project v onto v.

$$\frac{v \cdot v}{|v|^2} v \qquad \frac{7}{1} \langle 0, 1, 0\rangle$$

$= \langle 0, 7, 0\rangle$

63)



$\langle 3,5\rangle \cdot \langle 8,2\rangle = 24+10$

$\frac{34}{(\sqrt{64+4})^2} \cdot \langle 8,2\rangle$

$\frac{34}{68} \langle 3,5\rangle$

$\frac{34}{68} = \frac{1}{2}$

$\frac{1}{2}\langle 8,2\rangle$

$\langle 4,1\rangle$

$16+1$

$= \sqrt{17}$ is the magnitude

9/10/20 12.4 Homework

1, 5, 13, 21, 25, 27, 39, 41, 43, 45

1) $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$ det = $3 - 8 = -5$

5) $\begin{pmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

det = $1(-3) - 2(4) + 1(3)$

$-3 - 8 + 3$

$-8 = -8$

13) $(i+j) \times k$

$i \times k + j \times k$

$\langle 1,0,0 \rangle \times \langle 0,0,1 \rangle + \langle 0,1,0 \rangle \times \langle 0,0,1 \rangle$

$\begin{matrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$ $\begin{matrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

$-j \quad +i \qquad i$

21) $(v - 2w) \times (v + 2w)$

$v \times (v+2w) + -2w(v+2w)$

$(v \times v + v \times 2w - 2w \times v - 2w \times 2w)$

$0 + \langle 2,2,0 \rangle + \langle 2,2,0 \rangle$

$= \langle 4,4,0 \rangle$

$v \times v = \langle 1,1,0 \rangle$

$v \times w = \langle 0,3,1 \rangle$

$v \times w = \langle 2,-1,1 \rangle$

25) -v (right hand rule)

39) ~~60~~ $u = \langle 1, 0, 0 \rangle$

$v = \langle 0, 2, 0 \rangle$

$w = \langle 0, -4, 0 \rangle$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 2 \end{pmatrix}$$

$1(4) - 0 + 0$

4

41) $u = \langle 1, 0, 3 \rangle$ $v = \langle 0, 0, 4 \rangle$.

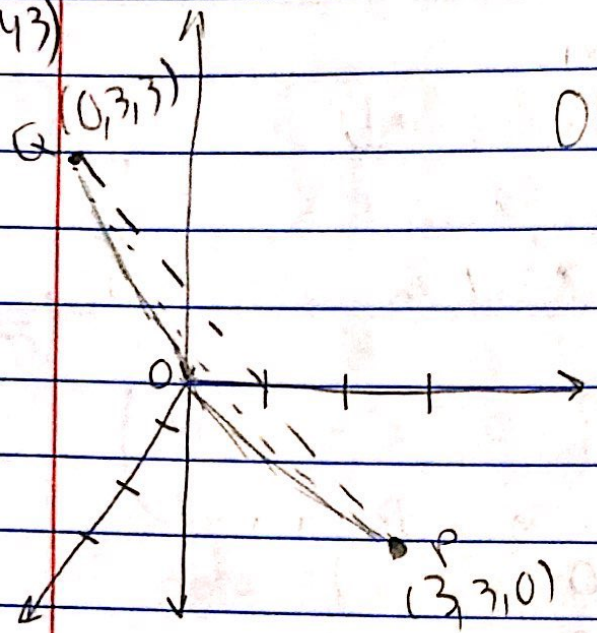
$$\begin{matrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{matrix}$$

$-3i + 5j - k$

$\|u \times v\| = \|-3, 5, -1\| =$

$= \sqrt{35}$

43)



$\overrightarrow{OP} = \langle 3,3,0 \rangle$

$\overrightarrow{OQ} = \langle 0,3,3 \rangle$

Area of triangle is $\frac{1}{2}$ of parallelogram

$$\frac{1}{2}\left\| \begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} \right\|$$

$$\frac{1}{2}\| 9i - 9j + 9k \|$$

$$\frac{\sqrt{3(81)}}{2} = \frac{9\sqrt{3}}{2}$$

45) A (1,2)
B (3,4)
C (-2,2)

Find Area.

$\overrightarrow{AB} = (2,2)$

$\overrightarrow{AC} = (-3,0)$

$$\begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ -3 & 0 & 0 \end{vmatrix} \quad +6k$$

Area of triangle = $\frac{1}{2}\| 6k \| = 3$

9/12/20 12.5 Homework 12 hrs Total

12.5 - 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

1) $n = \langle 1, 3, 2 \rangle, (4, -1, 1)$

$\langle 1, 3, 2 \rangle \cdot \langle 4, -1, 1 \rangle$

$4 \quad -3 \quad 2$

$d = 3$

$x + 3y + 2z = 3$

5) $n = i, \quad (3, 1, -9)$

$\langle 1, 0, 0 \rangle \cdot \langle 3, 1, -9 \rangle$

$3 = x$

9) $x = 0$

11) b and d are true.

13) $9x - 4y - 11z = 2$

$a = 9, \quad b = -4, \quad c = -11$

$n = \langle 9, -4, -11 \rangle$

15) $3(x-4) - 8(y-1) + 11z = 0$

$\langle 3, -8, 11 \rangle$

1
36
+16
52

17) Equation of plane given:

P(2, -1, 4) Q(1, 1, 1) R(3, 1, -2)

$\vec{PQ} = \langle -1, 2, -3 \rangle$

$\vec{PR} = \langle 1, 2, -6 \rangle$

$$\vec{PQ} \times \vec{PR} = \begin{pmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{pmatrix}$$

$det = i(-18) - j(3) + k(-4)$

$-18i \; -3j \; -4k$

$\langle -18, -3, -4 \rangle$

$d = \langle -18, -3, -4 \rangle \cdot \langle 2, -1, 4 \rangle$

$-36 + 3 - 16$

$-48 = d$

$-48 = -18(x) - 3y - 4z$

19) P(1, 0, 0) Q(0, 1, 1) R(2, 0, 1)

$\vec{QR} = \langle 1, 0, 1 \rangle$

$\vec{PQ} = \langle -1, 1, 1 \rangle$

$$\begin{pmatrix} i & j & k \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

$i + 2j + k$

$\langle 1, 2, 1 \rangle \cdot \langle 1, 0, 0 \rangle = 1$

$1, 0, 0$

$1 = x + 2y + z$

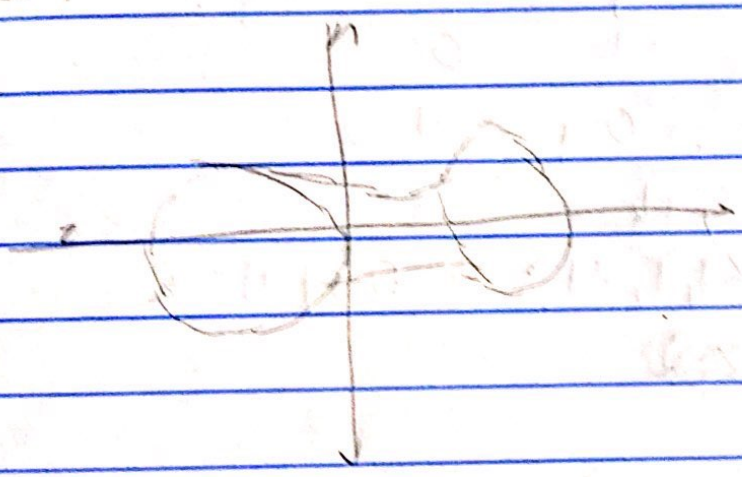
25) Passes through $(-2, -3, 5)$ normal $\langle 1, 0, 1 \rangle$

$\langle 1, 0, 1 \rangle \cdot (-2, -3, 5)$

$-2 + 5 = 3$

$3 = x + z$

31) $x + y + z = 4$



53) All planes in R^3 whose intersection with the xz plane is $3x + 2z = 5$

$ax + cz = d$

$(3\lambda)x + by + 2\lambda = 5\lambda, \quad \lambda \neq 0$

9/16/20 13.1
5, 17

5) $P = (3, -5, 7)$
$v = \langle 3, 0, 1 \rangle$
$(3, -5, 7) + t\langle 3, 0, 1 \rangle$
$\langle 3+3t, -5, 7+t \rangle$

17) $r(t) = (9\cos t)(i) + (9\sin t)(j)$.

$r(t) = 9(i\cos t + j\sin t)$
radius: 9
Center: (0, 0)
Plane: xy plane.

9/16/20

13.2

#3, 5, 7, 15, 31, 33, 41, 49.

3) $\lim_{t \to 0} \; e^{2t} \; i \; + \ln(t+1) \; j \; + 4k$

$= i + 4k$

5) $r(t) = \langle t^{-1}, \sin t, 4 \rangle$

$r'(t) = \langle -t^{-2}, \cos t, 0 \rangle$

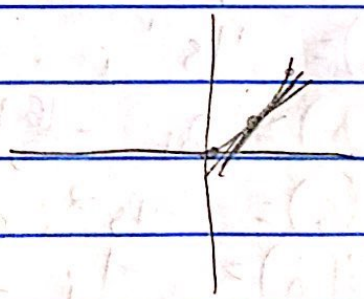
7) $r(t) = \langle t, t^2, t^3 \rangle$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

15) ~~$r_1(t) + r_2(t) =$~~

$r_1(t) = \langle t, t^2 \rangle$

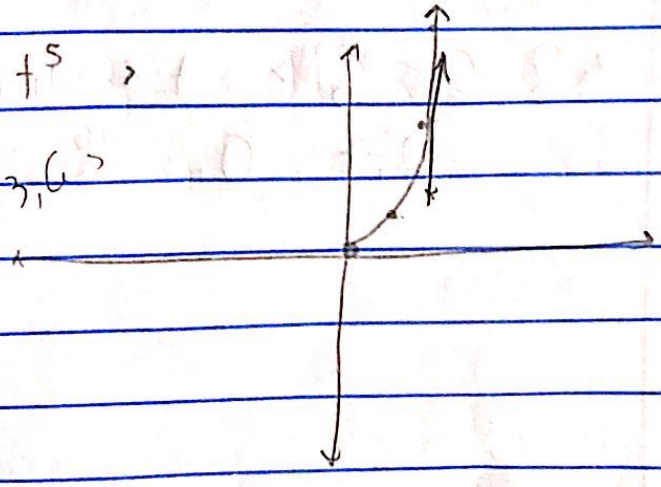
$r_1'(t) = \langle 1, 2t \rangle$



$r_2(t) = \langle t^3, t^6 \rangle$

$r_2'(t) = \langle 3t^2, 6t^5 \rangle$

$\langle 3, 6 \rangle$



31) $r(t) = \langle 1, -t^2, 5t, 2t^3 \rangle$, $t = 2$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

Tangent line $= r(t) + t(r'(t))$

$r(2) = \langle -3, 10, 16 \rangle$

$r'(2) = \langle -4, 5, 24 \rangle$

$r(t) + t(r'(t))$

$\langle -3, 10, 16 \rangle + t\langle -4, 5, 24 \rangle$

$\langle -3 - 4t, 10 + 5t, 16 + 24t \rangle$

33) $r(s) = 4s^{-1} i - \frac{8}{3} s^3 k$, $s = 2$

$r(2) = 2i - \frac{16}{3} k$

$r'(s) = -4s^{-2} i - 8s^2 k$

$r'(2) = -1 i - 32k$

$\langle 2, 0, -\frac{16}{3} \rangle + t\langle -1, 0, -32 \rangle$

$\langle 2 - t, 0, \frac{-16}{3} - \frac{32t}{3} \rangle$

41) $\int_{-2}^{2} (u^3 i + u^5 j)\,du$

$$\frac{u^4}{4} i + \frac{u^6}{6} j \Big|_{-2}^{2}$$

$$4i + \frac{32}{3} j$$

49) $r'(t) = t^2 i + 5t j + k, \quad r(1) = j + 2k$

$$\int r'(t)\,dt$$

$$\frac{t^3}{3} i + \frac{5t^2}{2} j + kt + C$$

$$\frac{1}{3} i + \frac{5}{2} j + k + C = j + 2k$$

$$C = -\frac{1}{3} i - \frac{3}{2} j + k$$

$$\frac{t^3}{3} - \frac{1}{3}, \quad \frac{5t^2}{2} - \frac{3}{2}, \quad t + 1 >$$