12.3 : 1, 13, 21, 29, 31, 57, 63

1) $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 15$

13) $\langle 1,1,1\rangle, \langle 1,-2,-2\rangle$ obtuse ; $\theta = 125.2644$

21) $\hat{i}+\hat{j},\ \hat{j}+2\hat{k} = \langle 1,1,0\rangle, \langle 0,1,2\rangle$ acute; $\theta = 71.5651$

29) $\langle b,3,2\rangle, \langle 1,b,1\rangle$; Make the vectors orthogonal

$\langle b,3,2\rangle \cdot \langle 1,b,1\rangle = 0$

$b+3b+2=0$

$4b=2$

$b=-2$

31) Find two vectors that are not multiples of each other, and are both orthogonal to $(2,0,-3)$

$\langle a,0,1\rangle \cdot \langle 2,0,-3\rangle = 0$

$2a+0-3=0$

$a=3/2$

$\langle 3/2, 0, 1\rangle$

$\langle a,2,5\rangle \cdot \langle 2,0,-3\rangle = 0$

$2a+0-15=0$

$a=7.5$

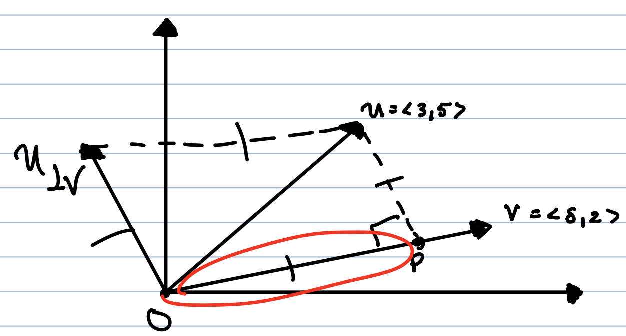
$\langle 7.5, 2, 5\rangle$

57) $\vec{u} = 5i+7j-4k$, $\vec{v} = k$

$$u_{\parallel v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|} \cdot e_v = \frac{\langle 5,7,-4\rangle \cdot \langle 0,0,1\rangle}{1} \cdot \langle 0,0,1\rangle$$

$$= -4 \cdot \langle 0,0,1\rangle = \langle 0,0,-4\rangle$$

63)



Find the length of $\overrightarrow{OP}$

$$\left[\frac{\vec{u}\cdot\vec{v}}{\|V\|}\right] e_v = \left[\frac{\langle 3,5\rangle \cdot \langle 8,2\rangle}{\sqrt{8^2+2^2}}\right] \left\langle \frac{8}{\sqrt{8^2+2^2}}, \frac{2}{\sqrt{8^2+2^2}} \right\rangle$$

$$= \frac{24+10}{\sqrt{68}} \left\langle \frac{8}{\sqrt{68}}, \frac{2}{\sqrt{68}} \right\rangle$$

$$\frac{34}{\sqrt{68}} \left\langle \frac{8}{\sqrt{68}}, \frac{2}{\sqrt{68}} \right\rangle$$

$$\left\langle \frac{272}{68}, \frac{68}{68} \right\rangle$$

$$\left\| \left\langle \frac{272}{68}, \frac{68}{66} \right\rangle \right\| = \sqrt{17}$$

12.4: 1, 5, 13, 21, 25, 27, 39, 41, 43, 45

1) $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$ det: $1(3) - 2(4) = -5$

5) $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix}$ det: $1\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix} =$

$1(-3) - 2(4) + 1(0 - -3)$

$-3 - 8 + 3 = -8$

13) $(\hat{i}+\hat{j}) \times \hat{k} = \hat{i}\times\hat{k} + \hat{j}\times\hat{k}$

$-j + i$

21) $u\times v = \langle 1,1,0\rangle$, $u\times w = \langle 0,3,1\rangle$, $v\times w = \langle 2,-1,1\rangle$

21) $(u-2v)\times(u+2v)$

$\cancel{(u\times u)} + (u\times 2v) + (-2v\times u) + (-2v\times 2v)$

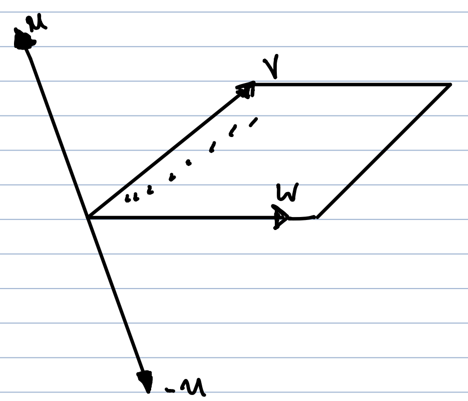
$+ 2(u\times v) + -2(v\times u) + \cancel{-2(v\times -v)}$

$\langle 2,2,0\rangle + -2\langle -1,-1,0\rangle$

$\langle 2,2,0\rangle + \langle 2,2,0\rangle$

$= \langle 4,4,0\rangle$

25) ?



$-u$

27) $\vec{v} = \langle 3,0,0\rangle$ and $\vec{w} = \langle 0,1,-1\rangle$

$\vec{u} = \vec{v}\times\vec{w} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0-0 \\ 0--3 \\ 3-0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$

$\|\vec{u}\| = \sqrt{3^2+3^2} = \sqrt{18}$ $\longrightarrow$ $\|\vec{v}\|\cdot\|\vec{w}\| \sin\frac{\pi}{2} = 1.5\sqrt{2}$

$3\cdot\sqrt{2}$

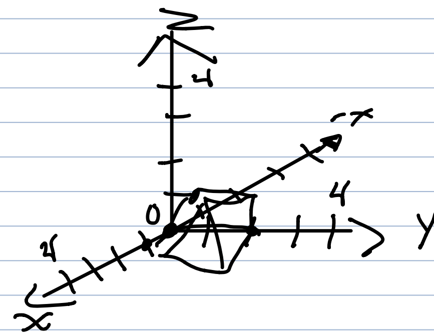
39) $\vec{u} = \langle 1,0,0\rangle$

$\vec{v} = \langle 0,2,0\rangle$ $\quad V = |\vec{u}\cdot(\vec{v}\times\vec{w})|$

$\vec{w} = \langle 1,1,2\rangle$

$\langle 1,0,0\rangle \cdot [\langle 0,2,0\rangle \times \langle 1,1,2\rangle]$

4



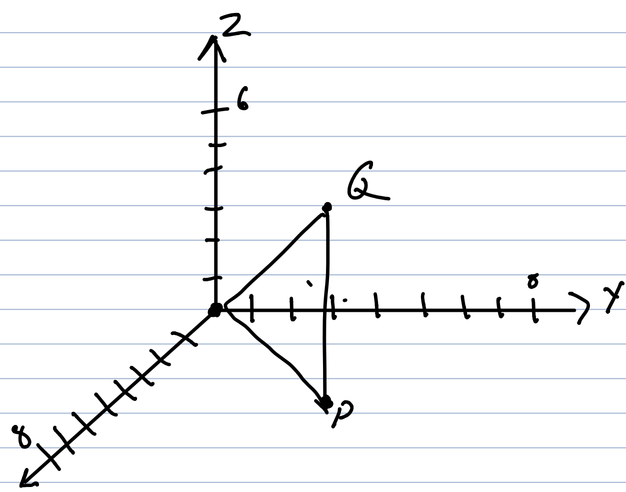
4.1) $\vec{u} = \langle 1,0,3\rangle$ $\vec{v} = \langle 2,1,1\rangle$

area(P) = $\|\vec{u} \times \vec{v}\|$ =>

$\vec{u} \times \vec{v} = \langle -3, 5, 1\rangle$

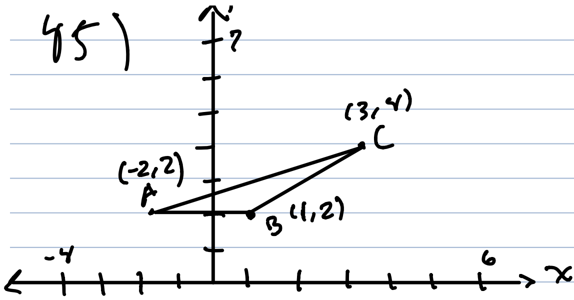
$\|\vec{u} \times \vec{v}\| = \sqrt{3^2+5^2+1^2} = \sqrt{35}$

4.3) Origin, $P = (3,3,0)$, $Q = (0,3,3)$



$\frac{\|\vec{P} \times \vec{Q}\|}{2}$ = area(T) ≈ 7.8

45)



$\overrightarrow{AC} = \langle 5,2\rangle = \vec{P}$

$\overrightarrow{AB} = \langle 3,0\rangle = \vec{Q}$

$$\frac{\|\vec{P} \times \vec{Q}\|}{2} = \text{area}(T) = 3$$

$$\begin{matrix}5\\2\\0\end{matrix} \times \begin{matrix}3\\0\\0\end{matrix} = \big(0 - 0$$

12.5 : 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

1) $\vec{n} = \langle 1, 3, 2 \rangle$, $P_0 = (4, -1, 1)$

$1(x-4) + 3(y--1) + 2(z-1) =$
$x - 4 + 3y + 3 + 2z - 2 =$
$x + 3y + 2z = 3$

5) $n = i = \langle 1, 0, 0 \rangle$, $P_0 = (3, 1, -9)$

$x - 3 = 0 \Rightarrow x = 3$

9) $3x + 2y + 5z = 0$

11) Which of the following statements are true of a plane that is parallel to the yz plane

a) $n = \langle 0, 0, 1 \rangle$ is a normal vector false
b) $n = \langle 1, 0, 0 \rangle$ is a normal vector true
c) The equation has the form $ay + bz = c$ false
d) The equation has the form $x = d$ true

13) $9x - 4y - 11z = 2$; $\hat{n} = \langle 9, -4, -11 \rangle$

15) $3(x-4) - 8(y-1) + 11z$; $\hat{n} = \langle 3, -8, 11 \rangle$

17) $P = (2, -1, 4)$, $Q = (1, 1, 1)$, $R = (3, 1, -2)$

$\overrightarrow{PQ} = \langle -1, 2, -3 \rangle$ $\overrightarrow{PR} = \langle 1, 2, -6 \rangle$

$$\hat{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} -6 \\ -9 \\ -4 \end{bmatrix}$$

$d = \hat{n} \cdot \overrightarrow{OP} = \langle -6, -9, -4 \rangle \cdot \langle 2, -1, 4 \rangle = -19$

$-6x - 9y - 4z = -19 \Rightarrow 6x + 9y + 4z = 19$

19) $P = (1, 0, 0)$ $Q = (0, 1, 1)$ $R = (2, 1, 1)$

$2y - 2z = 0 \Rightarrow y - z = 0$

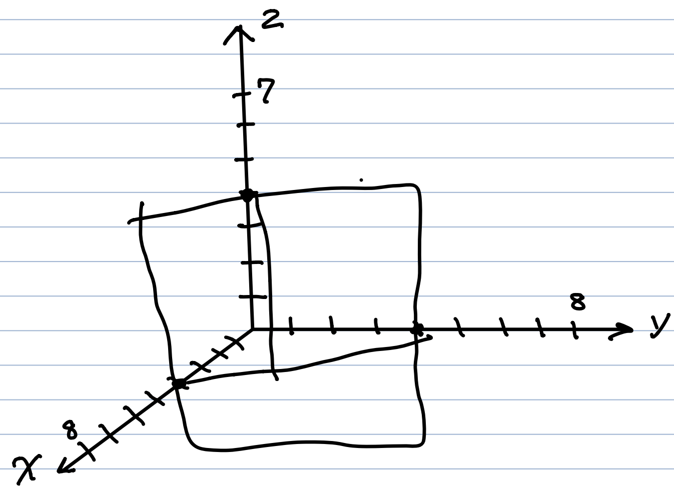
25) $P_0 = (-2, -3, 5)$ $\hat{n} = \langle 1, 0, 1 \rangle$

$1(x - -2) + 0(y - -3) + 1(z-5) \rightarrow x + z =$

$\overrightarrow{OP} = \langle -2, -3, 5 \rangle$
$d = \hat{n} \cdot \overrightarrow{OP} = \langle -2, -3, 5 \rangle \cdot \langle 1, 0, 1 \rangle = -2 + 5 = 3$

$1x + 0y + z = 3 \rightarrow \boxed{x + z = 3}$

31) $x+y+z=4$



53) Find all planes in $\mathbb{R}^3$ whose intersection with the x-z plane is the line with equation $3x+2z=5$

$(3\lambda)x+by+2\lambda z=5\lambda \; , \; \lambda \neq 0$

13.1 5 & 17

5) Find a vector parametrization of the line through $P = (3, -5, 7)$ the direction $\vec{v} = \langle 3, 0, 1 \rangle$

$\vec{r}(t) = P + t\vec{v}$

$= \langle 3, -5, 7 \rangle + \langle 3t, 0, t \rangle$

$= \langle 3+3t, -5, 7+t \rangle$

17) $r(t) = (9\cos t)i + (9\sin t)j$

radius : 9
center : origin
plane containing circle: xy plane

13.2 3, 5, 7, 15, 31, 33, 41, 49

3) $\lim_{t\to0} e^{2t}\hat{i} + \ln(t+1)\hat{j} + 4k = \left\langle \lim_{t\to0} e^{2t}, \lim_{t\to0} \ln(t+1), \lim_{t\to0} 4 \right\rangle = \langle 1, 0, 4\rangle = \hat{i} + 4\hat{k}$

5) $\lim_{h\to0} \frac{\vec{r}(t+h)-\vec{r}(t)}{h}$ for $\vec{r}(t) = \langle t^{-1}, \sin t, 4\rangle = \frac{d}{dt}\vec{r}(t) = \left\langle \frac{d}{dt} t^{-1}, \frac{d}{dt}\sin(t), \frac{d}{dt} 4 \right\rangle$

$\boxed{= \langle -t^{-2}, \cos(t), 0\rangle}$

or $\left\langle \lim_{h\to0} \frac{1/(t+h) - 1/t}{h}, \lim_{h\to0} \frac{\sin(t+h) - \sin(t)}{h}, \lim_{h\to0} \frac{4-4}{h} \right\rangle$

$\left\langle \lim_{h\to0} \frac{t-(t+h)}{ht(t+h)}, \lim_{h\to0} \frac{\sin(t)\cos(h) + \sin(h)\cos(t) - \sin(t)}{h}, \lim_{h\to0} \frac{0}{h} \right.$

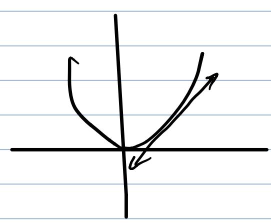
$\left\langle \lim_{h\to0} \frac{\cancel{t-t} - \cancel{h}}{\cancel{h}(t+h)}, \lim_{h\to0} \frac{\sin(t)\cos(h)}{h} + \lim_{h\to0} \frac{\sin(h)\cos(t)}{h} - \lim_{h\to0} \frac{\sin(t)}{h}, \lim_{h\to0} \frac{0}{h^0} = 0 \right.$

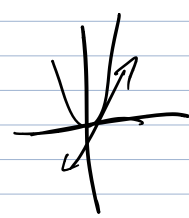
$\left\langle \lim_{h\to0} \frac{-1}{t^2+th} \right.$ $\cancel{\sin(t) \lim_{h\to0} \frac{\cos(h)}{h}} + \cos(t) \lim_{h\to0} \frac{\sin(h)}{h} - \cancel{\sin(t) \lim_{h\to0} \frac{1}{h}}$

$\langle -1/t^2, \cos(t), 0\rangle$

7) $r(t) = \langle t, t^2, t^3\rangle$ $r'(t) = \langle 1, 2t, 3t^2\rangle$ $r''(t) = \langle 0, 2, 6t\rangle$

15) $r_1(t) = \langle t, t^2\rangle$ $r_2(t) = \langle t^3, t^6\rangle$
$r_1'(t) = \langle 1, 2t\rangle$ $r_2'(t) = \langle 3t^2, 6t^5\rangle$





31) $r(t) = \langle 1-t^2, 5t, 2t^3\rangle$ $t = 2$
$r'(t) = \langle -2t, 5, 6t^2\rangle$

$r(2) = \langle -3, 10, 16\rangle$
$r'(2) = \langle -4, 5, 24\rangle$

$L(t) = r(2) + t\, r'(2)$
$= \langle -3, 10, 16\rangle + t\langle -4, 5, 24\rangle$
$= \langle -3, 10, 16\rangle + \langle -4t, 5t, 24t\rangle$
$= \langle -4t-3, 5t+10, 24t+16\rangle$

33) $r(s) = 4s^{-1} i - \frac{8}{3} s^{-3} k$, $s = 2$

$r(2) = 2i - \frac{1}{3}k$

$r'(s) = -4s^{-2} i + 8s^{-4} k$

$r'(2) = -i + \frac{1}{2}k$

$L(t) = r(2) + t\,r'(2)$

$= 2i - \frac{1}{3}k + t(-i + \frac{1}{2}k)$

$= 2i - \frac{1}{3}k + (-ti + t/2\,k)$

$= (2-t)i + (-1/3 + t/2)k$

41) $\int_{-2}^{2} (u^3 i + u^5 j)\,du = \left\langle \int_{-2}^{2} u^3 \hat{i}\,du, \int_{-2}^{2} u^5 \hat{j}\,du \right\rangle = \left\langle \frac{u^4}{4}\Big|_{-2}^{2}, \frac{u^6}{6}\Big|_{-2}^{2} \right\rangle = \langle 0, 0 \rangle$

49) $r'(t) = t^2 i + 5tj + k$, $r(1) = j + 2k$

$\int r'(t)\,dt = \left\langle \int t^2\,dt, \int 5t\,dt, \int 1\,dt \right\rangle = \left\langle \frac{t^3}{3} + C, \frac{5t^2}{2} + C, t + C \right\rangle = r(t)$

$r(1) = j + 2k$

$x(1) = \frac{1}{3} + C_1 = 0$ $\quad y(1) = \frac{5}{2} + C_2 = 1$ $\quad z(1) = 1 + C_3 = 2$

$C_1 = -1/3$ $\quad C_2 = -3/2$ $\quad C_3 = 1$

$r(t) = \left\langle \frac{t^3}{3} - \frac{1}{3}, \frac{5t^2}{2} - \frac{3}{2}, t+1 \right\rangle$